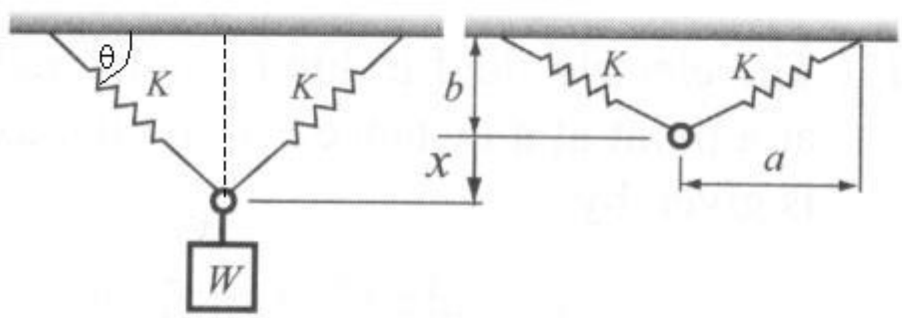
Applied Engineering Problem Solving (CHEN.3170)

Homework Assignment #5 -- Fall 2017

Root Finding Techniques and Applications

Problem #1: Comparison of Different Root Finding Methods (Weight on a Scale)

A scale is made from two springs as shown in the sketch. Initially, the springs are not stretched. When an object is attached to the ring, the springs stretch, and the ring is displaced downward a distance x.



The weight of the object placed on the scale can be expressed in terms of the vertical distance, x, by performing a force balance on the system with the weight, W, included. For equilibrium, the downward force due to the weight must be balanced by the upward force due to the springs, or

$$W = 2K\Delta L \sin\theta \qquad (1)$$

where ΔL is the stretch along the length of the springs, $K\Delta L \sin\theta$ is the vertical component of the spring force for one of the springs, and the factor of 2 accounts for the two springs.

Now, $\sin\theta$ can be written as the ratio of the length of the side opposite angle θ and the hypotenuse of the right triangle shown in the sketch, or

$$\sin\theta = \frac{b+x}{L} \qquad (2)$$

where

$$L = \sqrt{a^2 + (b+x)^2} \qquad (3)$$

Finally, we can write ΔL as

$$\Delta L = L - L_o \qquad (4)$$

with

$$L_o = \sqrt{a^2 + b^2} \qquad (5)$$

as the unstretched length. Substituting the appropriate expressions into eqn. (1) gives

$$W = \frac{2K}{L}(L - L_o)(b + x) \tag{6}$$

With L and L_o given by eqns. (3) and (5), this gives a straightforward ***explicit*** relationship for W as a function of x. However, if x is the unknown, for a given W, then eqn. (6) represents an ***implicit*** equation for the vertical deflection, x, as a function of W.

To illustrate your understanding of the difference between explicit and implicit equations, answer the following questions for a weight and scale system with the following design constants:

a = 0.15 m b = 0.05 m and K = 2800 N/m

a. What is the weight of an object placed on the scale if the observed value of x is 0.2 m?

b. For a known weight, W = 250 N, what will be the observed vertical deflection, x? As a comparison of the various root-finding methods discussed in class, you should use the ***bisection***, ***secant***, and ***fzero*** routines to answer this question. Do the solutions to the classical textbook methods (bisection and secant techniques) agree with Matlab's more sophisticated ***fzero*** routine? Note that **no changes are needed** to the ***bisection*** and ***secant*** routines available on the course website and, of course, this statement is also true for the Matlab built-in ***fzero*** routine.

c. Now, compare the relative efficiency of the various methods by tabulating the number of function evaluations needed to obtain the desired solution to Part b. Are your results as expected based on your knowledge of the different techniques? Explain...

Be sure to summarize your results and to include a copy of your m-file for this problem (**Note:** Please do not include printed version of the ***bisection*** and ***secant*** routines). Also be sure to document any programs you write with internal comments so that it is easy for a reviewer to understand what you have done.

Problem #2: Friction Factor Correlations for Smooth Pipes

There are several friction factor correlations that are used for internal fluid flow calculations. The universally accepted correlation for turbulent flow in smooth pipes is given by the Colebrook-White equation, as given below:

Colebrook-White Correlation: $$\frac{1}{\sqrt{f}} = 2\log_{10}\left(Re\sqrt{f}\right) - 0.8$$

where Re is the Reynolds number and f is the dimensionless friction factor.

Two other commonly used correlations are the Blasius equation and the Swamee-Jain correlation, which are given below:

Blasius Correlation: $$f = \frac{0.3164}{Re^{0.25}}$$

Swamee-Jain Correlation: $$f = \frac{0.25}{\left[\log_{10}\left(\frac{5.74}{Re^{0.9}}\right)\right]^2}$$

Compare these three expressions for the friction factor in a smooth pipe over a range of Reynolds numbers from 10^4 to 10^7. In particular, create a well-labeled plot that shows f vs. Re for the three cases. To highlight the variation of the alternate forms relative to the Colebrook-White correlation, you should also create a plot that gives the ratio f_B/f_{CW} and f_{SJ}/f_{CW} vs. Re, where f_B, f_{SJ}, and f_{CW} refer to the friction factors computed with the Blasius equation, Swamee-Jain expression, and the Colebrook White correlation, respectively. Be sure to comment on your method of calculation and your results. Note that the Colebrook-White correlation is considered the most accurate relationship, with a known uncertainty of about ±10% over the range of Reynolds numbers used here. Based on this statement and the results generated here, which relationship would you recommend using in routine applications? Explain…

Problem #3: Nonlinear Steady State Heat Transfer

The coating on a flat plate is cured by exposure to an infrared lamp providing a uniform surface irradiation of 2000 W/m^2. The coating absorbs 80% of the incident irradiation. The coating material has an emissivity ε = 0.50 and it is exposed to a combined convective and radiation environment with a bulk fluid temperature of 20 C and a surrounding temperature of 30 C.

In steady state, the rate of energy absorption in the coating material must equal the rate of energy loss by convection and radiation. We can write this mathematically as

$$\alpha G - q_{conv} - q_{rad} = 0 \qquad (1)$$

where G is the incident radiation per unit area from the lamps and α is the fraction of incident energy absorbed in the coating. We can write the heat transfer rates per unit area for convection and radiation heat transfer as

$$q_{conv} = h(T - T_\infty) \qquad \text{and} \qquad q_{rad} = \varepsilon\sigma(T^4 - T_{sur}^4) \qquad (2)$$

where T is the steady state coating temperature, T_∞ is the bulk fluid temperature, and T_{sur} is the temperature of the surroundings for radiation heat transfer between the coating material and the surroundings. In eqn. (2), h is the heat transfer coefficient that characterizes the convective cooling environment, and ε and σ are the material's emissivity and the Stefan-Boltzmann constant, respectively.

Putting the expressions in eqn. (2) into eqn. (1) gives

$$F(T) = \alpha G - h(T - T_\infty) - \varepsilon\sigma(T^4 - T_{sur}^4) = 0 \qquad (3)$$

which is clearly a nonlinear equation (because of the radiation heat transfer component). Equation (3) represents a classical root finding problem -- that is, "What is the value of T such that F(T) = 0?".

Given the Stefan-Boltzmann constant, σ = 5.67x10^{-8} W/m^2-K^4, and the above problem description, answer the following questions:

a. The wear and durability characteristics of the coating are sensitive to the temperature at which curing occurs. The cooling air velocity can be adjusted to change the convective heat transfer coefficient, which effectively changes the curing temperature. You are asked to provide this design data by computing and plotting the surface temperature as a function of h for $0 < h < 200$ W/m^2-K. What value of h would provide a cure temperature of about 40 C?

b. To get a better feel for the actual heat transfer processes that are important here, you should also create a formatted table of data that shows the three components in eqn. (1) and the resultant steady state temperature as a function of the heat transfer coefficient. From these data, can you explain what is happening for low h? What about for high h? Please elaborate a little on your results…

Note: For problems involving radiation heat transfer, all the temperatures must use an absolute scale during the calculational step, where T(K) = T(C) + 273.15. However, when displaying results, one often uses centigrade units for the temperatures (thus, use ^oC in the table and plot requested above).

Documentation

Documentation for this assignment should include a listing of the Matlab script and function files, the resultant Matlab plots and/or tabular data, as appropriate, and a brief description of the data and results of your analyses for each of the problems. An overall professional job is expected!

See HW#1 for a description of the expected format -- every HW in this course should follow these basic instructions…