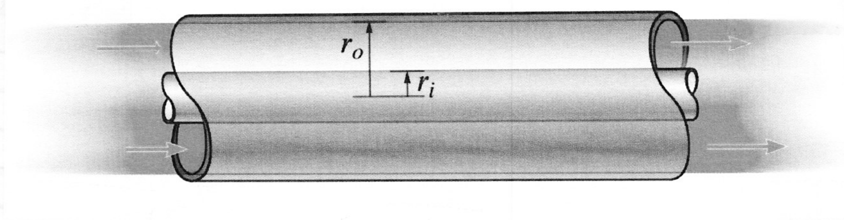
Example #1

Determine the Reynold's number for water flow at 30°c in the annulus region of the concentric pipes shown.

Let $Q = 0.01\, m^3/s$
with
$r_i = 0.04\, m$
$r_o = 0.06\, m$



- - - - -

$$Re_h = \frac{\rho V D_h}{\mu}$$

$$\text{where } D_h = \frac{4 A_f}{P_w} = \frac{4(\pi r_o^2 - \pi r_i^2)}{2\pi r_i + 2\pi r_o} = \frac{4(6.283 \times 10^{-3})}{0.6283}$$

$$= \frac{2(r_o^2 - r_i^2)}{r_o + r_i} = \frac{2\cancel{(r_o + r_i)}(r_o - r_i)}{\cancel{r_o + r_i}}$$

$$= 2(r_o - r_i)$$

$$= 2(0.06 - 0.04)\,m = \boxed{0.04\, m}$$

at 30°c (Append A)

$\rho = 995.7\, kg/m^3$ $\mu = 0.801 \times 10^{-3}\, N\text{-}s/m^2$

and

$$V = \frac{Q}{A} = \frac{0.01\, m^3/s}{6.283 \times 10^{-3}\, m^2} = \boxed{1.592\, m/s}$$

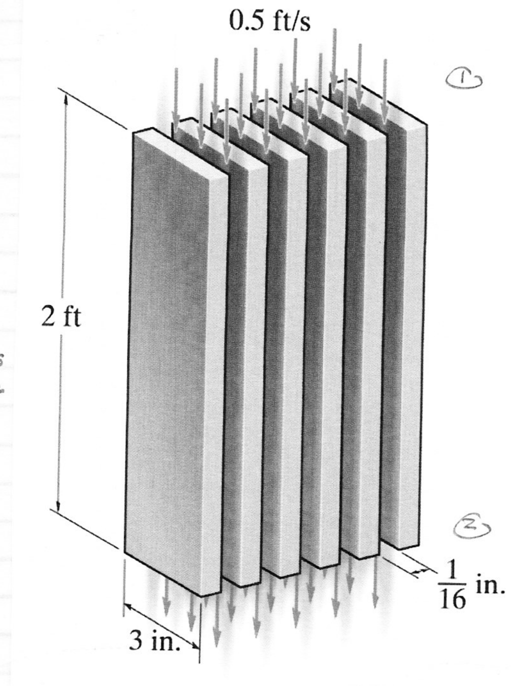
$$\therefore \quad Re = \frac{\left(995.7 \frac{kg}{m^3}\right)\left(1.592 \frac{m}{s}\right)(0.04\, m)}{0.801 \times 10^{-3}\, N\text{-}s/m^2}$$

$$= 7.916 \times 10^4$$

Note since $Re > 4000$, the flow is turbulent !!!

Consider the MTR fuel plates as shown. Water flows downward through the plates.

Neglecting end effects, estimate the ΔP over the length of the plates.



water properties

$\rho_w = 1.820 \frac{\text{slugs}}{ft^3}$ $\mu_w = 5.46 \times 10^{-6} \frac{lbf \cdot s}{ft^2}$

From the energy eqn

$$\frac{P_1}{\gamma} + \cancel{\frac{v_1^2}{2g}} + z_1 + \cancel{h_A} - \cancel{h_R} - h_L = \frac{P_2}{\gamma} + \cancel{\frac{v_2^2}{2g}} + z_2$$

$$\therefore P_2 - P_1 = \Delta P = ((z_1 - z_2) - h_L)\gamma$$

$$\Delta P = (2 ft - h_L)\left(58.60 \frac{lbf}{ft^3}\right)$$

$$\gamma = \rho g = \left(1.82 \frac{\text{slugs}}{ft^3}\right)\left(32.2 \frac{ft}{s^2}\right) = 58.60 \frac{lbf}{ft^3}$$

$\therefore$ need h_L ...

from development in class Notes (or HW) for $\boxed{\text{Laminar Flow}}$

$$\boxed{h_L = \frac{12 \mu L V}{\rho g a^2}} = \frac{12\left(5.46 \times 10^{-6} \frac{lbf \cdot s}{ft^2}\right)(2 ft)\left(0.5 \frac{ft}{s}\right)}{\left(58.6 \frac{lbf}{ft^3}\right)\left(\frac{0.0625}{12} ft\right)^2}$$

$a = \frac{1}{16}'' = 0.0625''$

$$= \frac{6.552 \times 10^{-5}}{1.590 \times 10^{-3}} ft$$

$$= 0.0412 ft$$

see back of page 2

$$\therefore \Delta P = (2 - 0.0412)(58.60) = \left(114.8 \ lbf/ft^2\right)\frac{ft^2}{144 in^2}$$

$$= 0.797 \ psi$$

---

check on lamina Flow

$$Re = \frac{\rho V a}{\mu} = \frac{\left(1.82 \frac{\text{slugs}}{ft^3}\right)\left(0.5 \frac{ft}{s}\right)\left(\frac{0.0625}{12} ft\right)}{5.46 \times 10^{-6} \frac{lbf \cdot s}{ft^2}}$$

← Hibbeler

$\boxed{Re = 868}$ OK

$Re_a < 1400 \therefore$ laminar flow

$\boxed{Re_h = 1736}$

$Re_h < 2800 \therefore$ laminar flow

An alternate soln to this problem that includes only the eqns given in the text is as follows
(No mention of h_L ...)

from Text

$$\boxed{Q = -\frac{ba^3}{12\mu}\left(\frac{dP}{dx} + \gamma\frac{dh}{dx}\right)}$$

for laminar flow in parallel channels

for this problem

$$\frac{dP}{dx} = \frac{P_2 - P_1}{L} = \frac{\Delta P}{L} \quad \text{and} \quad \frac{dh}{dx} = \frac{h_2 - h_1}{L} = \frac{-L}{L} = -1$$

$$\therefore \quad Q = \frac{ba^3}{12\mu}\left(\gamma - \frac{\Delta P}{L}\right)$$

where
$$Q = VA = \left(0.5\,\frac{ft}{s}\right)\left(\frac{1}{4}\,ft\right)\left(\frac{.0625}{12}\,ft\right)$$
$$= 6.510 \times 10^{-4}\ ft^3/s$$

$$\frac{ba^3}{12\mu} = \frac{\left(\frac{1}{4}\,ft\right)\left(\frac{0.0625}{12}\,ft\right)^3}{12\left(5.46 \times 10^{-6}\,\frac{lbf\text{-}s}{ft^2}\right)} = 5.391 \times 10^{-4}\,\frac{ft^6}{lbf\text{-}s}$$

$$\gamma = \rho g = \left(1.82\,\frac{slugs}{ft^3}\right)\left(32.2\,\frac{ft}{s^2}\right) = 58.60\,\frac{lbf}{ft^3}$$

$$\therefore \quad \Delta P = L\left(\gamma - \frac{Q}{ba^3/12\mu}\right)$$

$$= 2\,ft\left(58.60\,\frac{lbf}{ft^3} - \frac{6.510 \times 10^{-4}\ ft^3/s}{5.391 \times 10^{-4}\ ft^6/lbf\text{-}s}\right)$$

$$= 2\left(58.60 - 1.208\right)$$

$$= \boxed{114.8\,\frac{lbf}{ft^2}} \times \frac{1\,ft^2}{144\,in^2} = \boxed{0.797\,psi}$$

Same as before

OK

Note: If h_L is written in terms of the Darcy eqn

$$h_L = f \frac{L}{D_h} \frac{V^2}{2g}$$

where for flow in a rectangular channel with $b >> a$

$$f = \frac{96}{Re_h} \quad \text{and} \quad Re_h = \frac{\rho V D_h}{\mu}$$

with $D_h = \frac{4ab}{2a+2b} \approx 2a$

$$h_L = \left(\frac{96}{Re_h}\right) \frac{L}{D_h} \frac{V^2}{2g}$$

$$h_L = \left(\frac{96}{1736}\right)\left[\frac{2\ ft}{2\left(\frac{0.0625}{12}\right) ft}\right]\frac{(0.5\ ft/s)^2}{2(32.2\ ft/s^2)}$$

$$h_L = (0.05530)(192)(0.003882\ ft)$$

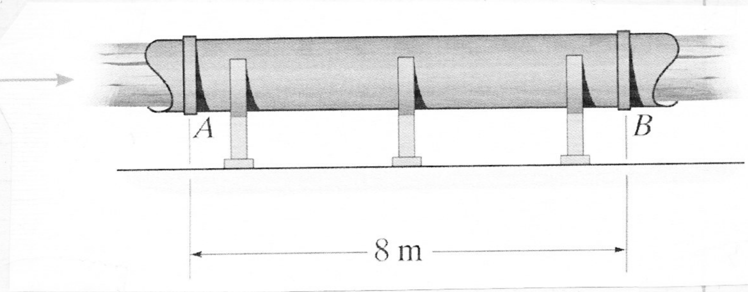
$$\boxed{h_L = 0.0412\ ft}$$ as before

Hibbeler
9-30

Example #3

the oil flow rate is
$Q = 0.004\ m^3/s$ and
the pipe diameter is 0.15m.

Determine ΔP for the 8-m long
section and find the
head loss per meter of pipe.



use $\rho = 900\ kg/m^3$ $\mu = 0.370\ N\text{-}s/m^2$

The relationship for head loss, h_L and the pressure drop, ΔP, can be developed from the energy eqn

$$\frac{P_A}{\gamma} + \cancel{\frac{V_A^2}{2g}} + \cancel{z_A} + \cancel{h_A} - \cancel{h_R} - h_L = \frac{P_B}{\gamma} + \cancel{\frac{V_B^2}{2g}} + \cancel{z_B}$$

$z_A = z_B$ same elevation

$V_A = V_B$ same pipe size

$h_A = h_R = 0$ no mechanical devices

$$\therefore\ h_L = \frac{P_A - P_B}{\gamma} \quad \text{and} \quad \boxed{\frac{h_L}{L} = \frac{P_A - P_B}{\gamma L}}$$

head loss per meter

Now, let's check if the flow is laminar???

$$Re = \frac{\rho V D}{\mu} = \frac{\rho Q D}{\mu A}$$

$$V = \frac{Q}{A} = \frac{0.004\ m^3/s}{\frac{\pi}{4}(0.15m)^2} = \frac{0.004}{0.01767} = 0.226\ m/s$$

$$= \frac{\left(900\ \frac{kg}{m^3}\right)\left(0.226\ \frac{m}{s}\right)(0.15m)}{0.370\ N\text{-}s/m^2}$$

$= 82.5$ ← laminar

Hagen Poiseuille Eqn

$$\Delta P = \frac{128 \mu L Q}{\pi D^4}$$ ← eqn 9.26 in Hibbeler

see back side

$$= \frac{(128)\left(0.370\ \frac{N\text{-}s}{m^2}\right)(8\ m)\left(0.004\ \frac{m^3}{s}\right)}{\pi\ (0.15)^4\ m^4} = \boxed{952.9\ Pa}$$

$$\frac{h_L}{L} = \frac{\Delta P}{\gamma L} = \frac{952.9\ N/m^2}{\left(900\ \frac{kg}{m^3}\right)\left(9.81\ \frac{m}{s^2}\right)(8\ m)} = \boxed{0.0135\ \frac{m}{m}}$$

Note for circular pipe

$$h_L = f \frac{L}{D} \frac{v^2}{2g} = \frac{64}{Re} \frac{L}{D} \frac{v^2}{2g}$$

where $Re = \frac{\rho v D}{\mu}$

$$h_L = \left(\frac{64}{82.5}\right)\left(\frac{8\ m}{0.15\ m}\right)\left(\frac{[0.226\ m/s]^2}{2(9.81\ m/s^2)}\right)$$

$$= (0.7758)(53.33)(0.002603\ m)$$

$$\boxed{h_L = 0.1077\ m}$$

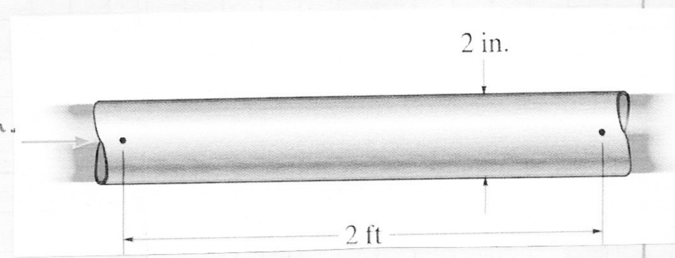
and $\frac{h_L}{L} = \frac{0.1077\ m}{8\ m} = \boxed{0.0135\ \frac{m}{m}}$

as before

Hibbeler 9-66

Example #4

Water flows in the 2" diameter smooth pipe.
$\Delta P = 1.5$ psi along the 2-ft length.



Determine wall shear stress and at the centerline.
Also compute u_{max} (at center of pipe)

water properties: $\rho = 62.4\ lbm/ft^3$ and $\nu = 16.6 \times 10^{-6}\ ft^2/s$

---

from the notes: $\tau(r) = \frac{r}{2} \frac{d}{dx}(P + \gamma h)$

$$= \frac{r}{2}\left(108 \frac{lbf}{ft^3}\right)$$

$\frac{dh}{dx} = 0$

$$\frac{dP}{dx} = \frac{\Delta P}{L} = \frac{\left(1.5\frac{lbf}{in^2}\right)\left(\frac{144 in^2}{ft^2}\right)}{2\ ft} = 108 \frac{lbf}{ft^3}$$

at wall $r = R = \frac{1}{12} ft$

$$\therefore \tau_w = \frac{108}{2(12)} = \boxed{4.5 \frac{lbf}{ft^2}}$$

at center $r = 0$ $\therefore \boxed{\tau(0) = 0}$

---

from the notes for turbulent flow

$$u(y) = u^*\left[2.5 \ln\left(\frac{u^* y}{\nu}\right) + 5.0\right]$$

where $y = R - r$ and $u^* = \sqrt{\tau_w/\rho}$ shear velocity

$$u_{max} = u(R) = u^*\left[2.5 \ln\left(\frac{u^* R}{\nu}\right) + 5.0\right]$$

where $u^* = \sqrt{\frac{\tau_w}{\rho}} = \sqrt{\frac{4.5\ lbf/ft^2}{62.4\ lbm/ft^3} \times \underbrace{\frac{32.2\ lbm\ ft/s^2}{lbf}}_{g_c}}$

$$= \sqrt{\frac{(4.5)(32.2)}{62.4} \frac{ft^2}{s^2}}$$

$\boxed{u^* = 1.524\ ft/s}$ ← shear velocity

$$\therefore u_{max} = 1.524 \left[2.5 \ln\left(\frac{1.524\ ft/s\ \left(\frac{1}{12} ft\right)}{16.6 \times 10^{-6}\ ft^2/s}\right) + 5.0\right]$$

$$= 1.524\left[22.36 + 5.0\right]$$

$\boxed{u_{max} = 41.7\ ft/s}$ ans

Note $Re_{max} = \frac{V_{max} D}{\nu} = 2.1 \times 10^5$ turbulent