Friction Effects in Pipe Flow

Consider the tank and pipe system shown in the sketch (not to scale). For a given height, h, of fluid in the tank and a given length of drainpipe, L, an estimate of the exit velocity, V, can be derived from a macroscopic energy balance between point 1 at the surface of the fluid and point 2 at the end of the pipe. Assuming steady flow, the general energy equation can be written as

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} + h_A - h_R - h_L = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

where the notation is standard (see eqn. 7-3 in Applied Fluid Mechanics (6th Ed.) by R. L. Mott, for example). Since there are no mechanical devices (i.e. $h_A = h_R = 0$), the velocity at point 1 is essentially zero (because of the large surface area), and $p_1 = p_2$ (both points are open to the atmosphere), the general energy equation for this problem reduces to

$$\frac{v_2^2}{2g} = z_1 - z_2 - h_L \tag{1}$$

Defining $h + L = z_1 - z_2$ and $V = v_2$, and using Darcy's equation for the friction loss term, h_L, (see eqn. 8-3 in Mott 6th Ed.),

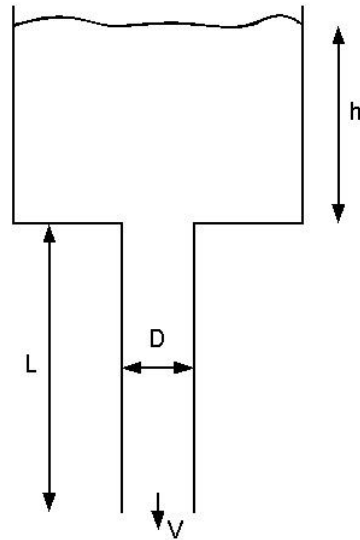$$h_L = f \frac{L}{D} \frac{V^2}{2g} \tag{2}$$

eqn. (1) becomes

$$\frac{V^2}{2g} = h + L - f \frac{L}{D} \frac{V^2}{2g}$$

or

$$V^2 = \frac{2g(h+L)}{\left(1 + f \dfrac{L}{D}\right)} \tag{3}$$

In eqn. (3), g is the gravitational constant (32.2 ft/s^2) and the friction factor, f, for turbulent flow is given approximately by the Swamee-Jain correlation (see eqn. 8-7 in Mott 6th Ed.),

$$f = \frac{0.25}{\left[\log_{10}\left(\dfrac{\varepsilon/D}{3.7} + \dfrac{5.74}{Re^{0.9}}\right)\right]^2} \tag{4}$$

where ε/D is the relative roughness, Re is the Reynolds number, $\text{Re} = VD/\nu$, and ν is the fluid's kinematic viscosity.

Now, we would like to study how the velocity varies as a function of the pipe length, L, and diameter, D. To do this, let's assume that the fluid of interest is water at 60 F (with ν = $1.21\text{x}10^{-5}$ ft^2/s). Also assume that the tank is large and that the water is being drained through Sch 40 steel pipe ($\varepsilon = 1.5\text{x}10^{-4}$ ft). With these parameters and h = 5 ft, our task is to generate a plot of the exit velocity versus length of the drainpipe. In particular, we will plot V(L) over the range 1/12 ft < L < 20 ft and, to see the influence of the pipe diameter, we will use three different Sch 40 pipe sizes:

$$1/2" \text{ pipe} \rightarrow D_1 = 0.0518 \text{ ft} \qquad 1" \text{ pipe} \rightarrow D_2 = 0.0874 \text{ ft} \qquad 1-1/2" \text{ pipe} \rightarrow D_3 = 0.1342 \text{ ft}$$

A Matlab program, **pipe_friction_1.m**, was written to accomplish the goals described above. Since eqn. (3) has a complicated nonlinear dependence on velocity (through the friction factor), Matlab's *fzero* routine was used to find V such that f(V) = 0. The function file for use by *fzero* is **pipe_friction_1a.m**. Both the main program and function file are listed in Table 1.

As in our other examples, the program structure is quite straightforward. After some initial comments to describe the overall program function, we first initialize the basic problem parameters, solve for the exit velocity using *fzero* for each length and diameter combination, and then do some minimal post processing and plotting of the key results. All the real computational work is being handled by the *fzero* root-finding routine and our user-defined function, **pipe_friction_1a.m**, that simply returns the value of the implicit function, f(V), for each input value of velocity. The double *for … end* loop in the main program allows us to compute the proper value of V for each L and D combination, with the result stored in a 2-D array. After the velocities are known, the friction factor and Reynolds number are re-computed in the main program and plotted along with the exit velocity -- to help explain the V vs. L and D results (recall that, since all the variables within a function are local variables, the intermediate results are not available to the main program, and any quantities of interest must be recomputed once the primary variables have been determined).

Table 1 Listing of pipe_friction_1.m and pipe_friction_1a.m.

```
%
%   PIPE_FRICTION_1.M   Fluid Flow Application Illustrating Pipe Friction
%
%   The goal of this problem is to compute and plot the exit velocity associated
%   with a tank and pipe system.  The problem is highly nonlinear because of the
%   dependence of the friction factor on the flow velocity.
%
%   The equation of interest is:
%           V^2 = 2g(h+L)/(1 + fL/D)
%   where f is the friction factor (given by the Swamee-Jain correlation)
%
%   Here we will use three different pipe sizes and let L vary to determine
%   V vs L.   We expect V to increase with increasing L since the overall head,
%   (h + L), is increasing.  However, with larger L, the friction in the pipe
%   tends to decrease the flow velocity V.  Thus, there is a trade-off here,
%   and the goal of this problem is to study this phenomenon.  Let's see if we
%   can discover this relationship via graphical analysis.  Also, we want to see
%   how this changes with the pipe diameter, D, where increasing D should
%   reduce the effect of friction.  Let's give it a try...
%
%   File last modified by J. R. White, UMass-Lowell (last update: Nov. 2017)
```

```
%
      clear all, close all, nfig = 0;
      global DD h LL g visc rough
%
%   initial data for problem
      D = [0.0518 0.0874 0.1342];          % different pipe diameters (ft)
      h = 5;                               % height of fluid in tank (ft)
      g = 32.2;                            % acceleration constant (ft/s^2)
      visc = 1.21e-5;                      % kinematic viscosity of 60 F water (ft^2/s)
      rough = 1.5e-4;                      % steel pipe roughness (ft)
      NL = 50;  L = linspace(1/12,20,NL)'; % setup dependent variable (pipe length)
%
%   let's loop over the number of pipe sizes
      ND = length(D);   V = zeros(NL,ND);
      for n = 1:ND
        DD = D(n);
%   now use fzero to solve for V for each value of L and D (uses previous V as guess)
        Vo = sqrt(2*g*h);        % initial guess for V for L = 0 ft
        LL = L(1);  V(1,n) = fzero(@pipe_friction_1a,Vo);
        for i = 2:NL
          LL = L(i);  V(i,n) = fzero(@pipe_friction_1a,V(i-1,n));
        end
      end
%
%   now plot the results
      nfig = nfig+1;   figure(nfig)
      plot(L,V(:,1),'r-',L,V(:,2),'b--',L,V(:,3),'g:','LineWidth',2)
      title('Pipe\_Friction\_1:  Exit Velocity vs Pipe Length')
      xlabel('Pipe Length (ft)'),ylabel('Exit Flow Velocity (ft/s)'), grid
      legend('Sch 40 1/2" pipe','Sch 40 1" pipe','Sch 40 1-1/2" pipe')
%
%   just for fun, let's calc Re and f for these cases
      Re = zeros(NL,ND);   f = zeros(NL,ND);
      for n = 1:ND
        Re(:,n) =  V(:,n)*D(n)/visc;                            % Reynolds number
        RR = rough/D(n);                                        % relative roughness
        f(:,n) = 0.25./(log10(RR/3.7 + 5.74./Re(:,n).^0.9)).^2; % friction factor
      end
%
%   now plot these results
      nfig = nfig+1;   figure(nfig)
      plot(L,Re(:,1),'r-',L,Re(:,2),'b--',L,Re(:,3),'g:','LineWidth',2)
      title('Pipe\_Friction\_1:  Reynolds Number vs Pipe Length')
      xlabel('Pipe Length (ft)'),ylabel('Reynolds Number'), grid
      legend('Sch 40 1/2" pipe','Sch 40 1" pipe','Sch 40 1-1/2" pipe')
%
      nfig = nfig+1;   figure(nfig)
      plot(L,f(:,1),'r-',L,f(:,2),'b--',L,f(:,3),'g:','LineWidth',2)
      title('Pipe\_Friction\_1:  Friction Factor vs Pipe Length')
      xlabel('Pipe Length (ft)'),ylabel('Friction Factor'), grid
      legend('Sch 40 1/2" pipe','Sch 40 1" pipe','Sch 40 1-1/2" pipe')
%
%   end of simulation


%
%   PIPE_FRICTION_1A.M   Function file for use in FZERO
%
%   Evaluates   F(V) = V^2 - 2g(h+L)/(1 + fL/D)
%   where f is friction factor (given by the Swamee-Jain correlation)
%
      function F = pipe_friction_1a(V)
      global DD h LL g visc rough
      Re = V*DD/visc;                      % Reynolds number
      RR = rough/DD;                       % relative roughness
      f = 0.25/(log10(RR/3.7 + 5.74/Re^0.9))^2;   % friction factor
      F = V^2 - 2*g*(h+LL)/(1 + f*LL/DD);
%
%   end of function
```

The key results of our analysis are presented in Figs. 1 and 2. Based on the functional dependence given in eqn. (3), we would expect V to increase with increasing L, since the overall head, (h+L), is increasing. However, with larger L, the increase in friction in the pipe tends to decrease the flow velocity, V. Thus, there is a trade-off, and the actual behavior is not so obvious! In fact, as seen in the figures, the V vs. L relationship is a strong function of the base friction in the system. Thus, we see that the smaller diameter pipes have a higher base friction and, for these cases, V is a decreasing function of L. However, for the 1.5″ Sch 40 pipe, where the larger pipe diameter has a lower base friction loss, we see that an increase in pipe length leads to an increase in flow. Thus, the behavior described by eqn. (3) is indeed rather complicated, and the graphical analysis presented here is essential for really understanding the flow behavior of this relatively simple tank and pipe system…
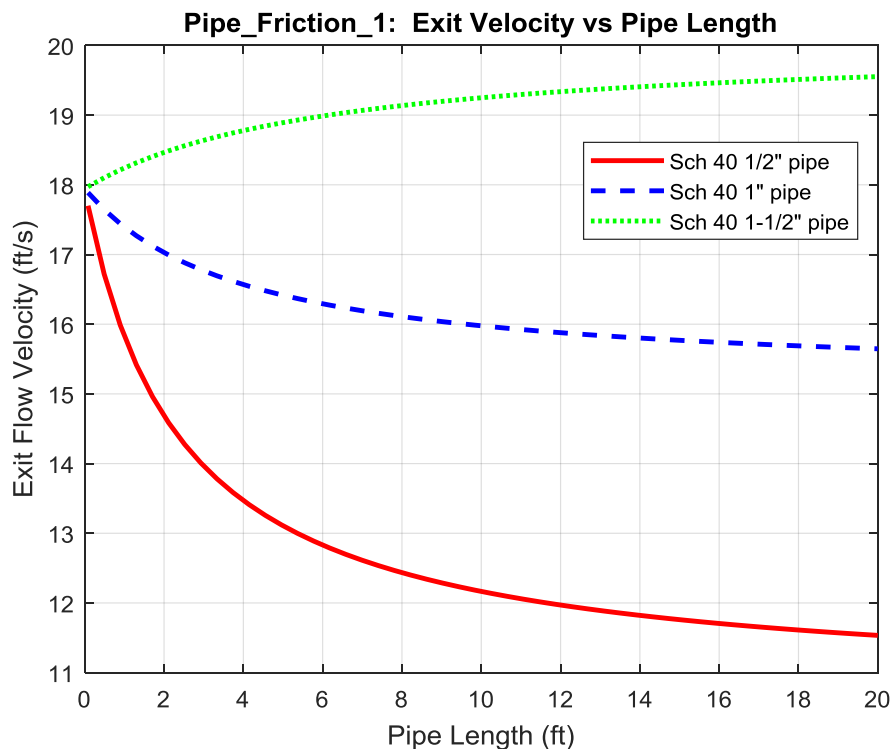

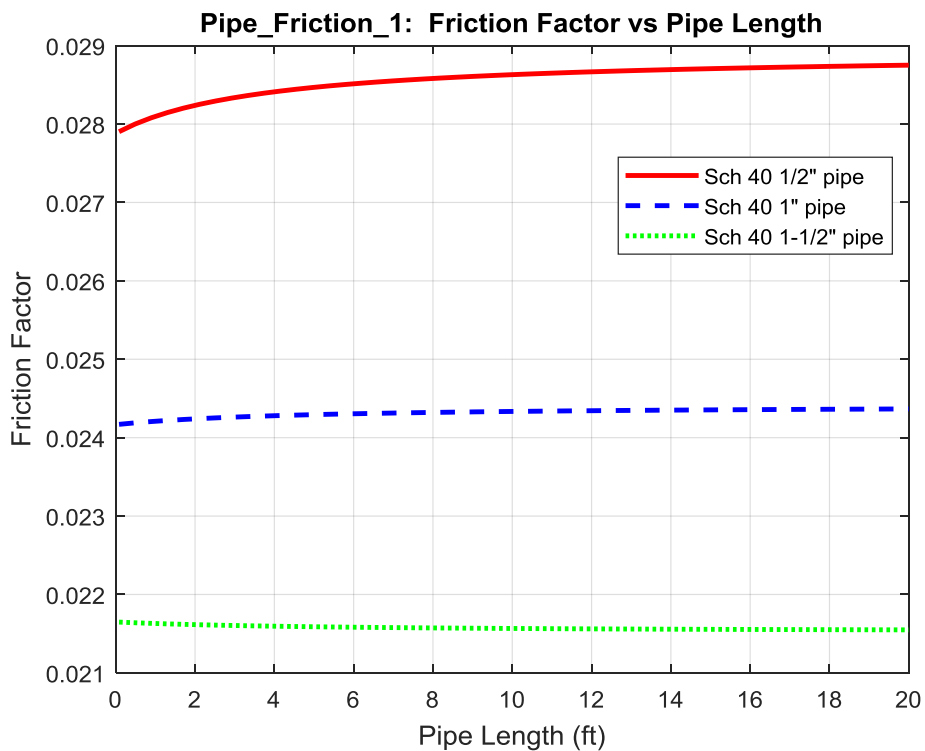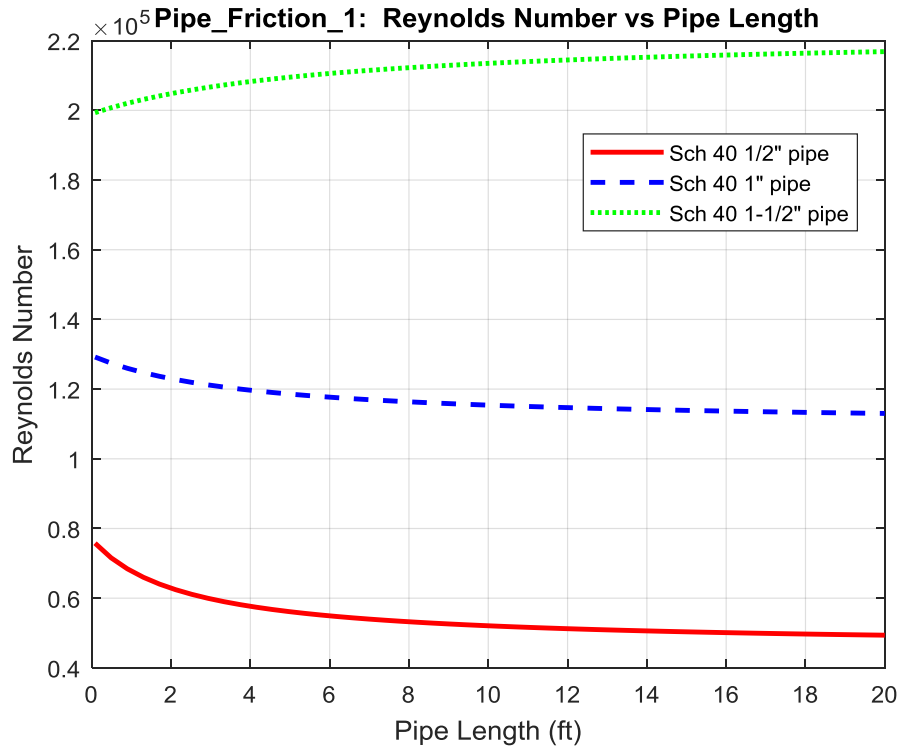**Fig. 1 Exit velocity vs. pipe length and diameter.**

Fig. 2 Reynolds number and friction factor vs. pipe length and diameter.