(a) Compute BE/A for O-16 and O-17 using the semi-empirical mass formula (use the form and coeffs from your text).

(b) Compare the "theoretical" result from Part a with the values computed using the measured masses (these were computed as part of HW#3).

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$$BE = \alpha A - \beta A^{2/3} - \gamma \frac{Z^2}{A^{1/3}} - \xi \frac{(A-2Z)^2}{A} - \frac{\delta}{\sqrt{A}}$$

where

$\alpha = 15.835$ MeV

$\beta = 18.33$ MeV

$\gamma = 0.714$ MeV

$\xi = 23.20$ MeV

$$\delta = \begin{cases} +11.2 \text{ MeV} & \text{odd } N, \text{ odd } Z \\ 0 & \text{even-odd pairing} \\ -11.2 \text{ MeV} & \text{even } N, \text{ even } Z \end{cases}$$

$\boxed{{}^{16}_{8}O}$

$$BE = 15.835(16) - 18.33(16)^{2/3} - \frac{0.714(8)^2}{(16)^{1/3}} - 23.20\frac{(0)^2}{16} + \frac{11.2}{(16)^{1/2}}$$

$$\begin{aligned} BE = \; & 253.36 \\ - \; & 116.39 \\ - \; & 18.13 \\ - \; & 0 \\ + \; & 2.80 \\ \hline & 121.64 \end{aligned}$$

$\therefore \frac{BE}{A} = \frac{121.64}{16} = \boxed{7.60 \text{ MeV}}$ per nucleon

→ from HW#3 $\boxed{BE/A = 7.98 \text{ MeV per nucleon}}$

~ 5% difference

Note: The semi-empirical mass formula under predicts BE/A by almost 5%. This is a little higher than expected, but one should also recall that O-16 is a double-magic isotope (both N and Z are magic numbers). Thus, the difference here is somewhat larger than expected because this is a somewhat special isotope

$^{17}_{8}O$ $BE = 15.835(17) - 18.33(17)^{2/3} - \frac{0.714\ (8)^2}{(17)^{1/3}}$

$- \frac{23.20\ (1)^2}{17} - 0$ ← odd, even pairing

$$
\begin{aligned}
BE = \ & 269.20 \\
& - 121.19 \\
& - 17.77 \\
& - 1.36 \\
& - 0 \\
\hline
& 128.88
\end{aligned}
$$

$\therefore \frac{BE}{A} = \frac{128.88}{17} = \boxed{7.58 \text{ meV}}$ per nucleon

→ from HW #3 $\boxed{\frac{BE}{A} = 7.75 \text{ meV}}$

~ 2% error

Data for Nuclides along Isobar A=149

For each nuclide along the isobar A=149 starting with Ba149 and ending with the first stable nuclide, identify the following data from the chart of the Nuclides:

nuclide, decay mode, half-life, $\sigma_{n,\gamma}$ thermal, U235 FP yield

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From the Chart of the Nuclides, we have the following information:

Nuclide	Decay Mode	T 1/2	Thermal $\sigma_{n,\gamma}$	U235 FP yield
$^{149}_{56}Ba$	β^-	0.344 s	—	1.0 e-5
$^{149}_{57}La$	β^-	1.05 s	—	8 e-4
$^{149}_{58}Ce$	β^-	5.3 s	—	0.0070
$^{149}_{59}Pr$	β^-	2.26 min	—	0.0030
$^{149}_{60}Nd$	β^-	1.728 hr	—	7 e-5
$^{149}_{61}Pm$	β^-	53.08 hr	1.400 e 3	3.9 e-8
$^{149}_{62}Sm$	stable	—	$\boxed{4.052 \text{ e } 4}$ *	1.7 e-12

→ This is a very large cross section and Sm149 can have an important contribution on the overall reactivity balance

we will study this data in the Reactor theory course

→ With Pm-149 having a 53 hr half life and Sm149 being stable, this FP chain represents a saturating FP — and its dynamics is indeed important

→ the effective yield of Pm149 is about 1% meaning that one atom of Sm149 will be form for each 100 fissions (not huge but definitely important)

Write out the formal reaction equation and compute the Q-value for the following two radioactive decay reactions. Assume that any neutrinos or anti-neutrinos that appear in the balance eqns have negligible rest mass. Also; be especially careful with the treatment of the number of electrons on each side of the reaction equation.

(a) β^- decay of Cs 137

$$\boxed{{}^{137}_{55}Cs \longrightarrow {}^{137}_{56}Ba^{+} + {}^{0}_{0}e^{-} + \bar{\nu}}$$

conceptual

$n \rightarrow p + e^- + \bar{\nu}$

$$Q = \left[M\left({}^{137}_{55}Cs\right) - M\left({}^{137}_{56}Ba\right)\right]c^2$$

here we assume that the free electron can be combined with the positive Ba ion to give neutral Ba 137

$$= (136.907084 - 136.905821)\,931.5$$

$$= (0.001263)(931.5)$$

$$= \boxed{1.176 \text{ MeV}}$$

ans

(b) EC in Ce 139

$${}^{139}_{58}Ce \longrightarrow {}^{139}_{57}La + \nu$$

conceptual

$p + e \rightarrow n + \nu$

$$Q = \left[M\left({}^{139}_{58}Ce\right) - M\left({}^{139}_{57}La\right)\right]c^2$$

here there are no spurious charged particles involved i.e. both the parent and daughter are neutral atoms

$$= (138.906647 - 138.906348)\,931.5$$

$$= (0.000299)(931.5)$$

$$= \boxed{0.279 \text{ MeV}}$$

ans

In class we developed an expression for the energy of the two particles emitted in a two-body radioactive decay scheme. In doing this we assumed that the energy and momentum of the daughter nucleus could be characterized by a classical treatment and that the lighter product needed, in general, a relativistic treatment. A key assumption in this analysis was that the kinetic energy of the reactants (parent nuclide) was negligible.

(a) Re-do this analysis for the case where both products can be treated as classical particles.

If the initial KE is small, then an energy balance says that

$$Q = E_1 + E_2 \quad (1) \qquad \text{where } E_1 = \tfrac{1}{2} m_1 v_1^2$$
$$E_2 = \tfrac{1}{2} m_2 v_2^2$$

Similarly for no initial momentum

note $\boxed{p^2 = 2mE}$

$$0 = \vec{p}_1 - \vec{p}_2$$

or $p_1 = p_2$ (magnitude only)

$$m_1 v_1 = m_2 v_2 \quad (2)$$

Squaring eqn (2)

$$m_1^2 v_1^2 = m_2^2 v_2^2$$

$$2m_1\left(\tfrac{1}{2} m_1 v_1^2\right) = 2m_2\left(\tfrac{1}{2} m_2 v_2^2\right)$$

$$\cancel{2}m_1 E_1 = \cancel{2}m_2 E_2 = \cancel{2}m_2 (Q - E_1)$$

$$m_1 E_1 + m_2 E_1 = m_2 Q$$

$$\boxed{E_1 = \left(\frac{m_2}{m_1 + m_2}\right) Q}$$

and

$$E_2 = Q - E_1 = \left(1 - \frac{m_2}{m_1 + m_2}\right) Q$$

$$\boxed{E_2 = \left(\frac{m_1}{m_1 + m_2}\right) Q}$$

(b) Is the result from Part a consistent with the relativistic treatment done in class? Explain

Yes – if we let $m_D = m_1$ and $m_2 = m_2$

Then

$$E_2 = \left(\frac{m_D}{m_2 + m_D}\right) Q \longrightarrow \left(\frac{m_1}{m_2 + m_1}\right) Q$$

which is the same as above

Thus, we see that the more detailed relativistic treatment was really not necessary, because we got the same result.

However, we would not know this unless we did the analysis both ways.

Thus, in general, when the energy and momentum of the reactants are negligible, then the two body problem gives the energies of the products as

$$\boxed{E_1 = \left(\frac{m_2}{m_1 + m_2}\right) Q} \quad \text{and} \quad \boxed{E_2 = \left(\frac{m_1}{m_1 + m_2}\right) Q}$$

These are very simple and very useful results and they apply to any two particles (even if the rest mass of one of the particles is zero)

This result is subject to this constraint...

Pu238 is an alpha emitter that decays to several different energy levels of the daughter. However, the most common decay is to the ground state of the daughter (this occurs nearly 71% of the time). For this particular decay mode, determine the following.

a. the daughter nucleus

$$^{238}_{94}Pu \longrightarrow \, ^{234}_{92}U^{2-} + \, ^{4}_{2}He^{2+}$$

∴ U234 is the daughter product

b. The Q value for decay to the ground state of the daughter.

$$Q = \left\{ M\left(^{238}_{94}Pu\right) - \left[M\left(^{234}_{92}U\right) + M\left(^{4}_{2}He\right) \right] \right\} c^2$$

$$= \left[238.049553 - (234.040946 + 4.002603) \right] 931.5$$

$$\boxed{Q = 5.593 \text{ meV}}$$

c. The kinetic energy of the alpha particle

$$E_\alpha = \left(\frac{m_D}{m_\alpha + m_D} \right) Q = \left(\frac{234.0}{4.0 + 234.0} \right) (5.593 \text{ meV})$$

$$\boxed{E_\alpha = 5.499 \text{ meV}}$$

d. The kinetic energy of the daughter nucleus

$$E_{U234} = Q - E_\alpha = 5.594 - 5.499$$

$$\boxed{E_{U234} = 0.095 \text{ meV}}$$