## Lab #6a -- Review for Exam #2 and Some Practice with Linear Equations

### Overview

This lab will have two distinct components -- the first part will focus on answering your questions concerning the material to be covered on Exam #2 (specifically the topics from Lessons 4 and 5), and the second part will include a couple of exercises that deal with systems of linear equations (from the first half of Lesson 6). About one hour will be devoted to each major goal -- that is, helping prepare you for the upcoming exam and giving you some further insight and confidence with the set up and computer solution of linear algebraic equations. The first part should be a direct help for the exam next week and the second part should be helpful when studying Lesson 6 and doing the associated HWs.

### Part I: Review for Exam #2

As preparation for the upcoming exam, the first part of the lab period will simply review many of the topics discussed in Lessons 4 and 5. In particular, after the initial quiz period, the remainder of the first hour will be spent answering specific student questions on any topics associated with these lessons (roundoff and truncation error, Taylor series, finite-difference methods, roots of nonlinear equations, etc.). This will be a great opportunity to resolve any specific weaknesses you may have or simply to review as many topics as time permits -- so take advantage of this review session and come prepared with your questions for the lab instructor…

If there are not enough student-generated questions to fill the first hour of the lab, the instructor will use the remaining time to review several of the quizzes, the lab exercises, and/or the HW problems performed on these topics. Thus, if you have any special requests to review one or more of these items, don't hesitate to ask the lab instructor.

The goal of this review session is to help everyone do as well as possible on the exam next week, so please take advantage of this opportunity to review/clarify as many topics as time permits!

### Part II: Some Practice with Linear Equations

The second hour of the lab will focus on topics from the first half of Lesson 6 with two separate exercises, as follows:

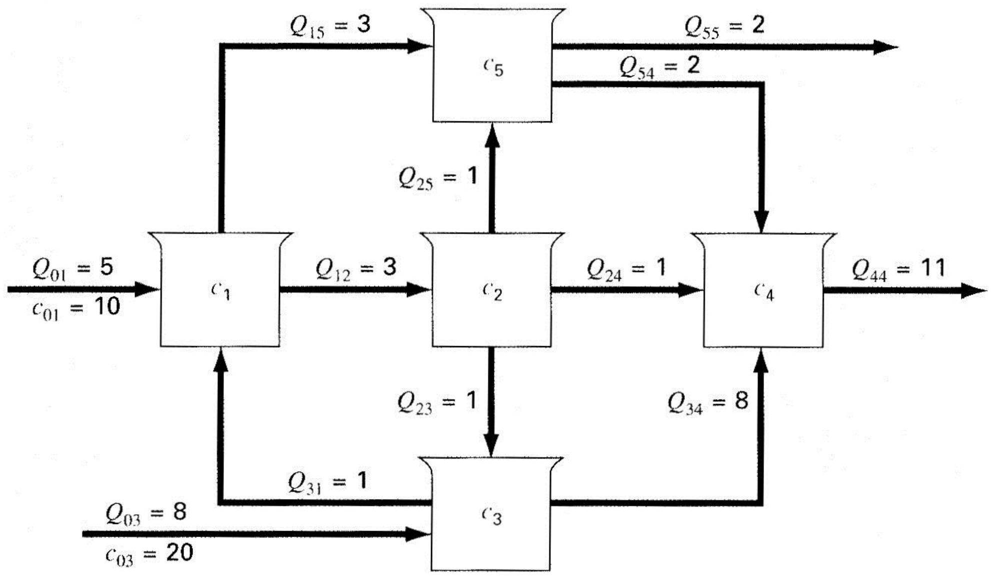
### Problem 1 -- Set up and Solution of Linear Systems (A Conservation of Mass Problem)

A series of five well-stirred tank reactors are interconnected as shown in the sketch. All the flow rates, $Q_{ij}$, are given in $m^3$/min, where the flow is from tank i to tank j. Note that a flow rate with a subscript 0j implies that this is an independent feed stream to tank j. The concentrations of the material of interest are given in milligrams/cubic meter ($mg/m^3$). Thus, the mass flow rate of the species of interest in each stream is given by the product of the volumetric flow rate and the mass concentration ($m^3/min \times mg/m^3 = mg/min$).

In steady state, mass conservation is given by

mass flow rate in – mass flow rate out = 0

With the data given in the sketch, set up and solve the steady state mass balance equations for the five-reactor system given here. The result should be the average steady state concentration of the material of interest in each of the five tanks (denoted by $c_j$ in the sketch). As before, a concentration denoted as $c_{0j}$ represents the concentration of the independent input stream to tank j.




**Problem #2: Diagonal Dominance and Convergence of the Gauss Seidel Method**

This exercise walks you through a series of simple computations that addresses diagonal dominance and convergence of the Gauss Seidel method (with some emphasis on proper ordering of the equations). In particular, you should solve the following three systems using both the **x = A\b** operation in Matlab and via the Gauss Seidel method implemented within the **sr.m** routine discussed in the Lecture Notes. For the Gauss Seidel method, however, we should note that the **sr.m** routine does not perform any internal row interchanges (this is a very simple algorithm that is used for demonstration purposes only and no partial pivoting in done internal to the code) -- so appropriate reordering of the equations to facilitate solution via the Gauss Seidel method may be needed before calling the **sr.m** routine!

The three systems of interest are:

Case 1:
$$\begin{aligned} x + y + 5z &= 7 \\ x + 4y - z &= 4 \\ 3x + y - z &= 3 \end{aligned}$$

Case 2:
$$\begin{aligned} 8x + 3y + z &= 12 \\ -6x + 7z &= 1 \\ 2x + 4y - z &= 5 \end{aligned}$$

Case 3:
$$\begin{aligned} -2x + 2y - 3z &= -3 \\ 2y - z &= 4 \\ -3x + 4y + 5z &= 6 \end{aligned}$$

Perform the following operations/calculations (and be sure to discuss your results):

a. Is Case 1 a diagonally dominant system as given? Attempt to solve Case 1 via the **x = A\b** operation and the **sr.m** routine and discuss your results -- is this what you expected?

b. For Case 1, interchange equations 1 and 3 and re-do Part a. Explain your observations…

c. Try solving Case 2 via the **x = A\b** operation and the **sr.m** routine. What happens here?

d. Would the interchange of rows 2 and 3 for Case 2 change things? Try it and explain your observations…

e. Now focus on Case 3. Is this system diagonally dominant? If not, can a row interchange make the system diagonally dominant? How about the solution to this system -- can you get the Gauss Seidel method to converge? Explain…

f. Finally, from the results obtained in the above manipulations, summarize how diagonal dominance, equation ordering (similar to partial pivoting), and convergence are interrelated when using the Gauss Seidel method to solve linear systems.

**Note:** For the Gauss Seidel method use $\alpha = 1$, a convergence criterion of 0.00001, a maximum number of iteration of 250, and an initial guess for the solution vector of **0** (i.e. a vector of zeros), for all cases. This will make for a consistent set of comparisons.