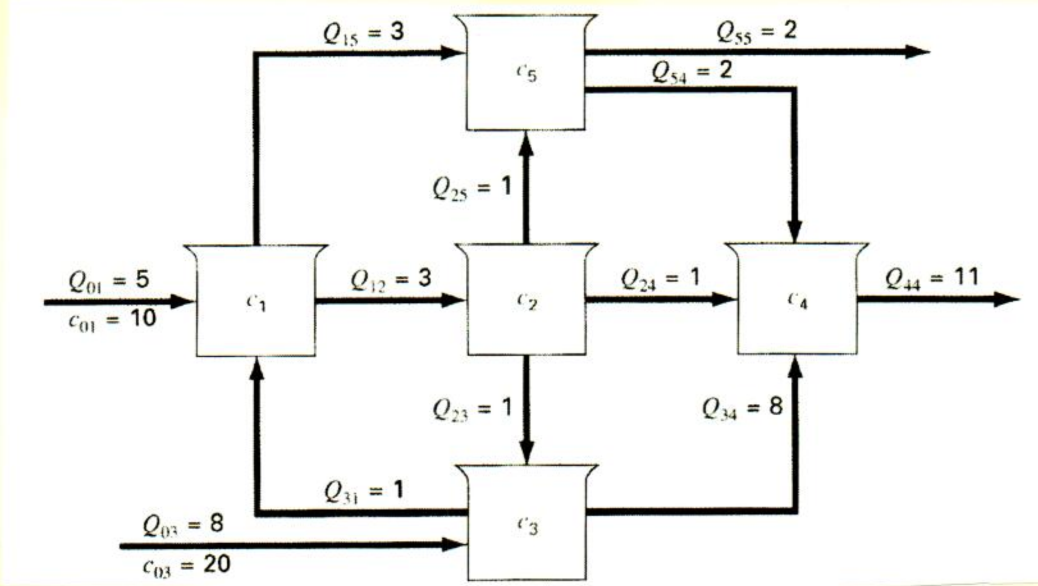
Conservation of mass



The steady state mass balance eqns for the above system are as follows.

tank 1 $\quad Q_{01}c_{05} + Q_{31}c_3 - Q_{12}c_1 - Q_{15}c_1 = 0$

tank 2 $\quad Q_{12}c_1 - Q_{23}c_2 - Q_{24}c_2 - Q_{25}c_2 = 0$

tank 3 $\quad Q_{03}c_{03} + Q_{23}c_2 - Q_{31}c_3 - Q_{34}c_3 = 0$

Tank 4 $\quad Q_{24}c_2 + Q_{34}c_3 + Q_{54}c_5 - Q_{44}c_4 = 0$

tank 5 $\quad Q_{15}c_1 + c_{25}c_2 - Q_{54}c_5 - Q_{55}c_5 = 0$

Let's put in numbers.

$$50 + c_3 - (3+3)c_1 = 0$$
$$3c_1 - (1+1+1)c_2 = 0$$
$$160 + c_2 - (1+8)c_3 = 0$$
$$c_2 + 8c_3 + 2c_5 - 11c_4 = 0$$
$$3c_1 + c_2 - (2+2)c_5 = 0$$

$\Rightarrow$

$$-6c_1 + c_3 = -50$$
$$3c_1 - 3c_2 = 0$$
$$c_2 - 9c_3 = -160$$
$$c_2 + 8c_3 + 2c_5 - 11c_4 = 0$$
$$3c_1 + c_2 - 4c_5 = 0$$

In matrix form

$$\begin{bmatrix} -6 & 0 & 1 & 0 & 0 \\ 3 & -3 & 0 & 0 & 0 \\ 0 & 1 & -9 & 0 & 0 \\ 0 & 1 & 8 & -11 & 2 \\ 3 & 1 & 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix} = \begin{bmatrix} -50 \\ 0 \\ -160 \\ 0 \\ 0 \end{bmatrix}$$

see mass_bal1.m

$$\underline{c} = \begin{bmatrix} 11.51 \\ 11.51 \\ 19.06 \\ 17.00 \\ 11.51 \end{bmatrix}$$

```
%
%   MASS_BAL1.M    Find the solution to the steady state mass balance
%                      equations for an interconnected 5-tank system
%
%   The coupled system of five equations is solved for a specific network
%   of five interconnected tank reactors (see notes for development of the eqns.).
%   The program simply illustrates the use of Matlab's backslash operator to solve a
%   system of linear equations...
%
%   Reference:  This problem was taken from Section 12.1 in the text, Numerical
%   Methods for Engineers," 3rd Ed. by Chapra and Canale, McGraw Hill (1998).
%
%   File prepared by J. R. White, UMass-Lowell  (last update: Nov. 2017)
%

      clear all,   close all
%
%   setup and solve 5x5 system (use numerical values for specific case given)
      A = [ -6 0 1 0 0;3 -3 0 0 0;0 1 -9 0 0;0 1 8 -11 2;3 1 0 0 -4];
      b = [-50 0 -160 0 0]';
      c = A\b;
      fprintf('\n\n  Mass_Ball:  --> Conc. in Tank #1 =  %8.5f mg/m^3  \n ',c(1))
      fprintf('              --> Conc. in Tank #2 =  %8.5f mg/m^3  \n ',c(2))
      fprintf('              --> Conc. in Tank #3 =  %8.5f mg/m^3  \n ',c(3))
      fprintf('              --> Conc. in Tank #4 =  %8.5f mg/m^3  \n ',c(4))
      fprintf('              --> Conc. in Tank #5 =  %8.5f mg/m^3  \n ',c(5))
%
%  end of problem
```

```
>> mass_ball

  Mass_Ball:  --> Conc. in Tank #1 =  11.50943 mg/m^3
              --> Conc. in Tank #2 =  11.50943 mg/m^3
              --> Conc. in Tank #3 =  19.05660 mg/m^3
              --> Conc. in Tank #4 =  16.99828 mg/m^3
              --> Conc. in Tank #5 =  11.50943 mg/m^3
```

```
>> dd_test1
Case 1a solution via x = A\b:
x =
    1.0000
    1.0000
    1.0000
Case 1a solution via Gauss Seidel:
x =
  1.0e+280 *
   -0.5582
    0.1681
   -1.5066
k =
   250
 *** Warning -- Case not converged ***
Case 1b solution via x = A\b:
x =
    1.0000
    1.0000
    1.0000
Case 1b solution via Gauss Seidel:
x =
    1.0000
    1.0000
    1.0000
k =
     7
 Hit any key to continue with Case 2

Case 2a solution via x = A\b:
x =
     1
     1
     1
Case 2a solution via Gauss Seidel:
   WARNING:  Check A matrix for a zero along the diagonal!!!

x =
     0
     0
     0
k =
     0
Case 2b solution via x = A\b:
x =
     1
     1
     1
Case 2b solution via Gauss Seidel:
x =
    1.0000
```

```
    1.0000
    1.0000
k =
    12
 Hit any key to continue with Case 3

Case 3a solution via x = A\b:
x =
    2.6500
    2.4250
    0.8500
Case 3a solution via Gauss Seidel:
x =
   1.0e+43 *
    1.3639
   -0.3720
    1.1159
k =
   250
 *** Warning -- Case not converged ***
Case 3b solution via x = A\b:
x =
    2.6500
    2.4250
    0.8500
Case 3b solution via Gauss Seidel:
x =
    2.6500
    2.4250
    0.8500
k =
    33
```

```
%
%   DD_TEST1.M   Study relationship of Diagonal Dominance on the convergence
%               of the Gauss Seidel method (equation order is also important)
%
%   This file has a sequence of steps that highlight the relationship of diagonal
%   dominance and the convergence of the Gauss Seidel method.  It uses the SR.M
%   program to implement the Gauss Seidel method with no row interchanges  --  thus,
%   the ordering of the equations can be important [normally proper ordering of the
%   equations (a form of partial pivoting) tends to remove or minimize this
%   dependency].
%
%   File written by J. R. White, UMass-Lowell (last update:  Nov. 2017)
%

     clear all,  close all
     format compact
%
%   set parameters for Gauss Seidel method (same for all cases)
     tol = 1e-5;  M = 250;  alf = 1;  xo = zeros(3,1);
%
%   Case 1a  --  original equations
     A = [1 1 5; 1 4 -1;3 1 -1];  b = [7 4 3]';
     disp('Case 1a solution via x = A\b: ');  x = A\b
     disp('Case 1a solution via Gauss Seidel: ');  [x,k] = sr(A,b,xo,alf,tol,M)
     if k == M, disp(' *** Warning -- Case not converged ***'); end
%
%   Case 1b  --  same as Case 1a with rows 1 & 3 interchanged to make system DD
     A = [3 1 -1; 1 4 -1;1 1 5];  b = [3 4 7]';
     disp('Case 1b solution via x = A\b: ');  x = A\b
     disp('Case 1b solution via Gauss Seidel: ');  [x,k] = sr(A,b,xo,alf,tol,M)
     if k == M, disp(' *** Warning -- Case not converged ***'); end
     disp(' Hit any key to continue with Case 2'), pause
     disp(' ')
%
%   Case 2a  --  original equations
     A = [8 3 1;-6 0 7;2 4 -1];  b = [12 1 5]';
     disp('Case 2a solution via x = A\b: ');  x = A\b
     disp('Case 2a solution via Gauss Seidel: ');  [x,k] = sr(A,b,xo,alf,tol,M)
     if k == M, disp(' *** Warning -- Case not converged ***'); end
%
%   Case 2b  --  interchanged rows 2 & 3 from original eqn. to make system DD
     A = [8 3 1;2 4 -1;-6 0 7];  b = [12 5 1]';
     disp('Case 2b solution via x = A\b: ');  x = A\b
     disp('Case 2b solution via Gauss Seidel: ');  [x,k] = sr(A,b,xo,alf,tol,M)
     if k == M, disp(' *** Warning -- Case not converged ***'); end
     disp(' Hit any key to continue with Case 3'), pause
     disp(' ')
%
%   Case 3:  Note that there is no row interchange that can make this system
%            diagonally dominant (DD), but reordering might help???
%   Case 3a  --  original eqns
     A =  [-2 2 -3;0 2 -1;-3 4 5];  b = [-3 4 6]';
     disp('Case 3a solution via x = A\b: ');  x = A\b
     disp('Case 3a solution via Gauss Seidel: ');  [x,k] = sr(A,b,xo,alf,tol,M)
     if k == M, disp(' *** Warning -- Case not converged ***'); end
%   Case 3b  -- interchange rows 1 and 3
```

```
  A =  [-3 4 5;0 2 -1;-2 2 -3];  b = [6 4 -3]';
  disp('Case 3b solution via x = A\b: ');  x = A\b
  disp('Case 3b solution via Gauss Seidel: ');  [x,k] = sr(A,b,xo,alf,tol,M)
  if k == M, disp(' *** Warning -- Case not converged ***'); end

%  end of program
```