CHEN.3170 Applied Engineering Problem Solving

A Short Quiz on

Working with Taylor Series -- Derivative Approximations

A Taylor series expansion for the functions f(x+h) and f(x–h) can be written in terms of the function f(x) and all its derivatives evaluated at the point x, where $h = |\Delta x|$. Using a discrete notation (i.e. f_i, f_{i+1}, etc.) for convenience, these are given as follows:

$$f_{i+1} = f_i + f_i' h + \frac{f_i'' h^2}{2!} + \frac{f_i''' h^3}{3!} + \frac{f_i'''' h^4}{4!} + \cdots \qquad \text{(Forward Taylor Series)}$$

and

$$f_{i-1} = f_i - f_i' h + \frac{f_i'' h^2}{2!} - \frac{f_i''' h^3}{3!} + \frac{f_i'''' h^4}{4!} - \cdots \qquad \text{(Backward Taylor Series)}$$

a. Now, using the expressions from above, **derive *forward***, ***backward***, and ***central*** difference approximations to df/dx (i.e. the **first derivative**) at the point x_i, and obtain an estimate of the order of error in each approximation. **This should be a formal development!**

b. The velocity profile in a pipe flow problem is measured in an experiment at several points between the pipe center at r = 0 and the pipe wall at r = 2.5 inches as noted in the table below. Assuming symmetry at r = 0, estimate the velocity gradient at each point (that is, compute dv/dr at each point and insert the value in the table). Explain the logic and equations used within the context of your solution to Part a.

Table I Experimental Data – Velocity vs. Position

Point	**Position (inches)**	**Velocity (in/s)**	**Velocity Gradient (in/s per in)**
1	0.0	5.0	
2	0.5	4.8	
3	1.0	4.2	
4	1.5	3.2	
5	2.0	1.8	
6	2.5	0.0	