CHEN.3170 Applied Engineering Problem Solving
Exam #2 Fall 2017

Problem 1 Working with Taylor Series -- Derivative Approximations (30 points)

A Taylor series expansion for the functions f(x+h) and f(x−h) can be written in terms of the function f(x) and all its derivatives evaluated at the point x, where $h = |\Delta x|$. Using a discrete notation (i.e. f_i, f_{i+1}, etc.), for convenience, these are given as follows:

$$f_{i+1} = f_i + f_i' h + \frac{f_i'' h^2}{2!} + \frac{f_i''' h^3}{3!} + \frac{f_i'''' h^4}{4!} + \cdots \qquad \text{(Forward Taylor Series)}$$

and

$$f_{i-1} = f_i - f_i' h + \frac{f_i'' h^2}{2!} - \frac{f_i''' h^3}{3!} + \frac{f_i'''' h^4}{4!} - \cdots \qquad \text{(Backward Taylor Series)}$$

a. Now, using the expressions from above, **derive *forward***, ***backward***, and ***central*** difference approximations to df/dx (i.e. the **first derivative**) at the point x_i, and obtain an estimate of the order of error in each approximation. **This should be a formal development!**

b. The temperature of an object is measured in an experiment at several different time points as noted in the table below. From the tabulated data, estimate the temperature gradient at each point (that is, compute dT/dt at each point and insert the value in the table). Explain the logic and equations used within the context of your solution to Part a (use the space below the table for your calculations and explanations, as needed).

Table I Experimental Data – Temperature vs Time

Point	**Time (sec)**	**Temperature (ºF)**	**Temperature Gradient (ºF/s)**
1	0	70.0	
2	3	74.2	
3	6	77.5	
4	9	81.1	
5	12	84.1	

Problem 2 Truncation Error versus Step Size (20 points)

When using the finite difference (FD) method to solve differential equations, one often needs to incorporate derivative approximations of different "order of error". As such, the composite truncation error behavior in a result that uses the FD solution can only be determined via numerical experiments, where the truncation error is given by $\varepsilon = O(h^n) = \alpha h^n$, with h being the step size used in the model.

a. For a particular case of interest, we have the following numerical results:

$h_1 = 0.100$ $\varepsilon_1 = 0.0200$ and $h_2 = 0.060$ $\varepsilon_2 = 0.0084$

Determine the "order of error", n, for this situation.

b. With the result from Part a, estimate the step size, h, that would be needed to reduce the error in the calculation to below $\varepsilon = 0.001$. (Note: If you were not successful with Part a, assume a reasonable value of n to get some partial credit for this part of the problem).

Problem 3 Finding Real Roots of Nonlinear Equations (25 points)

Use the **Bisection Method** to find a root of

$$f(x) = x - x^{1/3} - 2 = 0$$

within the range of 0 and 5. This should be a formal development that illustrates your understanding of the **Bisection Method** -- as applied to a specific problem. You should continue the basic algorithm until the value of x is known to within ±0.35 (that is, $b - a \le 0.7$, where a and b are the current lower and upper bounds on x, respectively). **Note that specific calculations, not a generic program, are required here.**

Note that my interest here is in evaluating your understanding of the solution methodology, not the specific answer to this root finding problem. Thus, an answer without a clear demonstration of the proper iteration process is not worth much!

Problem 4 Numerical Solution of ODEs (25 points)

Setup the finite difference (FD) equations for the following variable coefficient 2[rd] order initial value problem (IVP) where $0 \le x \le 10$. Also briefly outline/discuss the solution algorithm, with particular focus on getting the information needed for starting the algorithm. Your algorithm should give a good overview of the solution procedure, but **no formal code is needed here**.

$$y'' + 5xy' - 7y = 3x^2 - 2 \qquad \text{where} \qquad y(0) = 2 \quad \text{and} \quad y'(0) = -5$$

Note: The second-order FD derivative approximations needed for this problem are:

$$y''_i = \left.\frac{d^2y}{dx^2}\right|_i = \frac{y_{i-1} - 2y_i + y_{i+1}}{\Delta x^2} \qquad \text{and} \qquad y'_i = \left.\frac{dy}{dx}\right|_i = \frac{y_{i+1} - y_{i-1}}{2\Delta x}$$