```
%
%  ARRAY_MANIP_Lab2.M    Practice with various array manipulations in Matlab
%
%  This demo illustrates some common array manipulations with Matlab.  If we
%  consider a 2-D array as storage for various forms of information, then we need
%  to be able to extract and manipulate the data as needed.  This exercise simply
%  illutrates a few possibilities when the array simply holds numerical values...
%
%  Used for illustrative purposes for Lab 2a (Fall 2017)
%
%  File produced by Prof. J. R. White, UMass-Lowell (Sept. 2017)
%

    clear all, close all, nfig = 0;
    format short, format compact
%
%  create a 30x6 array of random numbers between -2 and 2
    disp('a.  30x6 array of random values between -2 and 2:')
    A = 4*rand(30,6) - 2
%
%  extract rows 4, 13, and 21 of original matrix
    disp('b.  rows 4, 13, and 21 of original matrix:')
    B = A([4 13 21],:)
%
%  print columns 2, 4, and 6 of original matrix to screen
    disp('c.  print columns 2, 4, 6 of A matrix to screen:')
    A(:,[2:2:6])
%
%  extract data from rows 2, 8, 30 and columns 1 and 6
    disp('d.  extract rows 2, 8, 30 and columns 1 and 6 of A matrix:')
    D = A([2 8 30],[1 6])
%
%  sum elements in column 3 of original array
    disp('e.  sum of elements in column 3 of original matrix:')
    sum(A(:,3))
%
%  sum all elements of original A array
    disp('f.  sum of all elements of original A matrix:')
    f = sum(sum(A))
%
%  find average value of all elements in A matrix
    disp('g.  average value of elements in A (two ways):')
    N = numel(A),  aveA1 = f/N,  aveA2 = mean(mean(A))
%
%  compute the inner product of rows 12 and 27
    disp('h.  inner product of rows 12 and 27 of original matrix (two ways):')
    A(12,:)*A(27,:)',     A(27,:)*A(12,:)'
%
%  compute the outer product of rows 12 and 27
    disp('i.  outer products of rows 12 and 27 of original matrix (order is important↙
here):')
    A(12,:)'*A(27,:),     A(27,:)'*A(12,:)  % transpose of each other
%
%  plot column 6 (dependent) versus column 2 (independent)
    plot(A(:,2),A(:,6),'ro','LineWidth',2),grid on  % see what happens with 'r-'
    title('Array\_Manip\_Lab2a Part j:  Plot of Coulmn 6 vs. Column 2 Data')
```

```
    xlabel('Column 2 Data'),ylabel('Coulmn 6 Data')
%
%   end of program
```

Given the following matrices

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 1 & 2 & 0 \\ 4 & 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} -2 & 3 \end{bmatrix} \quad X = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$$

Perform the indicated operations by hand calculation

a. $$B^T B = \begin{bmatrix} 3 & -1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} = \boxed{11} \text{ ans}$$

b. $$ABC = \begin{bmatrix} 1 & 2 & -3 \\ 1 & 2 & 0 \\ 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \\ 1 \\ 11 \end{bmatrix} \begin{bmatrix} -2 & 3 \end{bmatrix} = \boxed{\begin{bmatrix} 4 & -6 \\ -2 & 3 \\ -22 & 33 \end{bmatrix}} \text{ ans}$$

$3 \times 1 \quad 1 \times 2 \quad \Rightarrow \quad 3 \times 2$

c. $$X^T A X = \begin{bmatrix} -3 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -3 \\ 1 & 2 & 0 \\ 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 1 & 1 \end{bmatrix} \begin{bmatrix} -4 \\ -1 \\ -9 \end{bmatrix} = \boxed{2} \text{ ans}$$

d. $$X X^T = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} -3 & 1 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 9 & -3 & -3 \\ -3 & 1 & 1 \\ -3 & 1 & 1 \end{bmatrix}} \text{ ans}$$

e. $$A^2 = \begin{bmatrix} 1 & 2 & -3 \\ 1 & 2 & 0 \\ 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -3 \\ 1 & 2 & 0 \\ 4 & 2 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} -9 & 0 & -6 \\ 3 & 6 & -3 \\ 10 & 14 & -11 \end{bmatrix}} \text{ ans}$$

note that $A.*A$ is element by element multiplication

$$\begin{bmatrix} 1 & 2 & -3 \\ 1 & 2 & 0 \\ 4 & 2 & 1 \end{bmatrix} .* \begin{bmatrix} 1 & 2 & -3 \\ 1 & 2 & 0 \\ 4 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 9 \\ 1 & 4 & 0 \\ 16 & 4 & 1 \end{bmatrix}$$

These are quite different

```
>> matrix_ops_4
Matrices for the various exercises
A =
     1     2    -3
     1     2     0
     4     2     1
B =
     3
    -1
     1
C =
    -2     3
x =
    -3
     1
     1
Find B^T*B
ans =
    11
Find A*B*C
ans =
     4    -6
    -2     3
   -22    33
Find x^T*A*x
ans =
     2
Find x*x^T
ans =
     9    -3    -3
    -3     1     1
    -3     1     1
Find A*A
ans =
    -9     0    -6
     3     6    -3
    10    14   -11
Find A.*A
ans =
     1     4     9
     1     4     0
    16     4     1
>>
```

```
%
%   MATRIX_OPS_4.M  MATLAB file to verify some hand calculations (part of Lab #2a)
%
%   This file simply does a number of matrix multiplication tasks to verify
%   some hand calculations that were performed.  It should validate that you
%   undestand how to do the hand computations as well as give some further
%   experience with doing matrix computations in Matlab (although this part
%   is pretty straightforward, since this is what Matlab does best...).
%
%   File prepared by J. R. White, UMass-Lowell (last update: Sept. 2017)
%

      clear all,   close all
      format compact
%
%   define matrices for problems
      disp('Matrices for the various exercises ')
      A = [1 2 -3;1 2 0;4 2 1],  B = [3 -1 1]',  C = [-2 3],  x = [-3 1 1]'
%
%   do the desired calculations
      disp('Find B^T*B');       B'*B
      disp('Find A*B*C');       A*B*C
      disp('Find x^T*A*x');     x'*A*x
      disp('Find x*x^T');       x*x'
      disp('Find A*A');         A*A
      disp('Find A.*A');        A.*A
%
%   end of program
```

(a) Calc $\det A$ where $A = \begin{bmatrix} -1 & 1 & -3 \\ 3 & 0 & -1 \\ 5 & -2 & 5 \end{bmatrix}$

→ via Laplace's Expansion (along row 2) $\quad c_{ij} = (-1)^{i+j} M_{ij}$

$$\det A = a_{21}(-1)M_{21} + a_{22}(+1)M_{22} + a_{23}(-1)M_{23}$$

($a_{22} \to 0$)

$$= -3\begin{vmatrix} 1 & -3 \\ -2 & 5 \end{vmatrix} + 0 + (1)\begin{vmatrix} -1 & 1 \\ 5 & -2 \end{vmatrix}$$

$$= -3(-1) + (-3) = 0$$

→ via row operations

$$\begin{bmatrix} -1 & 1 & -3 \\ 3 & 0 & -1 \\ 5 & -2 & 5 \end{bmatrix}$$

multiply row 1 by 3 and add to row 2

$$\begin{bmatrix} -1 & 1 & -3 \\ 0 & 3 & -10 \\ 5 & -2 & 5 \end{bmatrix}$$

multiply row 1 by 5 and add to row 3

$$\begin{bmatrix} -1 & 1 & -3 \\ 0 & 3 & -10 \\ 0 & 3 & -10 \end{bmatrix}$$

note also that, at this point, row 2 and row 3 are clearly linearly dependent

subtract row 2 from row 3

$$\begin{bmatrix} -1 & 1 & -3 \\ 0 & 3 & -10 \\ 0 & 0 & 0 \end{bmatrix}$$

↑ determinant here is product of diagonal elements $\therefore$ $\det A = 0$

(b) Find the inverses of A and B

→ Inverse of A

$$A^{-1} = \frac{C^T}{\det A}$$ ⇒ however we just calculated $\det A = 0$

⇒ Therefore $A^{-1}$ does not exist

<u>A is singular</u>

<u>ans</u>

→ Inverse of B where

$$B = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 0 & -2 \\ 0 & -3 & 2 \end{bmatrix}$$

$$B^{-1} = \frac{C^T}{\det B}$$

Cofactor matrix ⇒ $$\begin{bmatrix} +(-6) & -2 & +(-3) \\ -(-2) & +4 & -(-6) \\ +2 & -(-4) & +(1) \end{bmatrix} = \begin{bmatrix} -6 & -2 & -3 \\ 2 & 4 & 6 \\ 2 & 4 & 1 \end{bmatrix}$$

along row 1 $\det B = 2(-6) - 1(-2) = -10$

$$\therefore B^{-1} = \frac{1}{(-10)} \begin{bmatrix} -6 & 2 & 2 \\ -2 & 4 & 4 \\ -3 & 6 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} .6 & -.2 & -.2 \\ .2 & -.4 & -.4 \\ .3 & -.6 & -.1 \end{bmatrix}}$$

<u>ans</u>

(c) Solve eqn $Bx = y$ where $y = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$

$$x = B^{-1}y = \begin{bmatrix} 0.6 & -0.2 & -0.2 \\ 0.2 & -0.4 & -0.4 \\ 0.3 & -0.6 & -0.1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 1 \\ 1 \\ 0.5 \end{bmatrix}}$$

<u>ans</u>

(d) Calculate the eigenvalues and eigenvectors of C where $C = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$.

→ eigenvalues

$$|C - \lambda I| = \begin{vmatrix} 1-\lambda & 0 \\ 2 & 3-\lambda \end{vmatrix} = (1-\lambda)(3-\lambda) = 0$$

$\therefore \lambda_1 = 1$ and $\lambda_2 = 3$ $\boxed{\lambda_{1,2} = 1, 3}$ ans

→ eigenvectors

$$(\underline{\underline{C}} - \lambda \underline{\underline{I}}) \underline{x} = 0$$

for $\lambda = \lambda_1 = 1$

$$\begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{matrix} 0x_1 + 0x_2 = 0 \\ 2x_1 + 2x_2 = 0 \end{matrix} \Rightarrow x_2 = -x_1 \qquad \therefore \boxed{\underline{x}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}}$$

ans

for $\lambda = \lambda_2 = 3$

$$\begin{bmatrix} -2 & 0 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{matrix} -2x_1 + 0x_2 = 0 \\ 2x_1 + 0x_2 = 0 \end{matrix} \Rightarrow x_1 = 0 \qquad \therefore \boxed{\underline{x}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}}$$

ans

```
>> linear_algebra_1
Matrices for Problems 2a - 2d
A =
    -1     1    -3
     3     0    -1
     5    -2     5
B =
     2    -1     0
     1     0    -2
     0    -3     2
C =
     1     0
     2     3
y =
     1
     0
    -2
Find det A
ans =
   1.3323e-15
Find inverses of A and B
Warning: Matrix is close to singular or badly scaled.
         Results may be inaccurate. RCOND =  5.286776e-18.
> In linear_algebra_1 at 20
AI =
   1.0e+16 *
   -0.1501    0.0751   -0.0751
   -1.5012    0.7506   -0.7506
   -0.4504    0.2252   -0.2252
BI =
    0.6000   -0.2000   -0.2000
    0.2000   -0.4000   -0.4000
    0.3000   -0.6000   -0.1000
Solve B*x = y
x =
    1.0000
    1.0000
    0.5000
Find eigenvalues & eigenvector of C
evec =
         0    0.7071
    1.0000   -0.7071
eval =
     3     0
     0     1
```

```
%
%   Linear_Algebra_1.M  MATLAB matrix tasks Prob. #2 in HW2  (also part of Lab #2a)
%
%   This file just does some simple linear algebra manipulations within Matlab.
%
%   File prepared by J. R. White, UMass-Lowell (last update: Sept. 2017)
%

     clear all,   close all
%
%   Define matrices for problems
     disp('Matrices for Problems 2a - 2d')
     A = [-1 1 -3;3 0 -1;5 -2 5],  B = [2 -1  0; 1 0 -2; 0  -3  2],  C = [1 0; 2 3]
     y = [1 0 -2]'
%
%   *** Problem 2a ***
     disp('Find det A');         det(A)
%
%   *** Problem 2b ***
     disp('Find inverses of A and B'),  AI = inv(A),  BI = inv(B)
%
%   *** Problem 2c ***
     disp('Solve B*x = y');      x = B\y
%
%   *** Problem 2d ***
     disp('Find eigenvalues & eigenvector of C'),  [evec,eval] = eig(C)
%
%   end of program
```