Applied Engineering Problem Solving (CHEN.3170)

Homework Assignment #6a -- Fall 2017

Applications Involving Systems of Linear Algebraic Equations

Problem #1: Reaction Stoichiometry

Find the appropriate stoichiometric coefficients for the following chemical reactions by conserving the number of atoms of each element between the reactants and products (select a_1 so that all the coefficients are integers):

a. $a_1 FeS + a_2 O_2 \rightarrow a_3 FeO + a_4 SO_2$

b. $$a_1\left(Cr\left(N_2H_4CO\right)_6\right)_4\left(Cr\left(CN\right)_6\right)_3 + a_2 KMnO_4 + a_3 H_2SO_4 \rightarrow$$
$$a_4 K_2Cr_2O_7 + a_5 MnSO_4 + a_6 CO_2 + a_7 KNO_3 + a_8 K_2SO_4 + a_9 H_2O$$

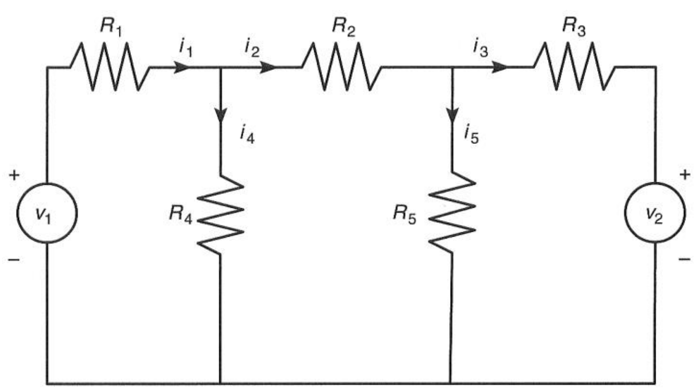
Be sure to show the development of the appropriate equations and the resultant balanced chemical reaction equation formula for each case.

Problem #2: Resistive Networks

Electric circuits containing only resistors lead to linear algebraic equations. Kirchhoff's voltage law, which states, "the algebraic sum of the voltage drops around a closed loop must be zero," and Kirchhoff's current law, which says, "the sum of the currents entering a node is zero," are often used to develop these equations. For resistors, the voltage drop is given by Ohm's law, $v = Ri$, where R is the resistance (ohms), i is the current (amperes), and v is the voltage drop (volts).

a. For the circuit shown, Kirchhoff's voltage and current laws give the following equations:

$$-v_1 + R_1 i_1 + R_4 i_4 = 0$$
$$-R_4 i_4 + R_2 i_2 + R_5 i_5 = 0$$
$$-R_5 i_5 + R_3 i_3 + v_2 = 0$$
$$i_1 - i_2 - i_4 = 0$$
$$i_2 - i_3 - i_5 = 0$$



Put these equations into standard matrix form, $\underline{\underline{A}}\,\underline{x} = \underline{b}$, where the individual currents, i_1 through i_5 are contained within the unknown x-vector.

b. Given fixed values for the resistances and the voltage source v_1:

$$R_1 = 5\text{ k}\Omega \quad R_2 = 100\text{ k}\Omega \quad R_3 = 200\text{ k}\Omega \quad R_4 = 150\text{ k}\Omega \quad R_5 = 250\text{ k}\Omega \quad \text{and} \quad v_1 = 100\text{ V}$$

compute and plot the currents, i_1 through i_5 versus v_2. Assume that the second voltage source can vary over the range $0 \le v_2 \le 400\ \text{V}$. Do the profiles make sense?

c. If the resistances are rated to carry a maximum current of 1 ma, what should we set for the allowed range of values for v_2?

Problem #3: Finite Difference Solution to a Conduction Heat Transfer Problem

Consider heat conduction through the wall of a thick circular pipe. In a certain case, the inside wall temperature is maintained at 200 ^oC and the outside wall temperature is 80 ^oC. The inner radius of a particular pipe is a = 0.05 m and the outer radius is b = 0.1m.

The differential equation that describes the temperature distribution in the pipe is given by

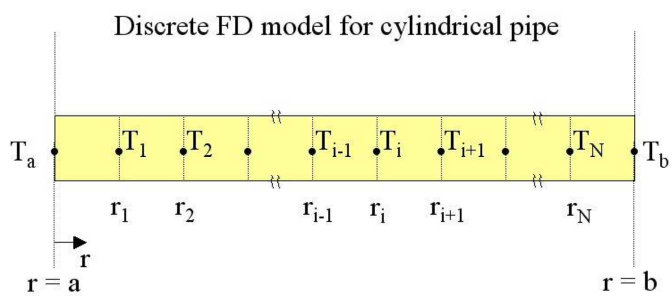
$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = 0$$

with boundary conditions

$$T(a) = T_a = 200\ ^oC \qquad \text{and} \qquad T(b) = T_b = 80\ ^oC$$

Although this boundary value problem (BVP) has a simple analytical solution, here we are primarily interested in developing and validating a numerical solution based on the Finite Difference (FD) Method (see the example application "Introduction to Finite Difference Methods" in the Notes for Lesson #4).

One always starts problems of this type by discretization of the independent variable. Recognizing that the temperatures at the inner and outer surfaces are known, we can develop a discrete representation of the spatial grid for this problem as shown in the sketch below,



where r_i represents the radial location of the i^{th} discrete unknown temperature, T_i. The vector of spatial points for the unknown temperatures can be generated with Matlab as follows:

N = number of unknown temperatures in the domain of interest

h = Δr = (b-a)/(N+1) = step size

r = a+h : h : b-h = vector of locations for unknown temperatures

a. Now, with the above discrete geometry representation and the following 2^{nd} order central FD approximations for the derivatives

$$\left.\frac{d^2T}{dr^2}\right|_i \approx \frac{T_{i-1} - 2T_i + T_{i+1}}{h^2} \qquad \text{and} \qquad \left.\frac{dT}{dr}\right|_i \approx \frac{T_{i+1} - T_{i-1}}{2h}$$

formally develop the discrete FD equations needed for this heat transfer (HT) problem, being careful to treat the boundary nodes as special cases (i.e. for i = 1 and i = N).

b. Now, implement the discrete equations developed in Part a along with the discrete geometry information and actually solve the resultant matrix equations, $\underline{\underline{A}}\,\underline{T} = \underline{B}$, for the unknown discrete temperature profile, $\underline{T}$, within the pipe. Plot the temperature profile (including the two known endpoint temperatures) for this problem. Does your solution make sense?

c. By simply integrating the ODE twice, also determine the analytical solution to this conduction HT problem, apply the BCs to determine the arbitrary coefficients within the general solution, and evaluate and plot this exact solution for the conditions given above. Put the numerical and exact analytical solutions on the same plot axes. Does the FD solution from Parts a and b compare favorably to the analytical solution? Explain any observed differences.

Documentation

Documentation for this assignment should include any hand calculations/derivations (i.e. be sure to show the development of any equations programmed into Matlab), answers to any specific questions given, a listing of the Matlab script and function files generated, the resultant Matlab plots, and a brief description of the data and results of your analysis for each problem. An overall professional job is expected!

See HW#1 for a description of the expected format -- every HW in this course should follow these basic instructions…