Shultis + Faw
Fund NSE
2nd Ed

Basic Notation + Atom Densities

(1.8) Find # neutrons and protons in following nuclides

ⓐ $^{10}B \rightarrow {}^{10}_{5}B$ 5 protons, 5 neutrons

ⓑ $^{24}Na \rightarrow {}^{24}_{11}Na$ 11 protons, 13 neutrons

ⓒ $^{59}Co \rightarrow {}^{59}_{27}Co$ 27 protons, 32 neutrons

ⓓ $^{208}Pb \rightarrow {}^{208}_{82}Pb$ 82 protons, 126 neutrons

ⓔ $^{235}U \rightarrow {}^{235}_{92}U$ 92 protons, 143 neutrons

(1.13) How many atoms of ^{234}U are there in 1 kg of natural U?

Table A.3 $U_{nat} \rightarrow M = 238.0289$ $\rho = 18.95\ g/cm^3$

Table A.4 $\gamma_{U234} = 0.0055$ a/o ← atom %

$$N_{U_{nat}} = 1000\,g\,U \times \frac{6.022\times10^{23}\ at\ U/gmole}{238.03\ g\,U/gmol}$$

$$= 2.530 \times 10^{24}\ at\ U/kg\,U$$

now

$$N_{U234} = 5.5\times10^{-5}\ \frac{at\ U234}{at\ U} \times 2.530\times10^{24}\ \frac{at\ U}{kg\,U}$$

$$= \boxed{1.391\times10^{20}\ \frac{at\ U234}{kg\ U_{nat}}}$$

(1.20) A concrete with a density of 2.35 g/cm² has a hydrogen of 0.0085 weight fraction. What is the atom density of hydrogen in the concrete.

$$N_H = \frac{2.35\ g\ concrete}{cm^3} \times \frac{0.0085\ g\,H}{g\ concrete} \times \frac{0.6022\times10^{24}\ at\ of\ H}{1.008\ g\ H}$$

$$\boxed{N_H = 1.193\times10^{22}\ \frac{at\ H}{cm^3} = 1.193\times10^{-2}\ \frac{at\ H}{b\text{-}cm}}$$

Lamarsh
3rd Ed

Atom Density
of UC Fuel

$\boxed{2.61}$ Given: UC fuel with 25 w/o enrichment
and a UC density of 13.6 g/cm^3

Calc M_u $\qquad \frac{1}{M_u} = \frac{0.25}{235} + \frac{0.75}{238} = 4.215\times10^{-3}$

$$\therefore \boxed{M_u = 237.2 \text{ g/gmole}}$$

Calc M_{uc} $\qquad M_{uc} = M_u + M_c = 237.2 + 12 = \boxed{249.2 \text{ g/gmole}}$

Calc N_i

N_u $\qquad 13.6 \frac{\text{g of UC}}{\text{cm}^3} * \frac{\cancel{237.2} \text{ g of U}}{249.2 \text{ g of UC}} * \frac{0.6022 \text{ at of U cm}^2/\text{b}}{\cancel{237.} \text{ g of U}} = \boxed{3.286\times10^{-2} \frac{\text{at U}}{\text{b-cm}}}$

N_{U235} $\qquad (13.6)\left(\frac{237.2}{249.2}\right)\left(\frac{.25}{1}\right)\left(\frac{.6022}{235}\right) = \boxed{8.293\times10^{-3} \frac{\text{at of U235}}{\text{b-cm}}}$

N_{U238} $\qquad (13.6)\left(\frac{237.2}{249.2}\right)\left(\frac{.75}{1}\right)\left(\frac{.6022}{238}\right) = \boxed{2.457\times10^{-2} \frac{\text{at of U238}}{\text{b-cm}}}$

check $\qquad N_{U235} + N_{U238} = 8.293\times10^{-3} + 2.457\times10^{-2}$

$$= \boxed{3.286\times10^{-2} \frac{\text{at of U}}{\text{b-cm}}}$$

ok

N_C $\qquad (13.6)\left(\frac{12}{249.2}\right)\left(\frac{.6022}{12}\right) = \boxed{3.286\times10^{-2} \frac{\text{at of C}}{\text{b-cm}}}$

1:1 ratio
ok

mass fractions

$w_u = \frac{237.2}{249.2} = 0.9518 \qquad \therefore w_c = 4.82$ w/o

$w_{U235} = (0.25)(0.9518) \qquad w_{U235} = 23.80$ w/o

and $\quad w_{238} = 71.38$ w/o

Weight % of components of UC

Lamarsh
3rd Ed

Atom Densities
of MOX fuel

$\boxed{2.63}$ given: mixed oxide fuel ($PuO_2 + UO_2$) with 30 w/o PuO_2

Pu isotopics ⇒ Pu239 - 70.5 w/o
Pu240 - 21.3 w/o
Pu241 - 5.5 w/o
Pu242 - 2.7 w/o
uranium is all U238

Calculate the # of atoms of each isotope per gram of fuel

$$M_{U238} = \boxed{238.05} \qquad M_{O_2} = 2(15.999) = \boxed{32.00}$$

$$\therefore M_{UO_2} = 238.05 + 32.00 = \boxed{270.05} \text{ g/gmole}$$

$$M_{Pu} \rightarrow \frac{1}{M_{Pu}} = \sum \frac{w_i}{m_i}$$

$$\therefore \frac{1}{M_{Pu}} = \frac{.705}{239.05} + \frac{.213}{240.05} + \frac{.055}{241.13} + \frac{.027}{242.06}$$

$$= .004176 \longrightarrow M_{Pu} = \boxed{239.46} \text{ g/gmole}$$

$$M_{PuO_2} = \boxed{271.46} \text{ g/gmole}$$

Pu

$$\text{now } \frac{0.3 \text{ g of } PuO_2}{\text{g of fuel}} * \frac{239.46 \text{ g of Pu}}{271.46 \text{ g of } PuO_2} = \boxed{\frac{.2646 \text{ g of Pu}}{\text{g of fuel}}}$$

$$\text{Pu239} \quad \frac{.2646 \text{ g of Pu}}{\text{g of fuel}} * \frac{.705 \text{ g of Pu239}}{\text{g of Pu}} * \frac{.6023 \times 10^{24} \text{ at of Pu239}}{239.05 \text{ g of Pu239}}$$

$$\Rightarrow \boxed{4.700 \times 10^{20} \text{ at of Pu239/g of fuel}}$$

ans

$$\text{Pu240} \quad (.2646)(.213)\left(\frac{.6023 \times 10^{24}}{240.05}\right) \Rightarrow \boxed{1.414 \times 10^{20} \text{ at of Pu240/g of fuel}}$$

$$\text{Pu241} \quad (.2646)(.055)\left(\frac{.6023 \times 10^{24}}{241.13}\right) \Rightarrow \boxed{3.635 \times 10^{19} \text{ at of Pu241/g of fuel}}$$

$$\text{Pu242} \quad (.2646)(.027)\left(\frac{.6023 \times 10^{24}}{242.06}\right) \Rightarrow \boxed{1,778 \times 10^{19} \text{ at of Pu242/g of fuel}}$$

U238

$$\text{now } \boxed{\frac{0.7 \text{ g of } UO_2}{\text{g of fuel}} \times \frac{238.05 \text{ g of U238}}{270.03 \text{ g of } UO_2}} \times \frac{.6023 \times 10^{24} \text{ at of U238}}{238.05 \text{ g of U238}} = \boxed{1,561 \times 10^{21} \frac{\text{at of U238}}{\text{g of fuel}}}$$

$$\Rightarrow \boxed{\frac{.6171 \text{ g of U238}}{\text{g of fuel}}}$$

Oxygen

$$(1 - .2646 - .6171)\frac{\text{g of O}}{\text{g of fuel}} * \frac{.6023 \times 10^{24} \text{ at of O}}{15.999 \text{ g of O}} = \boxed{4.453 \times 10^{21} \frac{\text{at of Oxygen}}{\text{g of fuel}}}$$

WPI Fuel Element Densities

To start, we first need to compute the densities in the individual fuel meat, structure, and water region.

Region Densities

Structure

$$N_{Al} = \frac{2.7\,g\,Al}{cm^3} * \frac{0.6022\times10^{24}\,at\,Al/gmole}{27\,g/gmole} * \frac{10^{-24}\,cm^2}{b}$$

$$= 6.022\times10^{-2}\,at/b\text{-}cm$$

Water

$$N_{H_2O} = \frac{1\,g\,H_2O}{cm^3} * \frac{0.6022\times10^{24}\,at\,H_2O/gmole}{18\,g/gmole} * 10^{-24}\,\frac{cm^2}{b}$$

$$= 3.346\times10^{-2}\,at/b\text{-}cm$$

$$\therefore\quad N_H = 6.692\times10^{-2}\,\frac{at}{b\text{-}cm} \qquad N_O = 3.346\times10^{-2}\,\frac{at}{b\text{-}cm}$$

Fuel meat This one is a lot more work. Let's start with what we know from the given design data for the WPI fuel plate.

$$\rho_{U235} = \frac{9.28\,g\,U235}{plate} \times \frac{1\,plate}{27.677\,cm^3} = \boxed{0.3353\,g^{U235}/cm^3 \text{ of fuel meat}}$$

$$\text{meat vol} = (0.0762)(6.085)(59.69) = 27.677\,cm^3$$

$$N_{U235} = \frac{0.3353\,g}{cm^3} \times \frac{0.6022}{235.04}\,\frac{at\,cm^2/b}{g} = \boxed{8.591\times10^{-4}\,\frac{at}{b\text{-}cm}}$$

$$\rho_U = \frac{0.3353\,g^{U235}}{cm^3} \times \frac{1\,g\,U}{0.1975\,g\,U235} = \boxed{1.698\,\frac{g\,U}{cm^3} \text{ of fuel meat}}$$

$$N_U = \frac{1.698\,g}{cm^3} * \frac{0.6022}{237.45} = 4.3063\times10^{-3}\,at/b\text{-}cm$$

$$\frac{1}{M_U} = \frac{0.1975}{235.04} + \frac{0.8025}{238.05} = 0.004211$$

$$\Rightarrow M_U = 237.45\,\frac{g}{gmole}$$

$$N_{U238} = N_U - N_{U235} = 4.3063\times10^{-3} - 8.591\times10^{-4}$$

$$= \boxed{3.447\times10^{-3}\,at/b\text{-}cm}$$

Now to find the Al density, we note that the U fraction in the UAl_x-Al meat is 45 w/o.

Thus,

$$N_{Al} = 1.698 \frac{g\,U}{cm^3} \times \frac{0.55\,g\,Al/g\,meat}{0.45\,g\,U/g\,meat} \times \frac{0.6022\ at\ cm^2/b}{26.982\ g\ at\ Al}$$

$$= \boxed{4.632 \times 10^{-2}\ at/b\text{-}cm}$$

To summarize, we have

$\boxed{\text{Region Densities}}$

	fuel meat	structure	water
U235	8.591×10^{-4}	—	—
U238	3.447×10^{-3}	—	—
Al	4.632×10^{-2}	6.022×10^{-2}	—
H	—	—	6.692×10^{-2}
O	—	—	3.346×10^{-2}

all densities in at/b-cm

Now to compute the region vol fractions (really area fractions at mid plane of fuel element).

$\boxed{\text{Volume Fractions}}$

assy area = $(7.7724)(7.7724) = 60.4102\ cm^2$

fuel meat area = $(0.0762)(6.085)(18) = 8.3462\ cm^2$

structure area

side plate = $(0.4572)(7.620)(2) = 6.9677\ cm^2$

plate - fuel meat = $(0.1524)\left[\underbrace{7.62 - 2(0.4572)}_{\text{inside side plates} \leftarrow 6.7056\ cm}\right](18) - 8.3462\ cm^2$

$= 18.3948 - 8.3462 = 10.0486\ cm^2$

total structure = $6.9677 + 10.0486$

$= 17.0163\ cm^2$

water area = total area − structure − fuel meat

$= 60.4102 - 17.0163 - 8.3462$

$= 35.0477 \text{ cm}^2$

Vol Fractions

To summarize, we have

	area (cm^2)	vol. frac
fuel meat	8.3462	0.1382
structure	17.0163	0.2817
water	35.0477	0.5801
total	60.4102	1.0

Now for the final assy averaged densities

$$N_i\Big|_{ass} = \sum_j \underset{\substack{\uparrow \\ \text{density for isotope } i \\ \text{in region } j}}{N_{ij}} f_j \longleftarrow \text{vol frac for region } j$$

$N_{U235} = (8.591\times10^{-4})(0.1382) = 1.187\times10^{-4}$

$N_{U238} = (3.447\times10^{-3})(0.1382) = 4.764\times10^{-4}$

$N_{Al} = (4.632\times10^{-2})(0.1382) + (6.022\times10^{-2})(0.2817) = 2.337\times10^{-2}$

$N_H = (6.692\times10^{-2})(0.5801) = 3.882\times10^{-2}$

$N_O = (3.346\times10^{-2})(0.5801) = 1.941\times10^{-2}$

Desired assy-averaged densities (at 15-cm)