Lamarsh 3rd Ed

Mass-Energy Relationships

(2.18) Electron at rest is accelerated across a potential of 5×10^6 volts

(a) what is its final kinetic energy?

$$\boxed{E = 5 \text{ MeV}}$$ ans

by definition: 1 eV is the increase in kinetic energy of an electron when it is accelerated by a potential of 1 V

(b) Total energy?

$$E_T = E + E_{rest}$$

and $E_{rest} = 0.511$ MeV for electron (Ex. 2.3 p. 12)

this is 8.186×10^{-14} J

$$\boxed{E_T = 5.511 \text{ MeV}}$$

(c) what is final mass?

$$E_T = mc^2$$

$$\therefore m = \frac{E_T}{c^2} = \frac{(5.511 \text{ MeV})(1.602 \times 10^{-13} \text{ J/MeV})\left(\frac{10^7 \text{ erg}}{\text{J}}\right)\left(\frac{\text{g-cm}^2/\text{s}^2}{\text{erg}}\right)}{(2.9979 \times 10^{10} \text{ cm/s})^2}$$

$$\boxed{m = 9.825 \times 10^{-27} \text{ g}} \times \frac{1 \text{ amu}}{1.66054 \times 10^{-24} \text{ g}} = \boxed{5.917 \times 10^{-3} \text{ amu}}$$ ans

note that m_0 (rest mass) $= 9.10956 \times 10^{-28}$ g

$$\therefore \boxed{\frac{m}{m_0} = 10.78}$$

mass of electron relative to an observer has increased by more than a factor of 10

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alternate formula

see Notes

$$\boxed{\frac{v}{c} = \sqrt{1 - \frac{E^2_{rest}}{E^2_{Total}}}} = \sqrt{1 - \left(\frac{.511}{5.511}\right)^2}$$

or $\frac{v}{c} = .9957$

and $$\boxed{m = \frac{m_0}{\sqrt{1 - (v/c)^2}}}$$

or $$\frac{m}{m_0} = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{1}{\sqrt{1-(.9957)^2}} = \boxed{10.78}$$

OK

Compute the speed and momentum of the following particles.

a. 1 MeV neutron

classical approach $E = \frac{1}{2} m v^2$ $p = mv$

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(1\,\text{meV})\left(\frac{1.602\times10^{-13}\,J}{\text{meV}}\right)\left(\frac{kg\text{-}m^2/s^2}{J}\right)}{1.674929\times10^{-27}\,kg}}$$

$$= \sqrt{1.9129\times10^{14}\,\frac{m^2}{s^2}}$$

since $c = 2.997925\times10^{8}\,m/s$

$$= \boxed{1.383\times10^{7}\,m/s} \Rightarrow \boxed{\frac{v}{c} = 0.0461}$$

< 5% the speed of light

$m = m_0$

$$p = m_0 v = (1.6749\times10^{-27}\,kg)(1.383\times10^{7}\,m/s)$$
$$= \boxed{2.316\times10^{-20}\,\frac{kg\,m}{s}} = \boxed{2.316\times10^{-20}\,N\text{-}s}$$

relativistic approach $E = E_t - E_{rest} = E_{rest}\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)$

$$\frac{v}{c} = \sqrt{1 - \frac{E_{rest}^2}{E_t^2}}$$

$E_{rest} = 939.5656$ MeV

$E_t = E + E_{rest} = 940.5656$ MeV

$$\frac{v}{c} = \sqrt{1 - \frac{939.5656^2}{940.5656^2}}$$

$$\frac{v}{c} = \sqrt{1 - 0.99787} = \boxed{0.0461}$$

Same as above

∴ classical formula is okay

b. 1 MeV electron

here we will definitely need the relativistic approach

note $m_0 = 9.109\times10^{-31}$ kg

$$\frac{v}{c} = \sqrt{1 - \frac{E_{res}^2}{E_t^2}}$$

$E_{rest} = 0.511$ MeV

$E_t = 1.511$ MeV

$$= \sqrt{1 - \frac{0.511^2}{1.511^2}}$$

$$= \sqrt{1 - 0.1144} = \boxed{0.941}$$

$$\boxed{V = 0.941c}$$

~94% the speed of light

$$p = mv = \frac{1}{c}\sqrt{E^2 + 2E\,E_{rest}}$$

$$= \frac{1}{c}\sqrt{1 + 2(1)(0.511)} = \frac{1.422\,\text{MeV}}{2.998\times10^{8}\,m/s} \times \frac{1.602\times10^{-13}\,N\text{-}m}{\text{MeV}}$$

$$= \boxed{7.599\times10^{-22}\,N\text{-}s}$$

c. 1 MeV gamma ray

$\frac{v}{c} = 1$ or $\boxed{v = c}$ electromagnetic radiation travels at the speed of light

$p = \frac{E}{c} = \frac{1\ MeV}{2.998 \times 10^{8}\ m/s} \times \frac{1.602 \times 10^{-13}\ N\text{-}m}{MeV}$

$\boxed{p = 5.344 \times 10^{-22}\ N\text{-}s}$

Summary for 1 MeV particles

	v/c	p (N-s)
neutron	0.0461	2.32×10^{-20}
electron	0.941	7.60×10^{-22}
photon	1.000	5.34×10^{-22}

We often refer to gamma radiation as a gamma ray particle (as in the previous problem)

Compute the deBroglie wavelength for a 1 MeV gamma ray and discuss whether it behaves more like a particle or an electromagnetic wave. Explain/justify your choice.

$$\lambda = \frac{h}{p}$$ where h = Planck's constant $= 6.626 \times 10^{-34}$ J-s

Thus for a 1 MeV gamma ray, we computed the momentum as $p = 5.344 \times 10^{-22}$ N-s

recall $1 J = 1 N\text{-}m$

$$\therefore \lambda = \frac{6.626 \times 10^{-34} \text{ N-m-s}}{5.344 \times 10^{-22} \text{ N-s}}$$

or $\boxed{\lambda = 1.240 \times 10^{-12} \text{ m}}$

For particle-like behavior, we argued that the wavelength must be much less than the characteristic dimension of an atom, with $R \simeq 2 \times 10^{-10}$ m

$$\therefore \frac{R}{\lambda} = \frac{2 \times 10^{-10}}{1.24 \times 10^{-12}} \simeq 161$$

The atomic radius is about 160 Times the wavelength of a 1 MeV gamma

$\therefore$ particle-like interactions are observed

ans

Prob 4.11

F-18 is a β^+ emitter and it is one of the primary isotopes used for generating positron emission Tomography (PET) images for medical diagnostics.

F-18 is usually produced in a cyclotron by bombarding O-18 with high energy protons. For production of F-18 via the $^{18}O(p,n)^{18}F$ reaction, determine the minimum allowable energy of the incident protons.

$$^{18}_{8}O + ^{1}_{1}p \longrightarrow ^{18}_{9}F^{+} + ^{1}_{0}n$$

Note That F-18 on The product side of The reaction eqn has been written with a +1 charge since it has 9 protons but only 8 electrons (since there were only 8 electrons available in the O-18 target).

However, the F-18 ion quickly captures an electron from The surrounding to neutralize itself.

In writing the mass-energy balance eqns, we can conceptually add one electron mass to each side of The equation to produce neutral H-1 on the left side and neutral F-18 on The right side — giving 9 electrons for both the products and reactants.

Thus, for computing the Q value, we have

$$Q\text{value} = \left\{\left[m(^{18}_{8}O) + m(^{1}_{1}H)\right] - \left[m(^{18}_{9}F) + m_n\right]\right\} c^2$$

$$= \left\{(17.999160 + 1.007825) - (18.000938 + 1.008665)\right\} * 931.5$$

$$= (19.006985 - 19.009603) * 931.5$$

$$= -(0.002618)(931.5)$$

$$= \boxed{-2.439 \text{ meV}} \quad \text{endothermic (i.e. threshold reaction)}$$

Thus, the incident proton energy must be greater than ~2.5 meV (usually much greater ...)

Neutron bombardment of Al27 can lead to several different nuclear reactions. In particular, for the following specific reactions of the form a(b, c)d, determine the reaction product d and the Q value for the reaction.

(a) $^{27}_{13}Al(n,\gamma)$?

$$^{27}_{13}Al + n \longrightarrow \boxed{^{28}_{13}Al} + \gamma$$

$$Q = \left\{\left[m(^{27}_{13}Al) + m_n\right] - \left[m(^{28}_{13}Al)\right]\right\}c^2$$

$$= \left\{\left[26.981538 + 1.008665\right] - \left[27.981910\right]\right\}931.5$$

$$= (0.008293)(931.5) = \boxed{7.72 \text{ MeV}}$$ exothermic

mass of reactants $^{27}_{13}Al + n$ = 27.990203 amu for all cases

(b) $^{27}_{13}Al(n,p)$?

$$^{27}_{13}Al + n \longrightarrow \boxed{^{27}_{12}Mg^-} + {}^1_1H^+$$

$$Q = \left\{\left[m(^{27}_{13}Al) + m_n\right] - \left[m(^{27}_{12}Mg) + m(^1_1H)\right]\right\}c^2$$

$$= \left\{27.990203 - \left[26.984341 + 1.007825\right]\right\}931.5$$

$$= (-0.001963)\,931.5$$

$$= \boxed{-1.83 \text{ MeV}}$$ endothermic

threshold reaction ⇒ ∴ need greater than 1.83 MeV of energy to initiate the reaction

(c) $^{27}_{13}Al(n,d)$?

$$^{27}_{13}Al + n \longrightarrow \boxed{^{26}_{12}Mg^-} + {}^2_1H^+$$

$$Q = \left\{\left[m(^{27}_{13}Al) + m_n\right] - \left[m(^{26}_{12}Mg) + m(^2_1H)\right]\right\}c^2$$

$$= \left\{27.990203 - \left[25.982593 + 2.014102\right]\right\}931.5$$

$$= (-0.006492)(931.5) = \boxed{-6.05 \text{ MeV}}$$ endothermic

(d) $^{27}_{13}Al\,(n, t)$?

$$^{27}_{13}Al + n \longrightarrow \boxed{^{25}_{12}Mg^{-}} + \, ^{3}_{1}H^{+}$$

$$Q = \left\{ \left[m(^{27}_{13}Al) + m_n \right] - \left[m(^{25}_{12}Mg) + m(^{3}_{1}H) \right] \right\} c^2$$

$$= \left\{ 27.990203 - \left[24.985837 - 3.016049 \right] \right\} 931.5$$

$$= (-0.011683)(931.5) = \boxed{-10.88 \text{ MeV}}$$

endothermic

(e) $^{27}_{13}Al\,(n, \alpha)$?

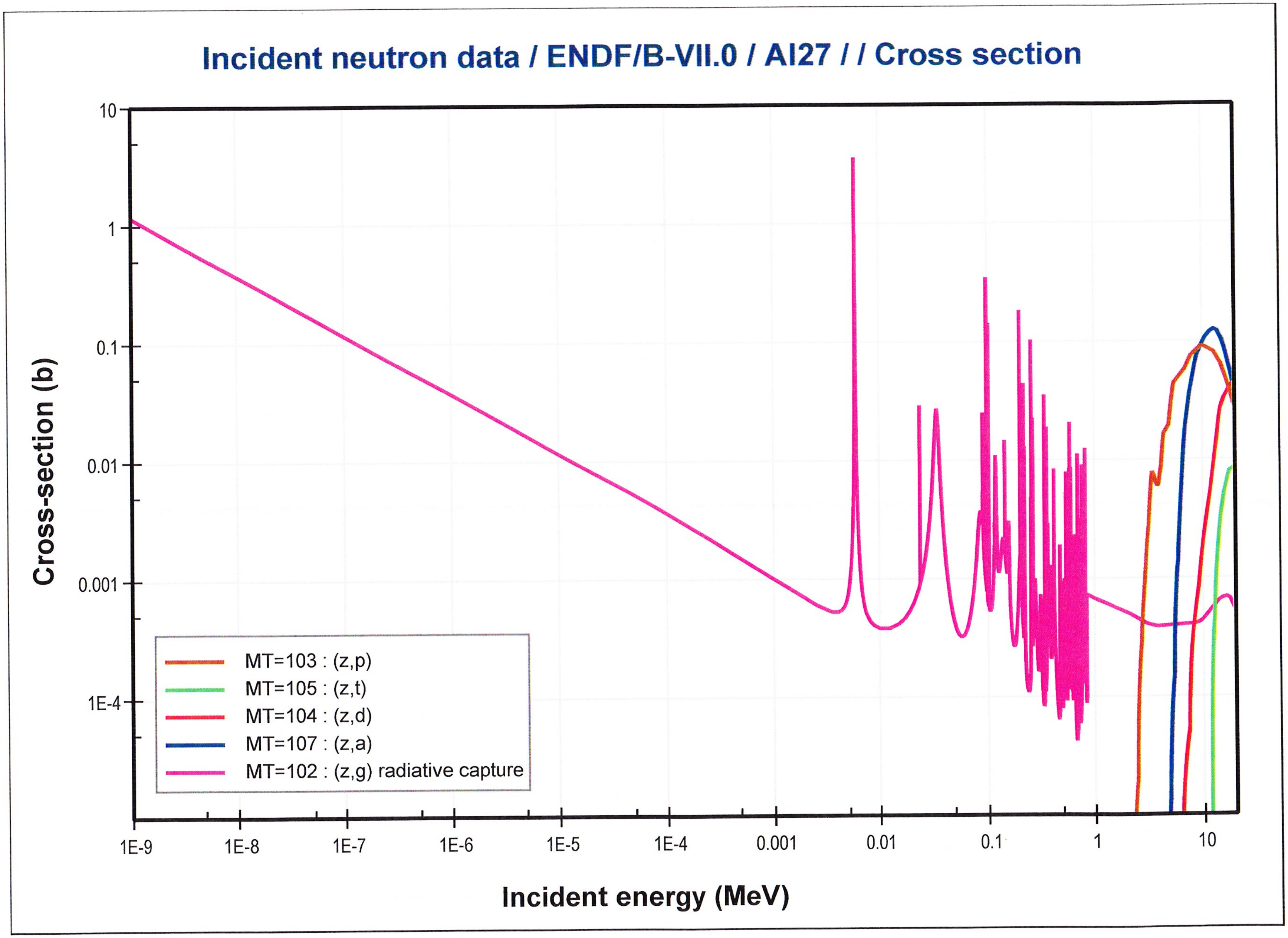
$$^{27}_{13}Al + n \longrightarrow \boxed{^{24}_{11}Na^{2-}} + \, ^{4}_{2}He^{2+}$$

$$Q = \left\{ \left[m(^{27}_{13}Al) + m_n \right] - \left[m(^{24}_{11}Na) + m(^{4}_{2}He) \right] \right\} c^2$$

$$= \left\{ 27.990203 - \left[23.990963 + 4.002603 \right] \right\} 931.5$$

$$= (-0.003363)(931.5) = \boxed{-3.13 \text{ MeV}}$$

endothermic


Incident neutron data / ENDF/B-VII.0 / Al27 / / Cross section
Cross-section (b)
10
1
0.1
0.01
0.001
1E-4
1E-9
1E-8
1E-7
1E-6
1E-5
1E-4
0.001
0.01
0.1
1
10
Incident energy (MeV)
MT=103 : (z,p)
MT=105 : (z,t)
MT=104 : (z,d)
MT=107 : (z,a)
MT=102 : (z,g) radiative capture

Prob 4.3

a. Determine the binding energy per nucleon for O-16 and O-17.

$\boxed{\text{O-16}}$

$$BE/A = \left[8\,m({}^1_1H) + 8\,m_n - m({}^{16}_{8}O)\right] c^2 / 16$$

$$= \left[8(1.007825) + 8(1.008665) - 15.994915\right] \frac{931.5}{16}$$

$$= (16.131920 - 15.994915)\frac{931.5}{16}$$

$$= \boxed{7.976 \text{ MeV per nucleon}}$$

$\boxed{\text{O-17}}$

$$BE/A = \left[8\,m({}^1_1H) + 9\,m_n - m({}^{17}_{8}O)\right] c^2 / 17$$

$$= \left[8(1.007825) + 9(1.008665) - 16.999132\right] \frac{931.5}{17}$$

$$= (17.140585 - 16.999132)\frac{931.5}{17}$$

$$= \boxed{7.751 \text{ MeV per nucleon}}$$

BE/A is higher for O-16 than O-17. This implies that O-16 is a more stable isotope. In particular, O-16 has an even number of proton and an even number of neutrons (even Z, even N) but O-17 has an odd neutron number.

In general, the even-even pairing leads to greater stability...

b. Determine the neutron separation energy for O-16 and O-17.

$\boxed{\text{O-16}}$ $\quad {}^{15}_{8}O + n \rightarrow {}^{16}_{8}O$

$$E_s = \text{Qvalue} = \left\{\left[m({}^{15}_{8}O) + m_n\right] - m({}^{16}_{8}O)\right\} c^2$$

$$= \left[(15.003065 + 1.008665) - 15.994915\right] 931.5$$

$$= (16.011730 - 15.994915)(931.5)$$

$$= \boxed{15.66 \text{ MeV}}$$ ← This is the amount of energy that would be needed to remove a neutron from O-16

$\boxed{\text{O-17}}$ $\quad {}^{16}_{8}O + n \rightarrow {}^{17}_{8}O$

$$E_s = \text{Qvalue} = \left\{\left[m({}^{16}_{8}O) + m_n\right] - m({}^{17}_{8}O)\right\} c^2$$

$$= \left[(15.994915 + 1.008665) - 16.999132\right] 931.5$$

$$= (17.003580 - 16.999132)\,931.5 = \boxed{4.143 \text{ MeV}}$$

Again, $E_{s\,O\text{-}16} > E_{s\,O\text{-}17}$ since O-16 is a more stable isotope...