

Laminar vs. Turbulent Flow in a Pipe

Many fluid flow applications involve the transport of a gas or liquid through circular pipes. The flow behavior is usually classified as being either laminar or turbulent, depending on the value of the dimensionless Reynolds number, Re , where

$$Re = \frac{\rho v D}{\mu} \quad (1)$$

with ρ = fluid density, v = average velocity, D = pipe diameter, and μ = fluid viscosity. For $Re < 2000$, the flow is said to be laminar and it is usually associated with the smooth deterministic flow of a highly viscous or low density fluid. For turbulent flow, where $Re > 4000$, one observes irregular random fluctuations in the fluid velocity caused by local mixing (or turbulence) within the fluid, which usually has a low viscosity and/or a high velocity. For Re values in the intermediate range between the usual cutoffs for laminar and turbulent flow given here, one often observes random transient behavior characteristic of both flow regimes. Thus, the transition region is often more difficult to characterize explicitly -- so here we will focus only on fluid flow well within the laminar or turbulent flow regimes.

For flow in a circular pipe, the “no-slip” boundary condition at the pipe walls forces the fluid velocity to be zero at $r = R$, where R is the pipe radius. Thus, the velocity will increase from zero at the wall to some maximum value at the center of the pipe [at $r = 0$, $u(r) = u_{\max}$].

Now, for laminar flow, the actual velocity profile can be developed analytically (see any Fluid Mechanics text) and it is given by

$$u_L(r) = u_{L_{\max}} \left[1 - (r/R)^2 \right] \quad (\text{Laminar Flow}) \quad (2)$$

and a formal averaging process gives

$$u_{\text{ave}} = \frac{1}{A} \int_0^R u_L(r) 2\pi r dr = \frac{u_{L_{\max}}}{2} \quad (3)$$

where A is the flow area ($A = \pi R^2$ for flow in a circular pipe).

For turbulent flow, the exact velocity profile cannot be determined analytically. However, it can be measured experimentally and these data are usually fit to a smooth curve of the form,

$$u_T(r) = u_{T_{\max}} \left[1 - r/R \right]^m \quad (\text{Turbulent Flow}) \quad (4)$$

and again, taking a formal average gives

$$u_{\text{ave}} = \frac{1}{A} \int_0^R u_T(r) 2\pi r dr = \frac{2u_{T_{\max}}}{(m+1)(m+2)} \quad (5)$$

where m is an exponent that is only weakly dependent on the degree of turbulence (i.e. magnitude of Re). For fully developed turbulent flow (relatively high Re), a value of $m = 1/7$ is commonly used to represent the turbulent flow profile in eqn. (4).

In the above equations, the average velocity, $v = u_{\text{ave}}$, is usually available from the volumetric flow rate (which can be easily measured in most cases). Since $Q = vA$, we have

$$v = u_{\text{ave}} = \frac{Q}{A} \quad (6)$$

Now, our goal in this example is simply to evaluate, plot, and compare the laminar and turbulent velocity profiles associated with fluid flow in a pipe for a typical situation. An algorithm to do this is given below:

1. Define the volumetric flow rate, Q , and the pipe diameter, D .
2. Compute the pipe area and the average velocity via eqn. (6).
3. Knowing u_{ave} , compute u_{max} from eqn. (3) and eqn. (5), respectively, for laminar and turbulent flow.
4. Define a discrete spatial grid, r , and evaluate the laminar and turbulent flow profiles via eqn. (2) and eqn. (4), respectively (use element-by-element vector arithmetic).
5. Plot both curves on a single axis for ease of comparison.
6. Print some summary data (Q , u_{ave} , u_{max} for both cases, etc.) and interpret the overall results...

Note that this algorithm assumes that the average velocity is the same for both the laminar and turbulent profiles (same Q and D). This was done so that we could focus on the shape of the profiles (not the actual magnitude of the velocity). In many cases, however, turbulent flow is associated with a higher average velocity (and higher Q) -- although it doesn't have to be, since we could have the same Q and D but a different viscosity.

In any case, the above algorithm was implemented in **pipe_flow_1.m**, as shown in Table 1. The program is quite straightforward and it produces a single plot that is given in Fig. 1 and a brief summary table of results that is reproduced in Table 2. Since the velocity profile is symmetric, the visualization focuses on only half of the pipe over the range $0 \leq r \leq R$. As apparent, the laminar flow case has a parabolic profile with a peak velocity that is twice the average (note that the flow rate was chosen to give $u_{\text{ave}} = 2$ m/s in this example). The turbulent flow profile is much flatter over most of the pipe, with a larger gradient near the pipe wall. This is characteristic of the local mixing that causes a more uniform profile in the center of the pipe. Since the turbulent profile is more uniform, the maximum-to-average value is much smaller than for the laminar flow case (about 1.22 for turbulent flow compared to 2.0 for laminar flow).

Actually, there is really nothing overly surprising here, since you are probably already familiar with this subject -- since a lot of emphasis is placed on the distinguishing aspects of laminar and turbulent flow in any introductory Fluid Mechanics course. However, the ease with which we were able to plot and visualize these profiles with Matlab does indeed make a detailed comparison quite an easy task.

Table 1 Program listing for pipe_flow_1.m.

```

%
% PIPE_FLOW_1.M   Plots Laminar and Turbulent Velocity Profiles in a Pipe
%
% This is another example that illustrates function evaluation and plotting in
% Matlab. The goal here is simply to evaluate and plot the velocity profiles
% for laminar and turbulent flow in a circular pipe geometry.
%
% For laminar flow, the velocity profile is parabolic, where u(r) is given by
%   u(r) = umax*[1 - (r/R)^2]
% For turbulent flow the velocity profile is given by
%   u(r) = umax*(1 - r/R)^m
% where m = 1/7 for fully developed turbulent flow.
%
% In evaluating these functions, we will again illustrate Matlab's vector
% processing capabilities and some simple 2-d plotting functions available
% in Matlab. It also represents our first use of the fprintf command
% in Matlab. This is a relatively straightforward example from the field
% of Fluid Mechanics.
%
% File prepared by J. R. White, UMass-Lowell (last update: August 2017)
%
%
%   clear all, close all, nfig = 0;
%%
% define problem parameters
%   Q1 = 64.25;           % volumetric flow rate (gal/min)
%   Q2 = Q1/15850;      % convert Q to m^3/s
%   D = 2*0.0254;      % pipe diameter (inches converted to m)
%   R = D/2;           % pipe radius (m)
%   m = 1/7;           % exponent in turbulent flow eqns.
%
% compute some derived quantities
%   A = pi*R^2;         % pipe flow area
%   uave = Q2/A;        % ave velocity
%   uLmax = 2*uave;     % max velocity for laminar flow
%   uTmax = (m+1)*(m+2)*uave/2; % max velocity for turbulent flow
%%
% evaluate the laminar and turbulent velocity profiles
%   Nr = 101;   r = linspace(0,R,Nr); % discrete r vector
%   uL = uLmax*(1 - (r/R).^2); % laminar velocity profile
%   uT = uTmax*(1 - r/R).^m; % turbulent velocity profile
%%
% now plot the results (both profiles on the same plot)
%   nfig = nfig+1; figure(nfig)
%   plot(r/R,uL,'r-',r/R,uT,'g--','LineWidth',2), grid
%   title('Pipe_Flow_1: Laminar vs. Turbulent Velocity Profiles');
%   xlabel('Normalized Radial Position - (r/R)'),ylabel('Fluid Velocity (m/s)')
%   legend('Laminar Flow','Turbulent Flow')
%
% finally, let's print out a few summary results
%   fprintf(1,'\n Summary Results for the Pipe Flow Problem \n\n');
%   fprintf(1,' Volumetric flow rate (gal/min): %10.3f \n',Q1);
%   fprintf(1,' Volumetric flow rate (m^3/s): %10.3e \n',Q2);
%   fprintf(1,' Pipe diameter (inches): %10.3f \n',D/0.0254);
%   fprintf(1,' Average velocity (m/s): %10.3f \n',uave);
%   fprintf(1,' LAMINAR flow peak velocity (m/s): %10.3f \n',uLmax);
%   fprintf(1,' TURBULENT flow peak velocity (m/s):%10.3f \n',uTmax);
%
% end of program

```

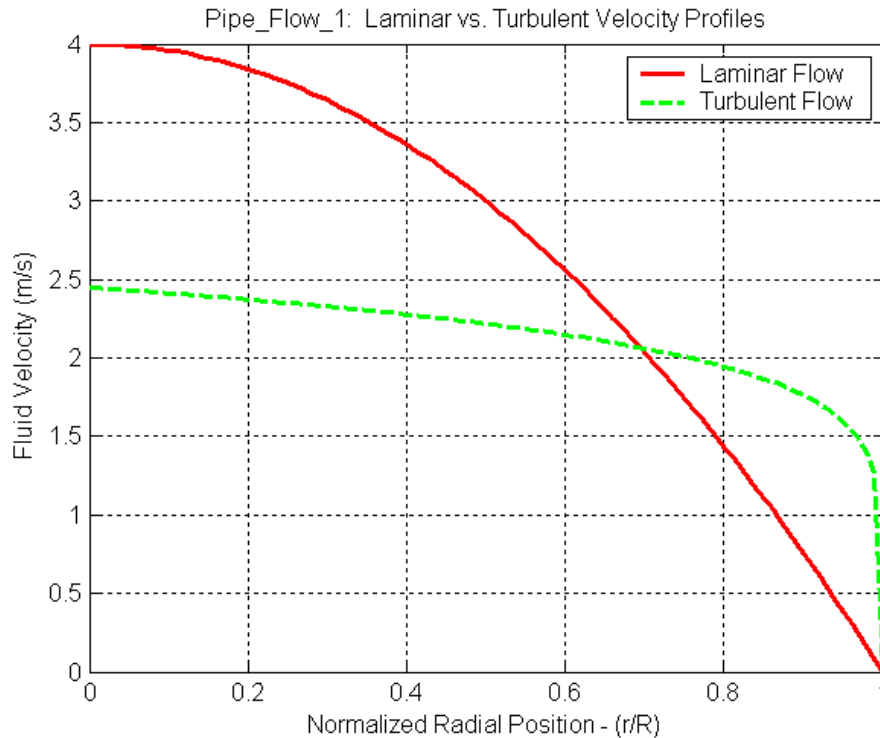


Fig. 1 Laminar and turbulent velocity profiles in a circular pipe.

Table 2 Summary results from the pipe_flow_1.m program.

```
>> pipe_flow_1

Summary Results for the Pipe Flow Problem

Volumetric flow rate (gal/min):      64.250
Volumetric flow rate (m^3/s):      4.054e-003
Pipe diameter (inches):             2.000
Average velocity (m/s):             2.000
LAMINAR flow peak velocity (m/s):   4.000
TURBULENT flow peak velocity (m/s): 2.449
```

The only new aspect in the **pipe_flow_1.m** program is the use of the *fprintf* command (see the last few lines of **pipe_flow_1.m**) to produce the summary quantitative results shown above in Table 2. This Matlab function is used to create formatted output to the screen or to an external file. Like a picture, “a specific example is worth a thousand words” -- so the intent here is for you to study this portion of the Matlab code so that you get a rough idea of how to use this relatively powerful (but somewhat awkward) Matlab function. We will use the *fprintf* command again in later examples to illustrate many additional capabilities, but this simple example is a good place to start (you can consult your texts or the Matlab *help* facility if you can't wait and really want to see further examples now...).

Well, this simple Fluid Mechanics application is now complete. Our focus here was on the evaluation, plotting, and comparison of the velocity profiles that occur during laminar and turbulent flow in a circular pipe. We will revisit this general subject area several times over the course of the semester, since many of the techniques that will be discussed in this course can be applied to a variety of practical Fluid Mechanics problems. Thus, as we learn how to use Matlab to solve practical engineering problems, we may also pick up some fluid mechanics fundamentals along the way -- and it is always nice to get a little extra mileage for your educational dollar!!!