Applied Engineering Problem Solving (CHEN.3170)

Homework Assignment #4 -- Fall 2017

Numerical Errors

Problem #1: Binary, Decimal, and Other Numbers

a. Find the decimal equivalent of $(1011101)_2$.

b. Find the binary equivalent to $(412)_{10}$.

c. Consider a number system with 6 digits -- a base 6 system. What is the equivalent of $(412)_{10}$ in this system?

Problem #2: Round Off vs. Truncation Error

Define and contrast the terms ***truncation error*** and ***round off error***. What are the primary mechanisms for reducing these two forms of error? Discuss the tradeoffs involved here and identify the dominant contribution to the total numerical error for most realistic applications. Be sure to fully explain the key concepts involved. Carefully make a labeled sketch of total error versus step size to help explain the tradeoff in round off error and truncation error for applications involving differential equations.

Problem #3: Taylor Series Derivative Approximations

Given the function: $f(x) = x^5 - 2x^4 + 3x^2 - 1$

a. Compute the exact derivative of f(x) at x = 2.1.

b. Estimate the first derivative of f(x) at x = 2.1 using a **forward** Taylor series approximation with the following step sizes:

$$h = [10^{-4}\ \ 10^{-3}\ \ 10^{-2}\ \ 10^{-1}\ \ 10^{0}\ \ 10^{1}]$$

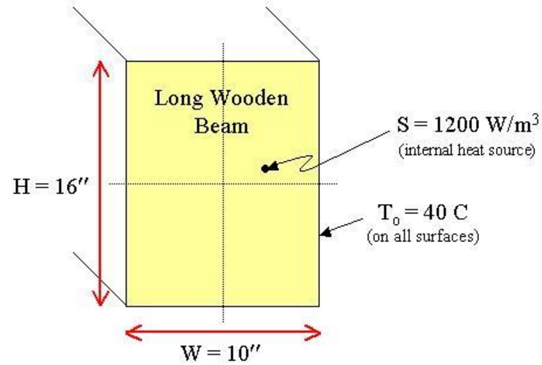
Using the exact result from Part a, make a plot of the magnitude of the relative error in $f'(x)\big|_{x=2.1}$ versus step size -- use a log-log plot. From the plot, determine the value of n in the error expression, $\varepsilon = \alpha h^n = O(h^n)$. Explain!!!

c. Repeat Part b using a **central** Taylor series approximation for $f'(x)\big|_{x=2.1}$. What is the "order of error" in this case? Plot the error vs. step size for this case in the **same plot as above**. Explain your results!!!

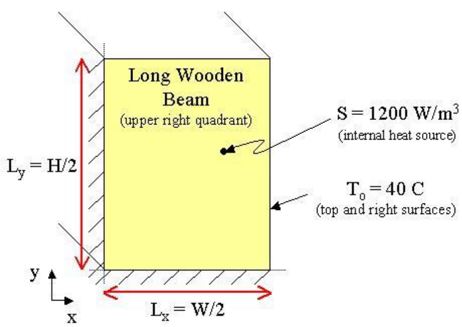
Note: You may want to use Matlab's ***polyval*** function to evaluate the polynomials for this problem (see ***help polyval***).

Problem #4: Evaluating and Plotting 2-D Temperature Distributions

Consider a long rectangular wooden beam that is heated by microwave radiation for drying. We will assume that this internal heating can be modeled as a uniformly distributed energy source, S = 1200 W/m^3. The surface of the wooden beam is kept at a fixed temperature of 40 C by passing air over all the wood surfaces. The wood has a thermal conductivity of about 0.16 W/m-C and it has a cross sectional area of 10″ x 16″. The beam's length is very large compared to the cross sectional area. A sketch of the beam's 2-D cross section is shown on the right.



Due to symmetry, we can position the origin of an x-y coordinate system at the center of the block and only treat the upper right quadrant in the above figure. This modeling technique is convenient in many practical applications, since it reduces the size of the computational domain, and the symmetry condition (i.e. no heat transfer) on the left and bottom surfaces of the new geometry are relatively easy to model. Thus, for this case, the sketch of the geometry becomes that shown on the left, where the defining energy balance equation and corresponding boundary conditions (BCs) for this situation can be summarized as follows:



$$\frac{\partial^2}{\partial x^2}T(x,y)+\frac{\partial^2}{\partial y^2}T(x,y)=-\frac{S}{k} \tag{1}$$

with

BC#1: $\left.\frac{\partial}{\partial x}T(x,y)\right|_{x=0}=0$ (no heat transfer on left surface)

BC#2: $\left.\frac{\partial}{\partial y}T(x,y)\right|_{y=0}=0$ (no heat transfer on bottom surface)

BC#3: $T(L_x,y) = T_R = 40$ C (right surface at 40 C)

BC#4: $T(x,L_y) = T_T = 40$ C (top surface at 40 C)

The solution to this PDE with the given BCs gives the desired temperature distribution, T(x,y), throughout the upper right quadrant of the long wooden beam. This problem can be solved analytically using the Separation of Variables (SOV) method, with the result given in the form of an infinite series expansion. The SOV solution can be summarized with the following expressions:

$$T(x,y)=v(x,y)+w(x) \tag{2}$$

where

$$w(x) = T_o + \frac{S}{2k}\left(L_x^2 - x^2\right) \tag{3}$$

$$v(x,y) = \sum_{n=1}^{\infty} B_n \cos(\lambda_n x)\cosh(\lambda_n y) \tag{4}$$

with

$$\lambda_n = \frac{(2n-1)\pi}{2L_x} \qquad \text{and} \qquad B_n = \frac{2S}{kL_x}\left[\frac{(-1)^n}{\lambda_n^3 \cosh(\lambda_n L_y)}\right] \tag{5}$$

where we have set $T_o = T_R = T_T = 40$ C for this case.

1. Now, your primary task for this problem is to evaluate the above expressions for T(x,y) and to plot the resultant temperature profile, as appropriate, so that one can easily visualize the 2-D temperature distribution within the wooden beam both quantitatively (with a 2-D plot) and qualitatively (with one or more 3-D views) -- and be sure to properly label your plots!!!

   Based on your plots, what is the maximum temperature in the beam and where does it occur? Are these reasonable? In general, you should rationalize that your plots make sense based on your physical understanding of this relatively simple heat transfer problem. Also address how many terms are needed in the series to converge to within a maximum error in the temperature profile of about 0.01 C.

2. **Optional (up to an extra 5 points):** With success with Part 1, you should now compute, evaluate, and plot the heat flux vector for this 2-D problem, where $\vec{q}(x,y)$ is given by

   $$\vec{q}(x,y) = q_x\hat{i} + q_y\hat{j} = -k\left(\frac{\partial T(x,y)}{\partial x}\hat{i} + \frac{\partial T(x,y)}{\partial y}\hat{j}\right)$$

   Since $\vec{q}(x,y)$ is a vector quantity, plotting this as a vector field with Matlab's **quiver** command often gives a useful visualization. In addition, we know that the heat flux vector at a point on an isothermal surface must be perpendicular to the surface, and it must point in the direction of decreasing temperature. Thus, a plot of the temperature contours on top of the heat flux vector field is particularly interesting. Therefore, for this problem, you should try to visualize the heat flux and temperature contours together as suggested here. Does your plot behave as expected? **Hint:** You might try **axis equal** or **axis image** to force the x and y axes to have the correct aspect ratio (i.e. to be representative of the real geometric dimensions).

Note: As a final comment (and strong hint), you might want to revisit Matlab's **meshgrid** command in formulating your solution to this problem. In addition, I would suggest that a thorough review of the **planewall_1.pdf** file, **Evaluating and Plotting Space-Time Temperature Distributions**, should be a great resource for addressing the solution to this problem. Also, be careful with units! Good Luck -- this should be fun…

Documentation:

Documentation for this assignment should include a listing of the Matlab script and function files, the resultant Matlab plots and/or tabular data, as appropriate, and a brief description of the data and results of your analyses for each of the problems. Note that no Matlab work is required for Problems 1 and 2, but be sure to include the hand calculations and complete discussions for these problems. For Prob. #3, be sure to include a good discussion of your results with a focus on the error vs step size relationships for both the forward and central derivative approximations. Finally, for Prob. #4, the challenge here is simply being successful in evaluating the given functions correctly and in presenting the T(x,y) distribution in a number of informative plots -- and, for the optional part of the problem, in figuring out how to calculate and present a 2-D vector quantity. As usual, an overall professional job is expected!

See HW#1 for a description of the expected format -- every HW in this course should follow these basic instructions…