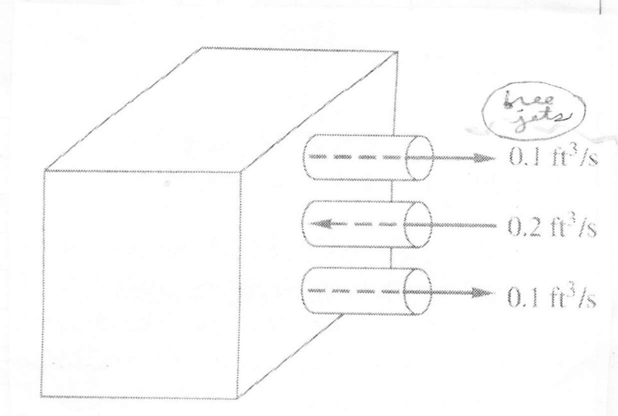
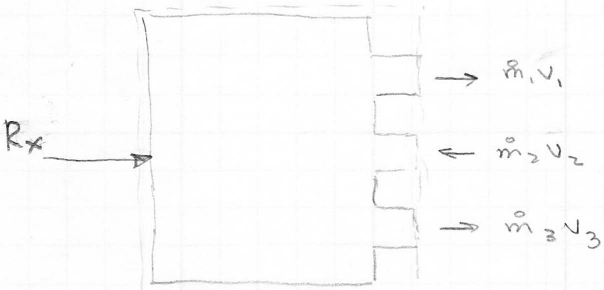
(3.65) The box shown has three 0.5 in holes on the right side. The volume flow rates are steady. The fluid is 20°C water. Compute the force, if any, which this flow situation causes on the box. Assume that the inlet and exit streams are free jets.





$$A = A_1 = A_2 = A_3$$
$$= \frac{\pi}{4} D^2 = \frac{\pi}{4}\left(\frac{0.5}{12}\right)^2$$
$$= 1.364 \times 10^{-3}\ ft^2$$

$$\rho = 62.4\ lbm/ft^3$$

x-directed momentum balance

$$\sum F_x = \sum_{out} \dot m \vec V_x - \sum_{in} \dot m \vec V_x$$

$$\boxed{\dot m = \rho Q \qquad Q = AV \qquad V = \frac{Q}{A}}$$

Note! There are no pressure forces because of the free jet assumption (i.e. $P_{gage} = 0$)

$$R_x = \dot m_1 \vec V_1 + \dot m_3 \vec V_3 - \dot m_2 \vec V_2$$

$$= \frac{(6.24)(73.3)}{32.2} + \frac{(6.24)(73.3)}{32.2} - \frac{(12.48)(-146.6)}{32.2} \quad \frac{slug\ ft}{s^2}$$

$$= 14.2 + 14.2 + 56.8$$

$$= \boxed{85.2\ lbf}$$

ans

note that V_2 is in $-x$ direction

$$V_1 = \frac{Q_1}{A_1} = 73.3\ \frac{ft}{s}$$

$$V_2 = \frac{Q_2}{A_2} = 146.6\ ft/s$$

$$V_3 = \frac{Q_3}{A_3} = 73.3\ ft/s$$

$$\dot m_1 = \rho Q_1 = 6.24\ \frac{lbm}{s}$$

$$\dot m_2 = \rho Q_2 = 12.48\ \frac{lbm}{s}$$

$$\dot m_3 = \rho Q_3 = 6.24\ \frac{lbm}{s}$$

$$\dot m_{in} = \dot m_{out}$$
$$\dot m_2 = \dot m_1 + \dot m_3$$

units

$$\frac{\left(\dot m\ \frac{lbm}{s}\right)\left(V\ \frac{ft}{s}\right)}{32.2\ \frac{lbm\text{-}ft/s^2}{lbf}} \Rightarrow \frac{\dot m V}{32.2}\ lbf \rightarrow F = \frac{\dot m V}{g_c}$$

English units

note: ① weight of fluid is not a factor here since weight is directed in the $-y$ direction (maybe a friction force of box on a table)

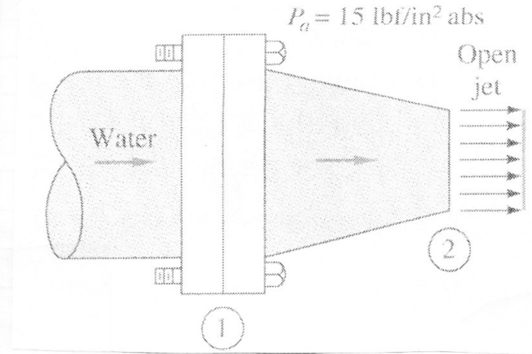
② Typically there is a pressure force, PA, that must be accounted for at each inlet/exit. However, the pressure in a free jet is 0 psig. ∴ PA = 0 here

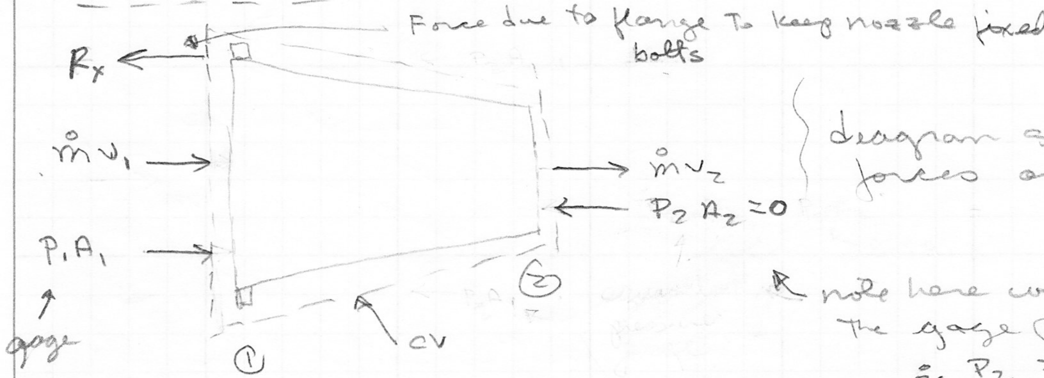
for subsonic flows

③ The force computed is required to hold the box in place. The force on the box due to the fluid has the same magnitude but is in the opposite direction...

3.49) The horizontal nozzle shown in the diagram has $D_1 = 12$ in and $D_2 = 6$ in. The inlet pressure at pt 1 is 38 psia and the exit speed at pt 2 is 56 ft/s.

For water at 20°C, compute the horizontal force provided by the flange bolts to hold the nozzle fixed.





} diagram shows all forces and momentum flows

note here we will use the gage pressure

$\therefore P_2 = 0$

$P_1 = 38 - 15 = 23$ psig

Note That you should always use gage press

can discuss this if There are any questions

atmospheric pressure on A_1 from both sides cancel

x-direction momentum balance

$$\sum F_x = \sum_{out} \dot{m}\vec{v}_x - \sum_{in} \dot{m}\vec{V}_x$$

$$-R_x + P_1A_1 - \underbrace{P_2A_2}_{\to 0} = \dot{m}\vec{V}_2 - \dot{m}\vec{v}_1$$

continuity eqn

$\dot{m} = \dot{m}_1 = \dot{m}_2$

$$\text{or } \boxed{R_x = \dot{m}(\vec{v}_1 - \vec{V}_2) + P_1A_1}$$

$A_1 = \frac{\pi}{4}(1\,ft)^2 = \frac{\pi}{4}\,ft^2$

$A_2 = \frac{\pi}{4}\left(\frac{1}{2}\right)^2 = \frac{\pi}{16}\,ft^2$

$\dot{m} = \rho V_2 A_2$

$= \left(62.4\,\frac{lbm}{ft^3}\right)\left(56\,ft/s\right)\left(\frac{\pi}{16}\,ft^2\right)$

$\boxed{\dot{m} = 686.8\text{ lbm/s}}$

now $v_1 = \frac{\dot{m}}{\rho A_1} = \frac{686.8}{(62.4)(\pi/4)}$

$\boxed{v_1 = 14\text{ ft/s}}$

$$\therefore R_x = \left(686.8\,\frac{lbm}{s}\right)\left(14 - 56\right)\frac{ft}{s} + 23\,\frac{lbf}{in^2} \times \frac{144\,in^2}{ft^2} \times \frac{\pi}{4}\,ft^2$$

$$= -\left(28846\,\frac{lbm\,ft}{s^2}\right)\left(\frac{1\,lbf}{32.2\,\frac{lbm\,ft}{s^2}}\right) + 2601\,lbf$$

both $\vec{V}_1$ and $\vec{V}_2$ are in + x direction

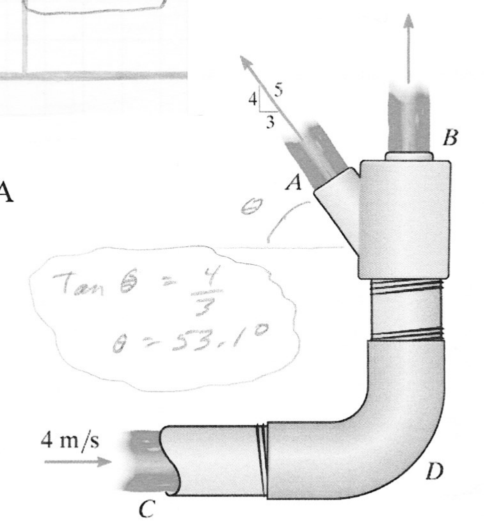
$\boxed{R_x = 1705\text{ lbf}}$

g_c always be careful with units

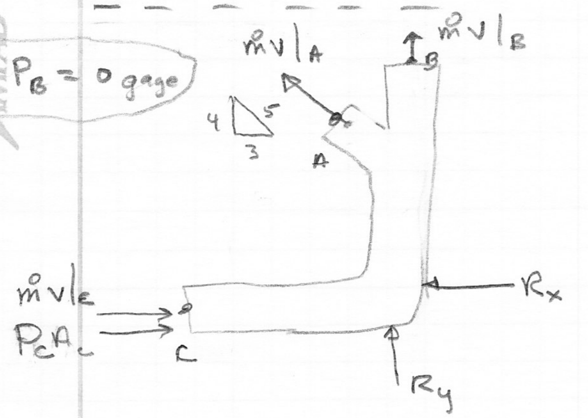
Water flows through pipe C at 4 m/s. The pipe diameter at C is 6 cm and, at A and B, both diameters are 2 cm. The streams at A and B are free jets.

Determine the horizontal and vertical components of force exerted by the elbow necessary to hold the pipe assembly in equilibrium.

Neglect the size and weight of the pipe and the water within…



$P_A = P_B = 0$ gage



Steady Flow Momentum Balance

$$\sum \vec{F} = \sum_{\text{outlets}} \dot{m}\vec{v} - \sum_{\text{inlets}} \dot{m}\vec{v}$$

$$\boxed{\sum \vec{F} = \dot{m}_A \vec{V}_A + \dot{m}_B \vec{V}_B - \dot{m}_c \vec{V}_c}$$

vector eqn

In x-direction

cos θ; in neg x-dir; no x-component

$$P_c A_c - R_x = \dot{m}_A \left(\tfrac{3}{5}\right)\left(-|\vec{V}_A|\right) + 0 - \dot{m}_c |\vec{V}_c|$$

$$\therefore \boxed{R_x = P_c A_c + \tfrac{3}{5}\dot{m}_A |\vec{V}_A| + \dot{m}_c |\vec{V}_c|} \qquad (1)$$

In y-direction

sin θ; no y component

$$R_y = \dot{m}_A \left(\tfrac{4}{5}\right)|\vec{V}_A| + \dot{m}_B |\vec{V}_B| - 0$$

$$\therefore \boxed{R_y = \tfrac{4}{5}\dot{m}_A |\vec{V}_A| + \dot{m}_B |\vec{V}_B|} \qquad (2)$$

Now, we need P_c, the mass flow rate, and velocities

$$Q_c = A_c V_c \qquad A_c = \frac{\pi D_c^2}{4} = \frac{\pi}{4}(0.06\text{m})^2$$

$$= 2.827\times10^{-3}\ \text{m}^2$$

$$= (2.827\times10^{-3})(4)$$

$$\boxed{Q_c = 1.131\times10^{-2}\ \frac{\text{m}^3}{\text{s}}}$$

$$\dot{m}_c = \rho Q = \left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(1.131\times10^{-2}\frac{\text{m}^3}{\text{s}}\right)$$

$$\boxed{\dot{m}_c = 11.31\ \text{kg/s}}$$

From continuity eqn

$$Q_c = Q_A + Q_B \quad \text{or} \quad \dot{m}_c = \dot{m}_A + \dot{m}_B = 2\dot{m}_A$$

but $\dot{m}_A = \dot{m}_B$ since points A and B have same areas

$$\therefore \dot{m}_A = \dot{m}_B = \frac{\dot{m}_c}{2} = \frac{11.31 \text{ kg/s}}{2} = \boxed{5.655 \text{ kg/s}}$$

$$Q_A = Q_B = \frac{Q_c}{2} = \frac{1.131 \times 10^{-2} \frac{m^3}{s}}{2} = \boxed{5.655 \times 10^{-3} \frac{m^3}{s}}$$

$$\therefore |V_A| = \frac{Q_A}{A_A} = \frac{5.655 \times 10^{-3}}{3.1416 \times 10^{-4}} = \boxed{18 \text{ m/s}} \qquad A_A = A_B = \frac{\pi}{4}(0.02 \text{ m})^2 = 3.1416 \times 10^{-4} \text{ m}^2$$

and $|\vec{V}_B| = |\vec{V}_A| = \boxed{18 \text{ m/s}}$

Finally, To find P_c, let's write the Bernoulli eqn $C \to B$

$$\frac{P_c}{\gamma} + \frac{V_c^2}{2g} + \cancel{z_c} = \cancel{\frac{P_B}{\gamma}}^{0} + \frac{V_B^2}{2g} + \cancel{z_B}$$

but $P_B = 0$ free jet and $z_c \approx z_B$ small elevation change

$$\therefore P_c = \frac{\gamma}{2g}\left(V_B^2 - V_c^2\right)$$

$$= \frac{9.81 \times 10^3 \text{ N/m}^3}{2(9.81 \text{ m/s}^2)}\left(18^2 - 4^2\right)\frac{m^2}{s^2} = 154 \times 10^3 \text{ Pa} = \boxed{154 \text{ kPa}}$$

Now from eqn (1)

$$R_x = \left(154 \times 10^3 \frac{N}{m^2}\right)\left(2.827 \times 10^{-3} \text{ m}^2\right) + \frac{3}{5}\left(5.655 \text{ kg/s}\right)\left(18 \text{ m/s}\right) + \left(11.31 \frac{kg}{s}\right)\left(4 \text{ m/s}\right)$$

or

$$R_x = 435.4 + 61.1 + 45.2 = \boxed{541.7 \text{ N}} \quad \text{ans}$$

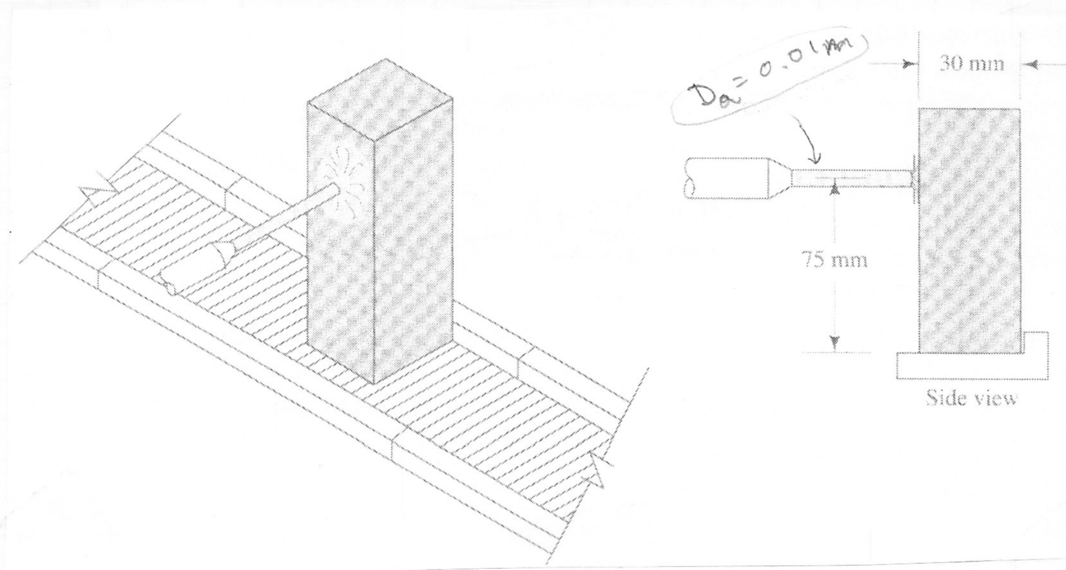
And from eqn (2)

$$R_y = \frac{4}{5}(5.655)(18) + (5.655)(18)$$

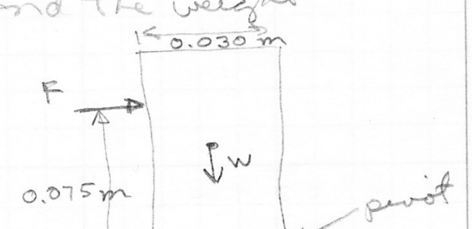
$$= 81.43 + 101.8 = \boxed{183.2 \text{ N}} \quad \text{ans}$$

16.21 A part of an inspection system in a packaging operation uses a jet of air to remove imperfect cartons from a conveyor line, as illustrated in the diagram below. The jet is initiated by a sensor and timed so that the product to be rejected is in front of the jet at the right moment. The product is to be tipped over a very small ledge on the side of the conveyor, as shown.

Compute the required velocity of the air jet needed to tip the carton off the conveyor. The density of air is 1.20 kg/m³. The carton has a mass of 0.10 kg. The jet has a diameter of 1 cm.



A side view of the carton shows the location of the force due to the air stream and the weight of the carton



a moment balance at the pivot gives

$$F(0.075\,m) = W\left(\frac{0.030}{2}\right)$$

$$F = (0.981\,N)\,\frac{0.015\,m}{0.075\,m}$$

$$\boxed{F = 0.196\,N}$$

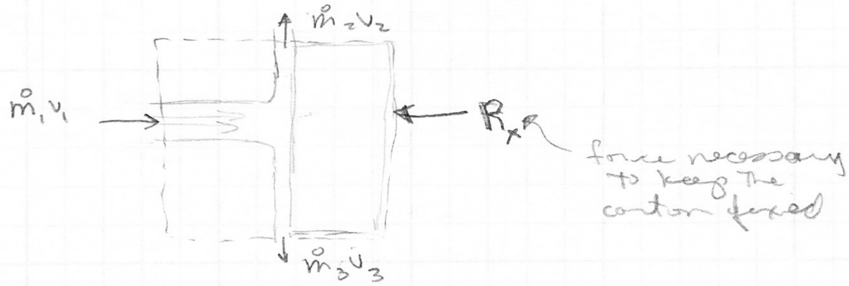
any force greater than this will toggle the carton about the pivot point

$$W = mg$$

$$= 0.10\,kg\left(\frac{9.81\,m}{s^2}\right)$$

$$= 0.981\,N$$

Now, for application of the momentum eqn, let's consider the CV shown below



x-directed momentum balance (not interested in other direction)

$$\sum F_x = \sum_{out} \dot{m} \vec{V}_x - \sum_{in} \dot{m} \vec{V}_x$$

$$-R_x = 0 - \dot{m}_1 \vec{V}_1 \qquad \underline{V_1 \text{ is in } +x \text{ direction}}$$

or $\boxed{R_x = \dot{m}_1 V_1}$ but $\dot{m} = \rho A V$

$\therefore$ $\boxed{R_x = \rho A_1 V_1^2}$ signs already included

$$\text{or } V_1 = \sqrt{\frac{R_x}{\rho A_1}} \qquad A_1 = \frac{\pi}{4}(0.01\,m)^2 = 7.854 \times 10^{-5}\,m^2$$

but $R_x = 0.196\,N$ from previous page

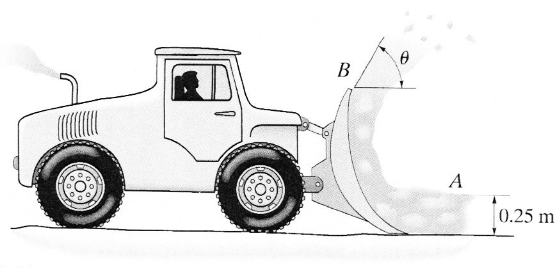
$$= \sqrt{\frac{0.196\,N}{(1.2\,kg/m^3)(7.854 \times 10^{-5}\,m^2)}} \qquad \underline{\text{units}} \quad \frac{kg\frac{m}{s^2}}{kg/m} \rightarrow \frac{m^2}{s^2}$$

$$= \sqrt{2079.6\ m^2/s^2}$$

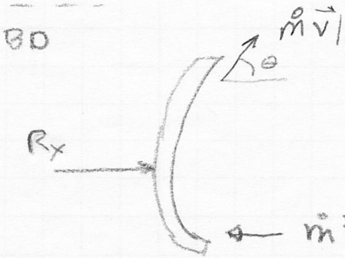
$\boxed{V_1 = 45.6\ m/s}$ ans pretty fast

velocity must be slightly greater than this value ...

6–53. The truck is traveling forward at 5 m/s, shoveling a liquid slush that is 0.25 m deep. If the slush has a density of 125 kg/m^3 and is thrown upwards at an angle of $\theta = 60°$ from the 3-m-wide blade, determine the traction force of the wheels on the road necessary to maintain the motion. Assume that the slush is thrown off the shovel at the same rate as it enters the shovel.



FBD



$\rightarrow \vec{V}_{cs} = 5\frac{m}{s}$

rel vel. at A

$\vec{V}_r = \underbrace{\vec{V}_{slush}}_{0 \text{ at rest}} - \vec{V}_{cs}$

$\vec{V}_r|_A = -5\,m/s$

momentum balance (x-dir) P=0 at control surfaces

$\sum F_x = R_x = \dot{m}\vec{V}_B \cos\theta - \dot{m}\vec{V}_A$

$\dot{m} = \rho A V_r = \left(125\frac{kg}{m^3}\right)(0.25)(3)\,m^2\,(5\,m/s)$

$= (125)(0.75)(5) = \boxed{468.75\ kg/s}$

note that $\dot{m}$ is always positive

assuming negligible losses and a small height change, the Bernoulli eqn says that $V_B = V_A$

$\frac{\cancel{P_A}}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{\cancel{P_B}}{\gamma} + \frac{V_B^2}{2g} + z_B$

$P_A \to 0$ open $P_B \to 0$ open

$z_B \approx z_A$

$P_A = P_B = 0$

$\Rightarrow \therefore V_B \simeq V_A$ magnitude

$\therefore R_x = 468.75\frac{kg}{s}\left((+5)\cos 60° - (-5)\right)\frac{m}{s}$

$= (468.75)(2.5 + 5) = \boxed{3.52\ kN}$

ans