

## Lab #4a -- Number Systems, Numerical Integration, and Plotting Functions written as Infinite Series

### Overview

Since we are just starting Lesson 4 on Numerical Error, today's lab will highlight a number of miscellaneous topics (as indicated in the above Lab #4a title). In particular, the discussion of different **number systems** is important since computers use a binary representation, which is quite different from the usual base-10 notation everyone is familiar with. Concerning **numerical integration**, this is a subject normally treated in some detail in a course like this but, unfortunately, because we spend so much time with the Matlab fundamentals portion of the class, we usually run out of time before we formally get to this topic. However, this lab will give you the opportunity to get some limited experience with the use of Matlab's *integral* routine to numerically evaluate continuous integrals -- and this experience will be a great addition to your toolbox of useful Matlab capabilities. Finally, the last exercise, will give you some experience with **evaluating and plotting 2-D functions written in the form of an infinite series**. The problem presented here is similar to the heat transfer example given in the Lesson 4 Illustrative Applications (see [planewall\\_1.pdf](#)), with a slightly more complicated set of resultant equations. This last task will build upon your prior experiences that include evaluating 2-D functions and storing the functional information in a 2-D array, working with infinite series, doing some numerical integrals (from earlier in today's lab), and plotting functions of two variables both quantitatively and qualitatively. Thus, the last exercise is a bit challenging since it brings together several separate topics from previous work, but the lab instructor will be there to help you get over any rough stops that may occur -- so don't hesitate to ask for help if needed...

Upon completion of this lab you should have added some new capabilities to your Matlab tool chest (i.e., the numerical integration capability) and be more comfortable with number systems and with working with functions written in the form of infinite series -- and these later two tasks will indeed prove to be useful when completing your next formal HW assignment...

### Problem 1 -- Binary, Decimal, and Other Numbers

- a. Find the decimal equivalent of  $(1101011)_2$ .
- b. Find the binary equivalent to  $(311)_{10}$ .
- c. Consider a numbering system with 8 digits -- a base 8 system. What is the equivalent of  $(311)_{10}$  in this system?

### Problem 2 -- Numerical Integration

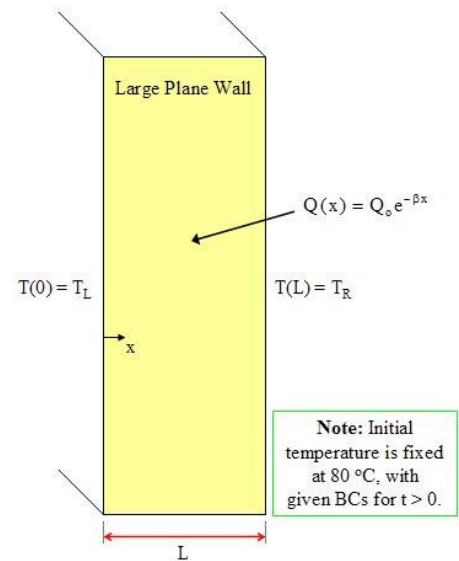
This exercise wants you to become familiar with the *integral* routine in Matlab for performing the numerical integration of continuous functions. In particular, integrate the following two functions analytically and with Matlab's *integral* routine and compare your solutions:

a.  $I_1 = \int_0^2 (3x^2 + 1) dx$                       b.  $I_2 = \int_0^2 te^{-t} dt$

**Hint:** The use of anonymous functions might prove useful here...

**Problem 3 -- Evaluating Infinite Series: Transient Heat Conduction in a Planar Wall**

As mentioned above, this problem is similar to the heat transfer example given in the Lesson 4 Illustrative Applications (see [planewall\\_1.pdf](#)), with a slightly more complicated set of resultant equations. The current case considers a large planar wall that is initially at a constant temperature of 80 C. The wall is suddenly subjected to gamma radiation from the left side, which manifests itself as an exponentially varying internal energy generation term of the form  $Q(x) = Q_0 e^{-\beta x}$ . This internal heat source is established nearly instantaneously at  $t = 0$ . The sides of the wall are maintained at the initial temperature value for all time. A rough sketch illustrating this situation is shown at the right.



This transient heat conduction problem can be formulated mathematically as

$$\frac{\partial}{\partial t} T(x, t) = \alpha \frac{\partial^2}{\partial x^2} T(x, t) + Q(x) \quad \text{with} \quad Q(x) = Q_0 e^{-\beta x}$$

with

$$\begin{aligned} \text{IC:} \quad & T(x, 0) = T_i = 80 \text{ C} && \text{(constant initial temperature)} \\ \text{BC\#1:} \quad & T(0, t) = T_L = 80 \text{ C} && \text{(left surface at 80 C for } t > 0) \\ \text{BC\#2:} \quad & T(L, t) = T_R = 80 \text{ C} && \text{(right surface at 80 C for } t > 0) \end{aligned}$$

where  $\alpha$  is the thermal diffusivity of the material,  $L$  is the thickness of the wall, and  $Q_0$  and  $\beta$  characterize the gamma ray internal heat source. For this application, we will assume that  $L = 0.20$  m,  $\alpha = 0.0015$  m<sup>2</sup>/s,  $Q_0 = 20$  C/s, and  $\beta = 5$  m<sup>-1</sup>. The goal here is to evaluate and plot the space-time temperature distribution,  $T(x, t)$ , in the 1-D wall.

A classical technique for solving problems of this type is the Separation of Variables (SOV) method. The formal SOV solution for the above situation can be summarized, as follows:

$$T(x, t) = V(x, t) + W(x)$$

where

$$W(x) = -\frac{Q_0}{\alpha\beta^2} e^{-\beta x} + c_1 x + c_2 \quad \text{and} \quad V(x, t) = \sum_{n=1}^{\infty} a_n \sin(\lambda_n x) e^{-\alpha\lambda_n^2 t}$$

with

$$\begin{aligned} c_1 &= \frac{1}{L} \left[ T_R - T_L - \frac{Q_0}{\alpha\beta^2} (1 - e^{-\beta L}) \right] && \text{and} && c_2 &= T_L + \frac{Q_0}{\alpha\beta^2} \\ \lambda_n &= \frac{n\pi}{L} && \text{and} && a_n &= \frac{2}{L} \int_0^L (T_i - W(x)) \sin(\lambda_n x) dx && \text{for } n = 1, 2, \dots \end{aligned}$$

Now, your task for this problem is to evaluate the above expressions for  $T(x,t)$  and to plot the resultant space-time temperature distribution, as appropriate, so that one can easily visualize the time-dependent temperature distribution within the wall, both quantitatively (with a 2-D plot containing  $T(x,t_i)$  for several time points,  $t_i$ ) and qualitatively (with one or more 3-D views) -- and be sure to properly label your plots. Rationalize that your solutions physically make sense!

**Note:** The primary difference between this problem and the heat transfer example given in the Lesson 4 Illustrative Applications involves the computation of the  $a_n$  expansion coefficients. In particular, here the coefficients are left in their integral form (see above equation for  $a_n$ ) and, in the Lectures Notes, the integrals were evaluated for you and the simple analytical result was given explicitly. In the current problem, the integrals are a little tedious to evaluate analytically, so you might try the use of Matlab's *integral* routine to compute the desired expansion coefficients (since, after Prob. #2 from above, you should already be familiar with numerical integration of continuous functions...).

Finally, since your program should be very similar to the **planewall\_1.m** file, you should make sure you are familiar with this sample illustrative problem first -- then, you can simply make changes, as needed, to solve the specific problem requested here.

Good luck -- this should be both challenging and fun...