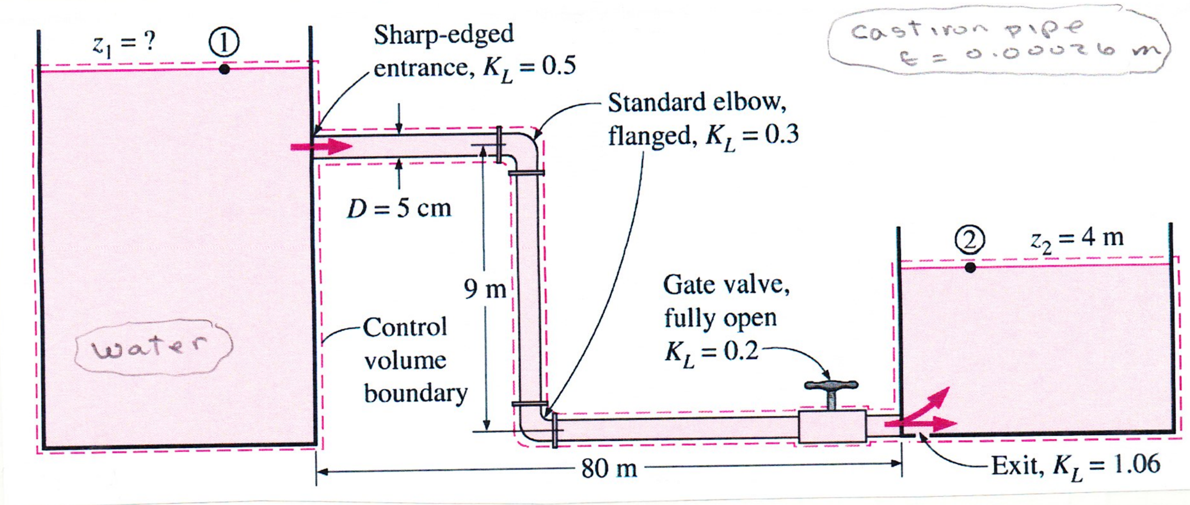
Example 8.8 in Cengel

Major + Minor losses

Example #1

Cast iron pipe
$\varepsilon = 0.00026$ m



Find the elevation z_1 needed for a flow rate of $6\times10^{-3}\ m^3/s$

Energy eqn between pts 1 and 2

$$\cancel{\frac{P_1}{\gamma}}^{0} + \cancel{\alpha\frac{V_1^2}{2g}}^{0} + z_1 + \cancel{h_A} - \cancel{h_R} - h_L = \cancel{\frac{P_2}{\gamma}}^{0} + \cancel{\alpha\frac{V_2^2}{2g}} + z_2$$

free surface — no mechanical devices — free surface

$$\therefore\ z_1 = z_2 + h_L = z_2 + f\frac{L}{D}\frac{V^2}{2g} + \left(\sum_i k_i\right)\frac{V^2}{2g}$$

for $V = \frac{Q}{A} = \frac{6\times10^{-3}\ m^3/s}{\left(\frac{\pi}{4}\right)(0.05m)^2} = \frac{6\times10^{-3}}{1.963\times10^{-3}} = \boxed{3.056\ \frac{m}{s}}$

$$\frac{V^2}{2g} = \frac{(3.056\ m/s)^2}{2(9.81\ m/s^2)} = \boxed{0.476\ m}$$

$\rho = 1000\ kg/m^3$

$\mu = 1.31\times10^{-3}\ \frac{N-s}{m^2}$

$$Re = \frac{\rho V D}{\mu} = \frac{\left(1000\frac{kg}{m^3}\right)\left(3.056\frac{m}{s}\right)(0.05m)}{1.31\times10^{-3}\frac{N-s}{m^2}}$$

$\boxed{Re = 1.166\times10^5}$ turbulent flow

$$\frac{\varepsilon}{D} = \frac{2.6\times10^{-4}\ m}{0.05\ m} = 0.0052$$

Haaland Eqn.

$$\frac{1}{\sqrt{f}} = -1.8 \log\left[\left(\frac{\epsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right]$$

$$= -1.8 \log\left[\left(\frac{0.0052}{3.7}\right)^{1.11} + \frac{6.9}{1.166\times10^5}\right]$$

$$= -1.8 \log\left[0.000644 + 0.000059\right]$$

$$\frac{1}{\sqrt{f}} = 5.653 \qquad \therefore \boxed{f = 0.0313}$$

use this

moody chart $f \simeq 0.032$

Now

major losses

$$f\frac{L}{D}\frac{V^2}{2g} = (0.031)\left(\frac{89\text{ m}}{0.05\text{ m}}\right)(0.476\text{ m}) = \boxed{26.3\text{ m}}$$

from diagram

For the minor losses, we have (from diagram)

$$\left(\sum_i k_i\right) = \underbrace{0.5}_{\text{entrance}} + \underbrace{2(0.3)}_{\text{2 elbows (flanged)}} + \underbrace{0.2}_{\text{gate valve (open)}} + \underbrace{1.06}_{\text{exit}} = 2.36$$

minor losses

$$\left(\sum_i k_i\right)\frac{V^2}{2g} = 2.36\,(0.476\text{ m}) = \boxed{1.12\text{ m}}$$

Thus, putting these values back into the reduced energy eqn gives

$$z_1 = z_2 + h_{L\,major} + h_{L\,minor}$$

$$= 4\text{ m} + 26.3\text{ m} + 1.1\text{ m}$$

$$\boxed{z_1 = 31.4\text{ m}}$$

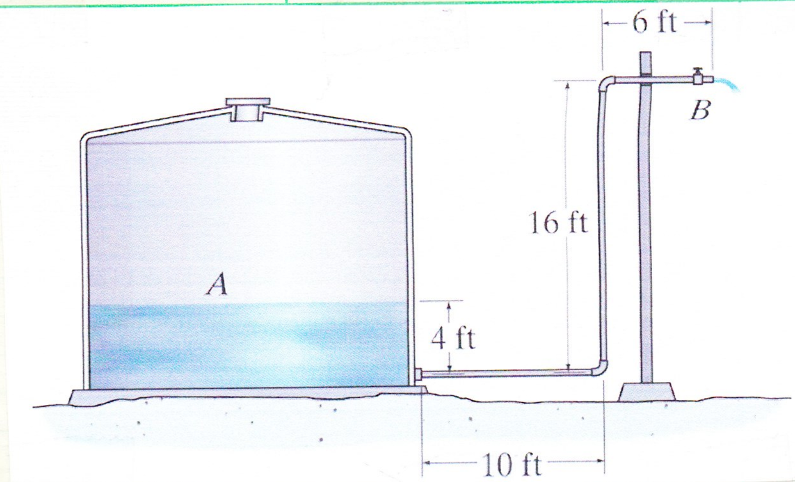
ans

The tank shown is pressurized to 40psig.

Minor losses in the system include a flush entrance, 2 elbows, and a fully open gate valve.

The pipe has a 2" diameter and is galvanized iron.

Estimate the water flow rate, Q. in gpm



Energy eqn

$$\frac{P_A}{\gamma} + \cancel{\frac{V_A^2}{2g}}^{\,0} + z_a + \cancel{h_A}_{\to 0} - \cancel{h_R}_{\to 0} - h_L = \cancel{\frac{P_B}{\gamma}}^{\,0} + \frac{V_B^2}{2g} + z_B$$

large reservoir — free jet

$$\frac{P_A}{\gamma} + (z_a - z_B) = h_L + \frac{V_B^2}{2g} = \left[f\frac{L}{D} + \left(\sum_i k_i\right) + 1\right]\frac{V^2}{2g}$$

$$\frac{V^2}{2g} = \frac{\frac{P_A}{\gamma} + (z_A - z_B)}{\left[f\frac{L}{D} + \left(\sum_i k_i\right) + 1\right]}$$

$\rho_w = 1.94$ slug/ft^3

$g = 32.2$ ft/s^2

$\gamma = \rho g = 62.4$ lbf/ft^3

some values

$$\frac{P_A}{\gamma} = \frac{\left(40\,\frac{lbf}{in^2}\right)\left(\frac{144\,in^2}{ft^2}\right)}{62.4\,\frac{lbf}{ft^3}} = 92.31\text{ ft}$$

$$z_A - z_B = 4 - 16 = -12\text{ ft}$$

$$\frac{L}{D} = \frac{32}{2/12} = 192$$

$$\frac{\epsilon}{D} = \frac{0.0005\text{ ft}}{(2/12)\text{ ft}} = 0.003$$ galvanized

$$\left(\sum_i k_i\right) = \underbrace{0.5}_{\text{sudden contraction}} + \underbrace{2(0.9)}_{90^\circ\text{ elbow}} + \underbrace{0.19}_{\text{open gate valve}} = 2.49$$

see pg 531 Hibbeler

$$V = \left(2\left(32.2\,\frac{ft}{s^2}\right)\left[\frac{92.3 - 12}{192f + 2.49 + 1}\right]ft\right)^{1/2}$$

$$\boxed{V = \sqrt{\frac{5171.3}{192f + 3.49}}\ \text{ft/s}}$$

but clearly f is related to V

∴ nonlinear eqn.

assume turbulent flow and Swamee-Jain correlation

$$f = \frac{0.25}{\left[\log\left(\frac{\epsilon/D}{3.7} + \frac{5.74}{Re^{0.9}}\right)\right]^2}$$

$$\frac{\epsilon/D}{3.7} = \frac{0.003}{3.7} = 8.108 \times 10^{-4}$$

$$Re = \frac{\rho V D}{\mu} \qquad \mu = 20.2 \times 10^{-6}\ \frac{lbf\cdot s}{ft^2}$$

$$Re = \frac{\left(\frac{2}{12}\right) V}{10.4 \times 10^{-6}} = 1.603 \times 10^4\, V \qquad \nu = \frac{\mu}{\rho} = 10.4 \times 10^{-6}\ \frac{ft^2}{s}$$

sequence of calcs

① assume f

② calc $V = \sqrt{\frac{5171.3}{192f + 3.49}}$

③ calc $Re = 1.603 \times 10^4\, V$

④ calc $f = \frac{0.25}{\left[\log\left(8.108\times10^{-4} + \frac{5.74}{Re^{0.9}}\right)\right]^2}$

⑤ check convergence

if needed

turbulent flow was a good assumption

iteration	f	V (ft/s)	Re	calc f
1	0.0262	24.64	3.950×10^5	0.0266
2	0.0266	$\boxed{24.53}$	3.931×10^5	0.0266

OK — done

note f_T for $\epsilon/D = 0.003 \approx 0.026$ from moody chart

also $f_T = \frac{0.25}{\left[\log\left(\frac{\epsilon/D}{3.7} + \frac{5.74}{\infty}\right)\right]^2} = \frac{0.25}{\left[\log(8.108\times10^{-4})\right]^2}$

$f_T = 0.0262$ ← use this as first guess

with $V = 24.53$ ft/s

$$Q = VA = (24.53)\left(\frac{\pi}{4}\right)\left(\frac{2}{12}\right)^2 = \boxed{0.535\ \frac{ft^3}{s}}$$

$$= 0.535\ \frac{ft^3}{s} \times \frac{448.83\ gpm}{1\ ft^3/s} = \boxed{240\ gpm} \quad \text{ans}$$

Note: You can always turn a type II problem into a type I case and then do a parametric study by varying Q

For example, for Example 2 let's twist the problem by asking the question, "For a given Q, what is the pressure at Point A"?

this is now a type I problem — and we write the Energy Eqn as

$$\frac{p_A}{\gamma} = (z_B - z_A) + \left[f\frac{L}{D} + \left(\sum_i k_i\right) + 1 \right] \frac{V^2}{2g}$$

Now, the sequence of calcs is

given Q, calc V, calc Re, calc f

Then calc $h_L = h_{L\,major} + h_{L\,minor}$

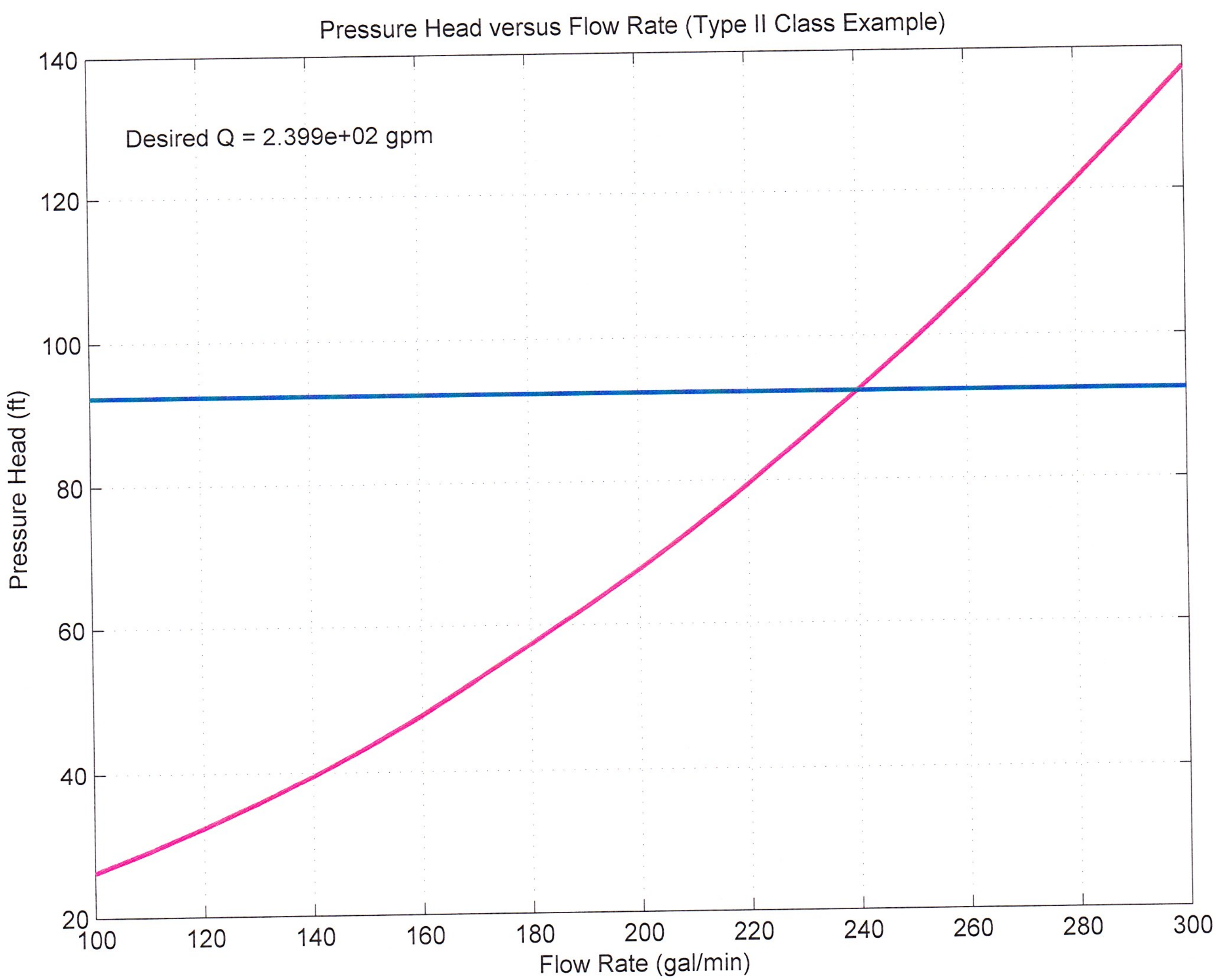
Finally calc p_A/γ for given Q

repeat for and Q of interest

In practice, we do this for a whole range of Q values and then plot p_A/γ vs Q. Then with the desired (actual) pressure head we can get the desired Q for the given p_A/γ.

Thus, we have solved a type II problem by performing a parametric study of a type I problem

see pipe_phead_typeII.m



Pressure Head versus Flow Rate (Type II Class Example)
Desired Q = 2.399e+02 gpm
Pressure Head (ft)
Flow Rate (gal/min)
140
120
100
80
60
40
20
100
120
140
160
180
200
220
240
260
280
300

```
>> pipe_phead_TypeII
   Results for Type II Class Example
     Geometry:
       pipe dia (ft):               1.667e-01
       flow area (ft^2):            2.182e-02
       channel length (ft):         3.200e+01
       surface roughness (ft):      5.000e-04
       relative roughness :         3.000e-03
       elevations head (ft):        1.200e+01
     Fluid Properties:
       density (slugs/ft^3):        1.940e+00
       viscosity (lbf-s/ft^2):      2.020e-05

     Calculated Parameters:
```

flow rate (gpm)	ave vel (ft/s)	Reynolds #	fric factor	vel head (ft)	hL major (ft)	hL minor (ft)	press head (ft)
1.000e+02	1.021e+01	1.635e+05	2.718e-02	1.620e+00	8.453e+00	4.033e+00	2.611e+01
1.100e+02	1.123e+01	1.798e+05	2.710e-02	1.960e+00	1.020e+01	4.880e+00	2.904e+01
1.200e+02	1.226e+01	1.962e+05	2.703e-02	2.332e+00	1.211e+01	5.808e+00	3.225e+01
1.300e+02	1.328e+01	2.125e+05	2.697e-02	2.737e+00	1.418e+01	6.816e+00	3.573e+01
1.400e+02	1.430e+01	2.289e+05	2.692e-02	3.175e+00	1.641e+01	7.905e+00	3.949e+01
1.500e+02	1.532e+01	2.452e+05	2.688e-02	3.644e+00	1.881e+01	9.074e+00	4.353e+01
1.600e+02	1.634e+01	2.616e+05	2.684e-02	4.146e+00	2.137e+01	1.032e+01	4.784e+01
1.700e+02	1.736e+01	2.779e+05	2.680e-02	4.681e+00	2.409e+01	1.166e+01	5.243e+01
1.800e+02	1.838e+01	2.943e+05	2.677e-02	5.248e+00	2.698e+01	1.307e+01	5.729e+01
1.900e+02	1.940e+01	3.106e+05	2.675e-02	5.847e+00	3.003e+01	1.456e+01	6.243e+01
2.000e+02	2.043e+01	3.270e+05	2.672e-02	6.479e+00	3.324e+01	1.613e+01	6.785e+01
2.100e+02	2.145e+01	3.433e+05	2.670e-02	7.143e+00	3.661e+01	1.779e+01	7.354e+01
2.200e+02	2.247e+01	3.597e+05	2.667e-02	7.839e+00	4.015e+01	1.952e+01	7.951e+01
2.300e+02	2.349e+01	3.760e+05	2.666e-02	8.568e+00	4.385e+01	2.133e+01	8.575e+01
2.400e+02	2.451e+01	3.923e+05	2.664e-02	9.329e+00	4.771e+01	2.323e+01	9.227e+01
2.500e+02	2.553e+01	4.087e+05	2.662e-02	1.012e+01	5.174e+01	2.521e+01	9.907e+01
2.600e+02	2.655e+01	4.250e+05	2.660e-02	1.095e+01	5.593e+01	2.726e+01	1.061e+02
2.700e+02	2.758e+01	4.414e+05	2.659e-02	1.181e+01	6.028e+01	2.940e+01	1.135e+02
2.800e+02	2.860e+01	4.577e+05	2.658e-02	1.270e+01	6.480e+01	3.162e+01	1.211e+02
2.900e+02	2.962e+01	4.741e+05	2.656e-02	1.362e+01	6.948e+01	3.392e+01	1.290e+02
3.000e+02	3.064e+01	4.904e+05	2.655e-02	1.458e+01	7.432e+01	3.630e+01	1.372e+02

```
     Interpolated Result for Tank Pressure of 40.0 psi:  Q =  2.399e+02 gpm
>>
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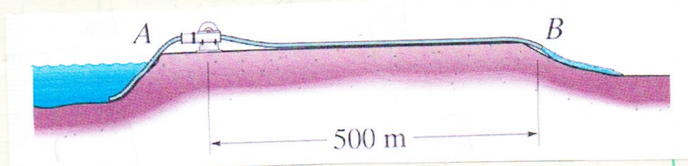


Water is to be delivered from Point A To Point B along the ground at $0.04 \frac{m^3}{s}$

the pump supplies 40 kW To fluid.

Assuming no elevation change, determine the smallest diameter flexible tubing ($\epsilon = 0.00002$ m) that can be used for this application.

— — — — —

($\alpha = 1.0$)

From A To B, the energy eqn gives

$$\cancel{\frac{P_A}{\gamma}}^{0} + \cancel{\frac{V_A^2}{2g}}^{0} + \cancel{z_A} + h_A - \cancel{h_R}_{0} - h_L = \cancel{\frac{P_B}{\gamma}}^{0} + \frac{V_B^2}{2g} + \cancel{z_B}$$

$z_A = z_B$ — no elevation change

$P_A = 0$ and $V_A \simeq 0$ at free surface of reservoir

$P_B = 0$ free jet let $V_B = V$ for ease

$P_A = \dot{W}_A = \gamma Q h_A$ ∴

$$h_A = h_L + \frac{V^2}{2g}$$

power added by pump

$$\frac{\dot{W}_P}{\gamma Q} = \left(f \frac{L}{D} + 1 \right) \frac{V^2}{2g}$$ (neglect minor losses)

but $Q = VA$

$$\frac{\dot{W}_P}{\gamma Q} = \left(f \frac{L}{D} + 1 \right) \frac{Q^2}{2gA^2}$$ $A = \frac{\pi}{4} D^2$

where, assuming turbulent flow, f is given by the Swamee-Jain correlation: $f = \phi\left(\frac{\epsilon}{D}, Re\right)$

and $Re = \frac{\rho V D}{\mu} = \frac{\rho Q D}{\mu A}$

Thus, clearly this represents a rather ugly nonlinear problem That needs to be solved via iteration. $= \frac{4 \rho Q}{\pi \mu D}$

A and f are both functions of D

OR

We solve This as a parametric Type I problem where we loop over a number of candidate values of D and compute h_A — which is Then compared to the actual head added in the system.

easier to do and it gives more info...

Algorithm (given Q and fluid properties)

select D

calc $A = \frac{\pi}{4} D^2$ and ϵ/D

calc $V = \frac{Q}{A}$

calc $Re = \frac{\rho V D}{\mu}$

Swamee-Jain

calc $f = \phi\left(\frac{\epsilon}{D}, Re\right) = \frac{0.25}{\left[\log\left(\frac{\epsilon/D}{3.7} + \frac{5.74}{Re^{0.9}}\right)\right]^2}$

calc $h_A(D) = \left(f \frac{L}{D} + 1\right) \frac{V^2}{2g}$

repeat for a series of reasonable diameters and Then select The smallest value of D that meets The given head added by the given pump.

This is done in pipe-headadded-TypeIII.m

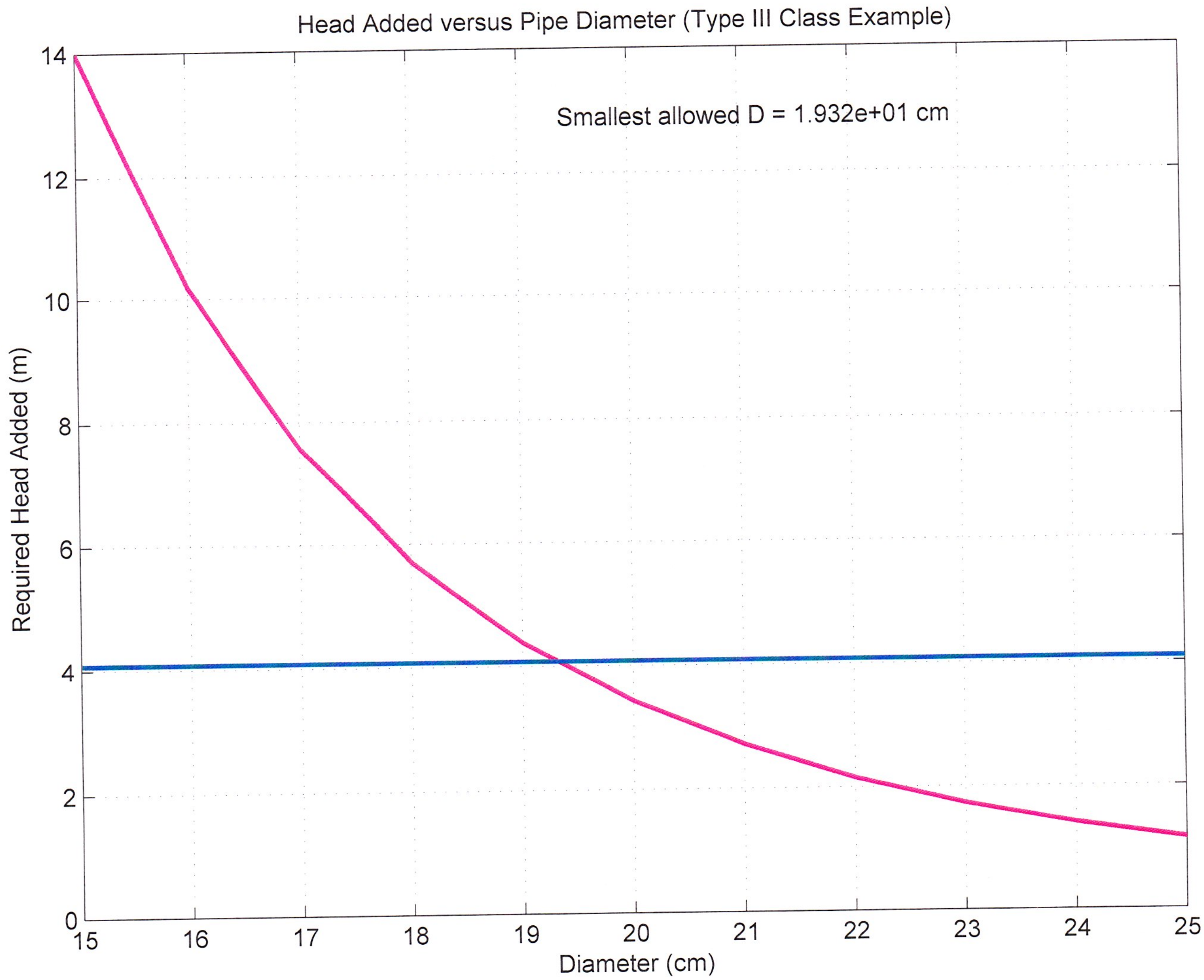
data for problem:

$Q = 0.04\ m^3/s$

$P_A = \dot{W}_p = 40\ kW$

$\rho = 1000\ kg/m^3$, $\mu = 1.15 \times 10^{-3}\ N\text{-}s/m^2$

$L = 500\ m$, $\epsilon = 0.00002\ m$



Head Added versus Pipe Diameter (Type III Class Example)
Smallest allowed D = 1.932e+01 cm
Required Head Added (m)
Diameter (cm)
0
2
4
6
8
10
12
14
15
16
17
18
19
20
21
22
23
24
25

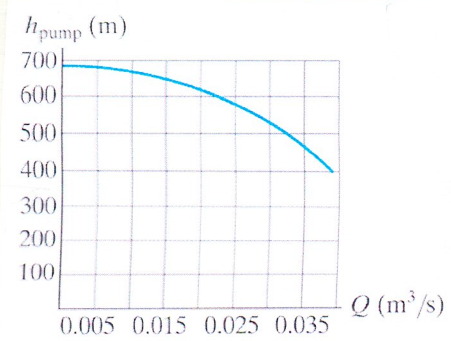
```
>> pipe_head_added_TypeIII
```

Calculated Parameters:

diameter (m)	ave vel (m/s)	Reynolds #	fric factor	vel head (m)	hL major (m)	hL minor (m)	hA (m)
1.500e-01	2.264e+00	2.952e+05	1.578e-02	2.611e-01	1.373e+01	0.000e+00	1.399e+01
1.600e-01	1.989e+00	2.768e+05	1.584e-02	2.017e-01	9.984e+00	0.000e+00	1.019e+01
1.700e-01	1.762e+00	2.605e+05	1.590e-02	1.583e-01	7.404e+00	0.000e+00	7.563e+00
1.800e-01	1.572e+00	2.460e+05	1.598e-02	1.259e-01	5.589e+00	0.000e+00	5.715e+00
1.900e-01	1.411e+00	2.331e+05	1.605e-02	1.014e-01	4.286e+00	0.000e+00	4.387e+00
2.000e-01	1.273e+00	2.214e+05	1.613e-02	8.263e-02	3.333e+00	0.000e+00	3.415e+00
2.100e-01	1.155e+00	2.109e+05	1.622e-02	6.798e-02	2.625e+00	0.000e+00	2.693e+00
2.200e-01	1.052e+00	2.013e+05	1.630e-02	5.644e-02	2.091e+00	0.000e+00	2.147e+00
2.300e-01	9.628e-01	1.926e+05	1.639e-02	4.724e-02	1.683e+00	0.000e+00	1.730e+00
2.400e-01	8.842e-01	1.845e+05	1.647e-02	3.985e-02	1.367e+00	0.000e+00	1.407e+00
2.500e-01	8.149e-01	1.771e+05	1.656e-02	3.384e-02	1.121e+00	0.000e+00	1.155e+00

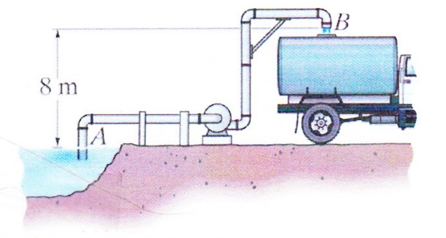
Interpolated Result for Pump Power of 40.0 kW: Dmin = 1.932e+01 cm

Water is pumped from the lake to the Tanker truck Through a 5 cm diameter galvanized pipe.



$h = 50$ m and there are 5 90° elbows

Find the operating point for this system using the given pump curve



at 20°C $\rho = 1000$ kg/m³
$\mu = 1.0 \times 10^{-3}$ N-s/m²

The system curve:

$$\underbrace{\frac{P_A}{\gamma}}_{0} + \underbrace{\frac{\alpha V_A^2}{2g}}_{0} + z_A + h_A - \cancel{h_R}^{\,0} - h_L = \underbrace{\frac{P_B}{\gamma}}_{\text{free jet}} + \frac{\alpha V_B^2}{2g} + z_B$$

free surface

let $\alpha \approx 1.0$

$$\therefore\ h_A = (z_B - z_A) + h_L + \frac{V_B^2}{2g} \qquad V = \frac{Q}{A}$$

$$= (z_B - z_A) + \left[f\frac{L}{D} + \left(\sum_i k_i\right) + 1\right]\frac{Q^2}{2gA^2}$$

system curve

$$\frac{L}{D} = \frac{50\text{m}}{0.05\text{m}} = 1000 \qquad \left(\sum_i k_i\right) = 5(0.9) = 4.5$$

$$z_B - z_A = 8\text{ m} \qquad A = \frac{\pi}{4}(0.05\text{m})^2 = 1.963 \times 10^{-3}\text{ m}^2$$

$$\frac{e}{D} = \frac{0.15 \times 10^{-3}}{.05} = 0.003 \qquad 2gA^2 = 2(9.8\ \text{m/s}^2)(1.963 \times 10^{-3}\text{ m}^2)^2$$

$$= 7.556 \times 10^{-5}\ \text{m}^5/\text{s}^2$$

$$\boxed{h_A = 8 + (1.323 \times 10^4)\left[1000 f + 5.5\right] Q^2} \text{ m}$$

$f = \phi(e/D, Re)$

Pump Curve (from diagram)

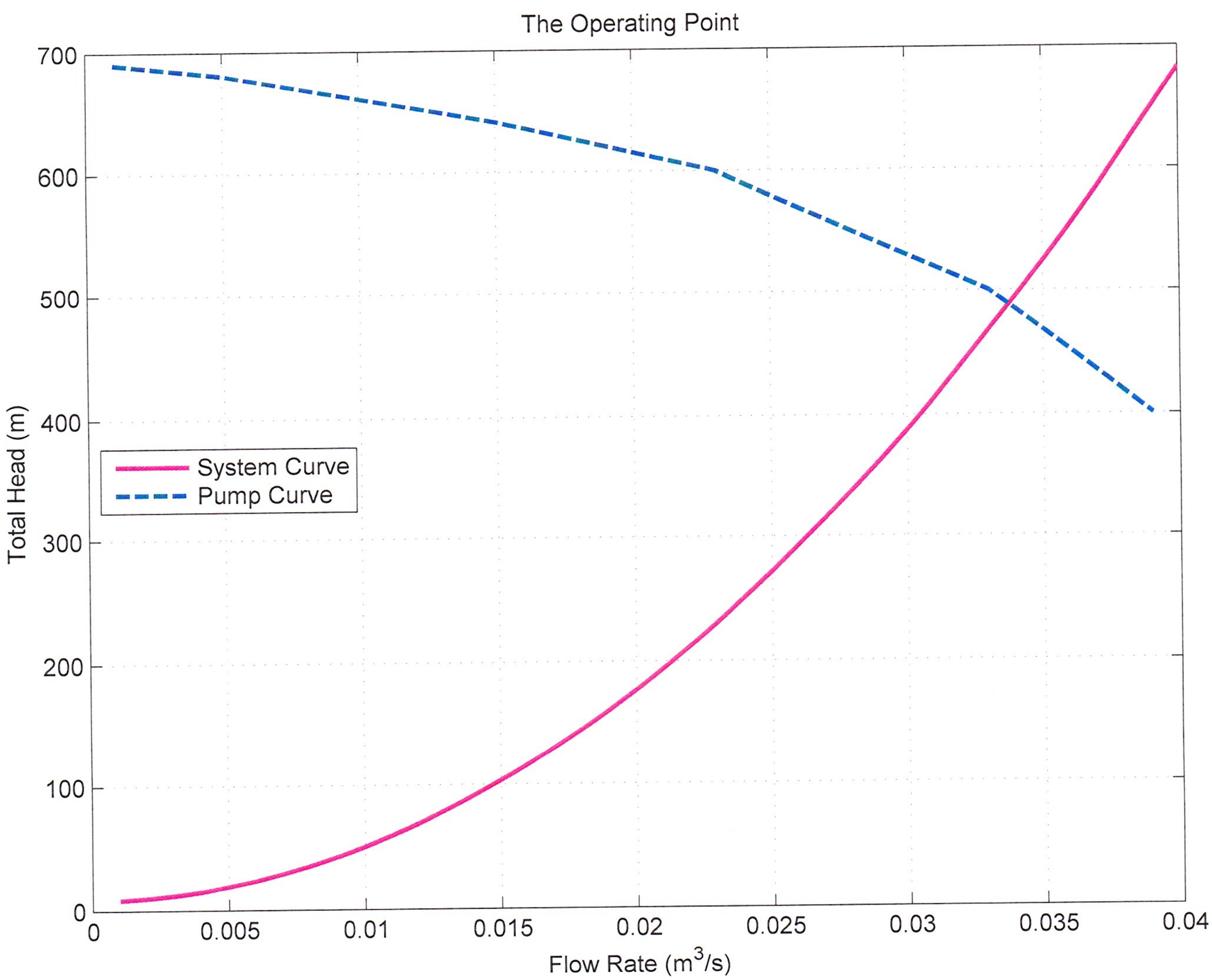
h_{pump} (m)	690 / 680	640	600	500	400
Q (m³/s)	0.005 (0.001)	0.015	0.023	0.033	0.039

very rough

plot The two curves...

$\dot{W}_p = \gamma Q h_A$ and $\eta = \frac{P_A}{P_B}$

$Q_{op} = 0.0331$ m³/s



The Operating Point
700
600
500
400
300
200
100
0
Total Head (m)
System Curve
Pump Curve
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
Flow Rate (m3/s)