CHEN.3030 Fluid Mechanics

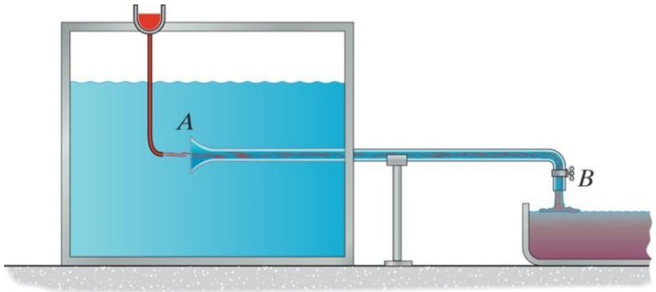
VII. Internal Viscous Flows

Prof. John R. White
Chemical and Nuclear Engineering
UMass-Lowell, Lowell MA

See Chapter 9 (all sections except 9.2 & 9.4) in your text by Hibbeler

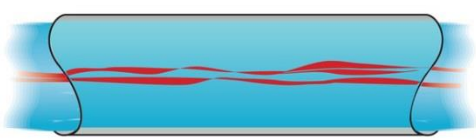
Laminar vs. Turbulent Flow

UMASS LOWELL
Learning with Purpose

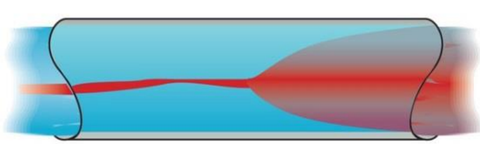


(a)

Laminar flow
(b)



Transitional flow
(c)



Turbulent flow
(d)

$$Re = \frac{\rho v L}{\mu} = \frac{vL}{\nu}$$

where L is a characteristic dimension associated with the flow geometry

L $\rightarrow$ D = pipe diameter for pipe flow

L $\rightarrow$ D_h = hydraulic diameter for general internal flows

$$D_h = \frac{4A_f}{P_w}$$

where A_f is the flow area and P_w is the wetted perimeter

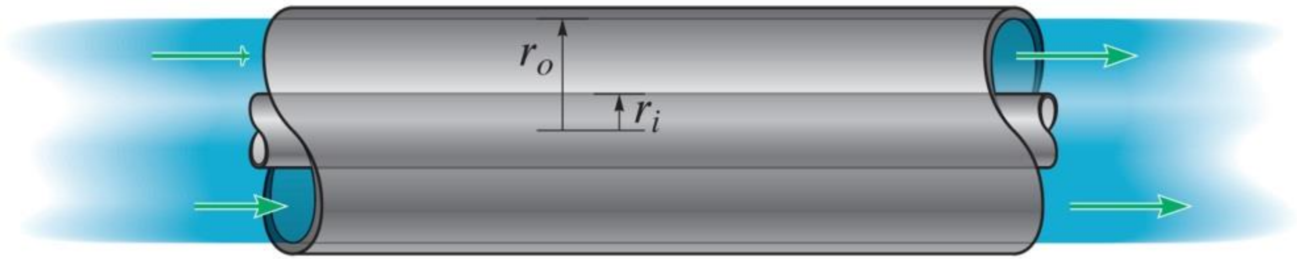
Ex. #1 -- hydraulic diameter…

Consider water flow within the annulus shown in the sketch for T = 30 °C and Q = 0.01 m^3/s with r_i = 0.04 m and r_o = 0.06 m.

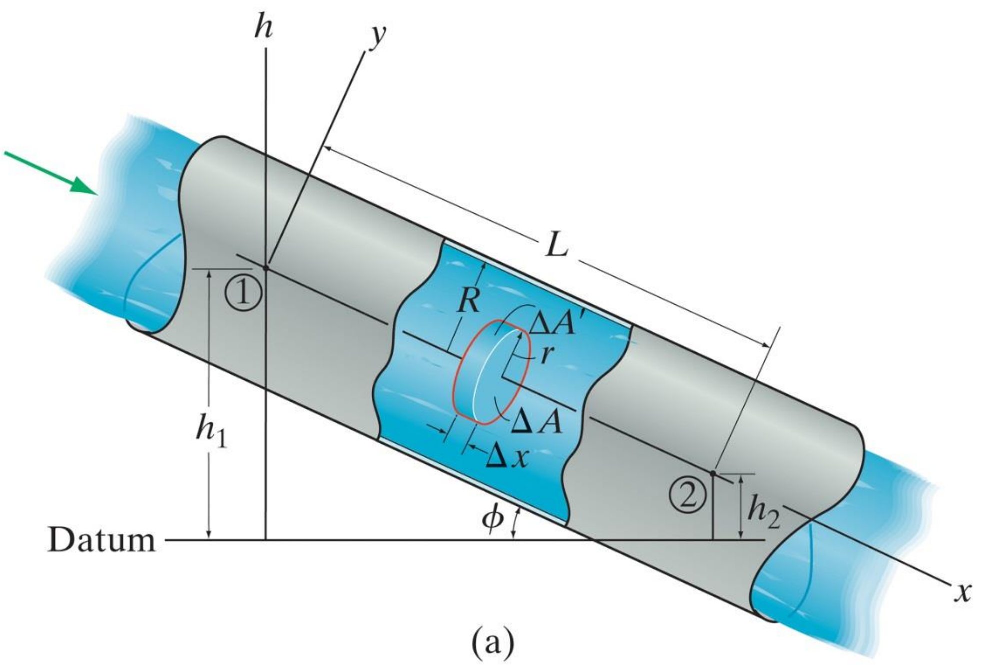
Is the flow laminar or turbulent?

Water properties at 30 °C from Appendix A :

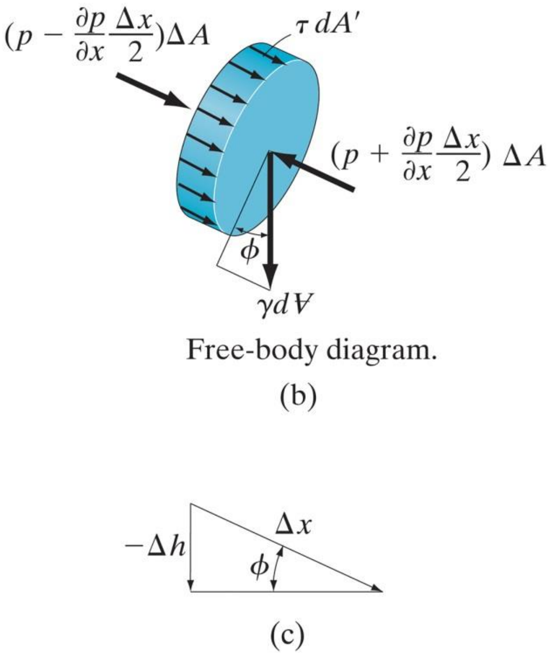
ρ = 995.7 kg/m^3 and $\mu = 0.801\times10^{-3}$ $N\text{-}s/m^2$



Steady Laminar Flow – Smooth Pipe



(a)



Basic Configuration and Coordinate Definitions

Free Body Diagram

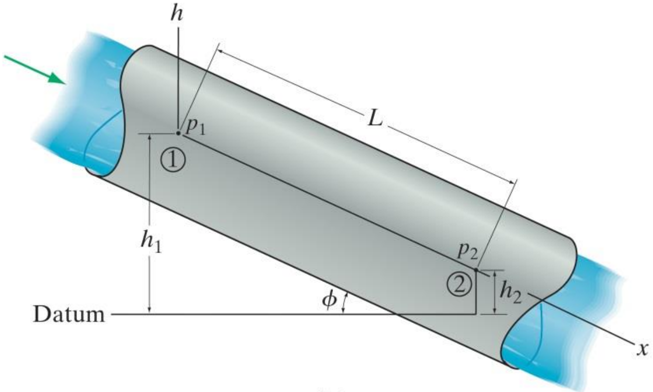
Steady Laminar Flow – Smooth Pipe

$$\tau(r) = \frac{r}{2}\frac{d}{dx}(P + \gamma h)$$

Shear Stress Distribution (Laminar +Turbulent)

$$u(r) = -\frac{(R^2 - r^2)}{4\mu}\frac{d}{dx}(P + \gamma h)$$

Velocity Profile (Laminar Flow Only)



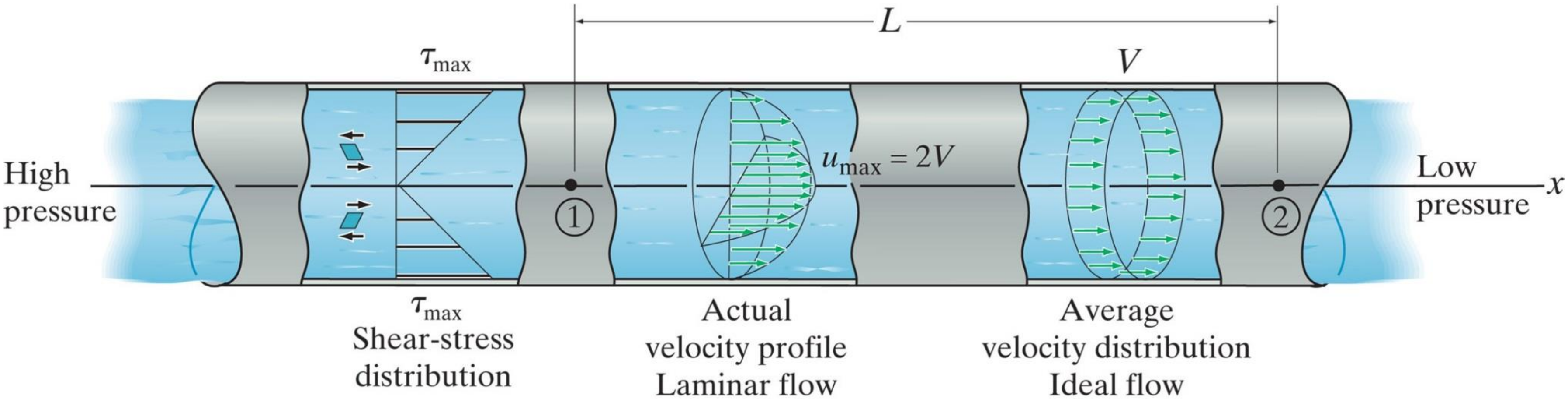
$$\frac{d}{dx}(P + \gamma h) = \frac{P_2 - P_1}{L} + \gamma\frac{h_2 - h_1}{L}$$

or

$$-\frac{d}{dx}(P + \gamma h) = \frac{P_1 - P_2}{L} + \gamma\frac{h_1 - h_2}{L}$$

Pressure and Elevation Gradients

Steady Laminar Flow – Smooth Pipe



Shear Stress and Velocity Profile for a Negative Pressure Gradient

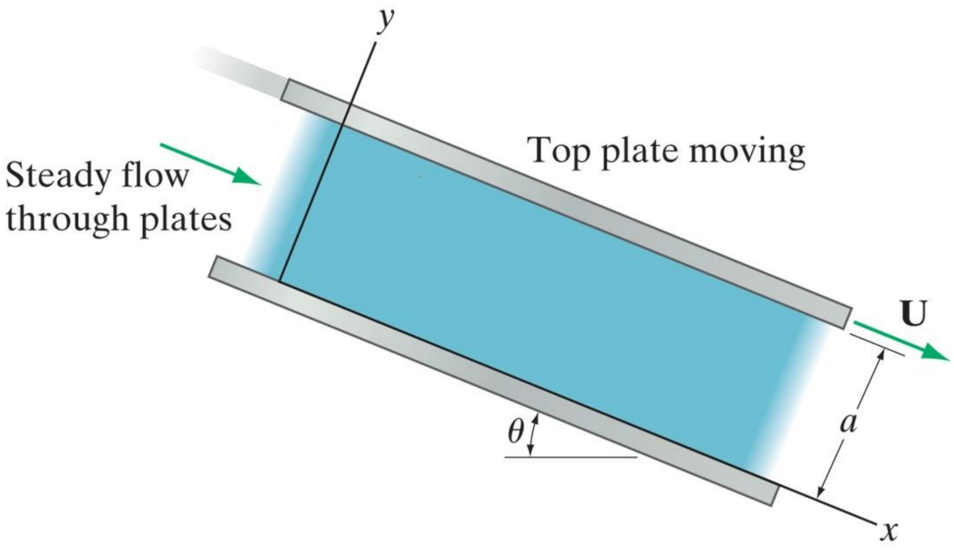
Also $$Q = \int_A \vec{v}\cdot\hat{n}\,dA = \int_A u\,dA = -\frac{\pi D^4}{128\mu}\frac{d}{dx}(P+\gamma h)$$

and $$-\frac{d}{dx}(P+\gamma h) = \frac{\gamma h_L}{L}$$

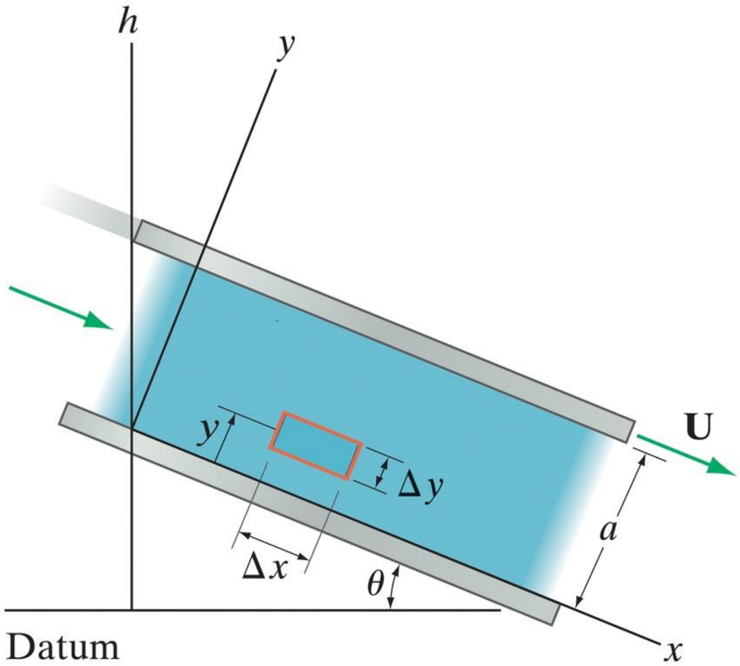
from Energy Eqn.

Thus, $$h_L = \frac{128\mu L Q}{\rho g \pi D^4} = \frac{32\mu L v}{\rho g D^2}$$

Steady Laminar Flow – Parallel Plates

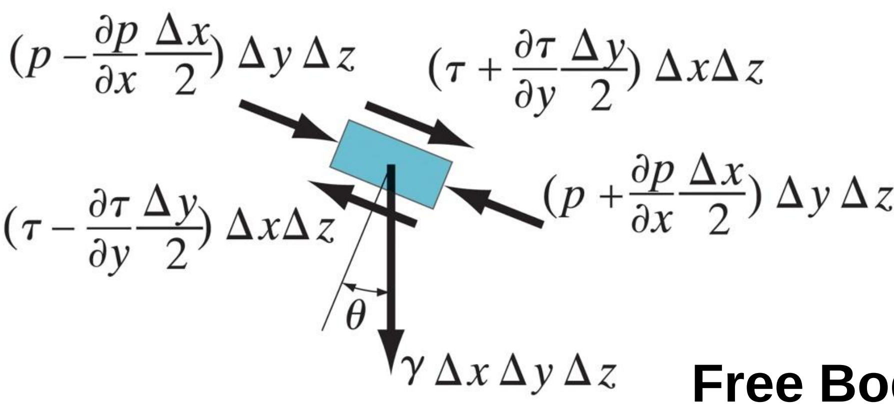


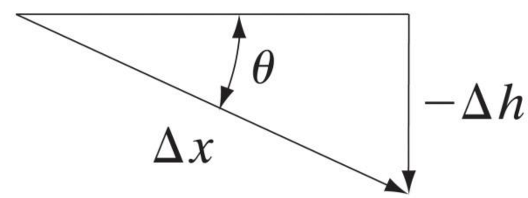
Basic Configuration



Coordinate Definitions

Steady Laminar Flow – Parallel Plates





Free Body Diagram

$$\tau(y) = \frac{U\mu}{a} + \left[\frac{d}{dx}(P + \gamma h)\right]\left(y - \frac{a}{2}\right)$$

Shear Stress Distribution
(Laminar +Turbulent)

$$u(y) = \frac{U}{a}y - \frac{1}{2\mu}\left[\frac{d}{dx}(P + \gamma h)\right](ay - y^2)$$

Velocity Profile
(Laminar Flow Only)

Steady Laminar Flow – Parallel Plates

Note that the above development used the fact that the sum of the pressure and elevation gradients are independent of y!!!

$$\frac{d}{dx}(P+\gamma h)=\frac{P_2-P_1}{L}+\gamma\frac{h_2-h_1}{L}$$

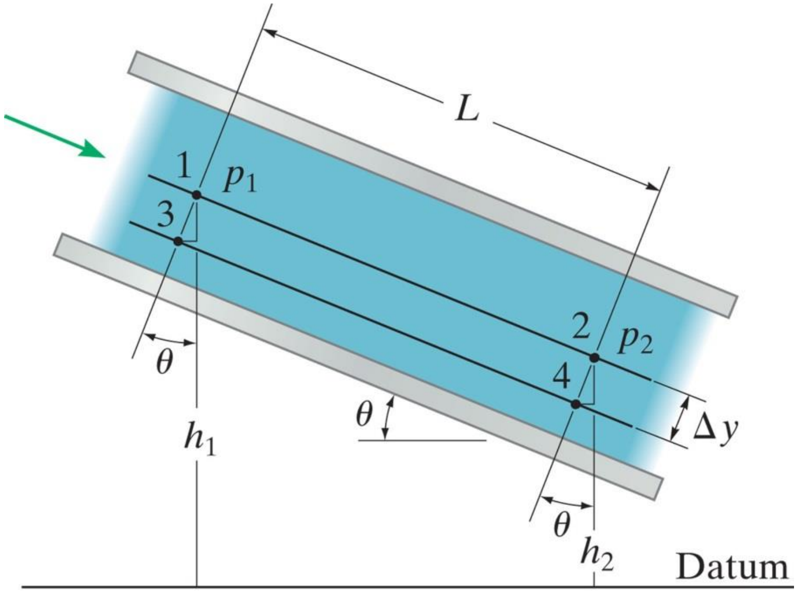
or

$$\frac{d}{dx}(P+\gamma h)=\frac{P_4-P_3}{L}+\gamma\frac{h_4-h_3}{L}$$

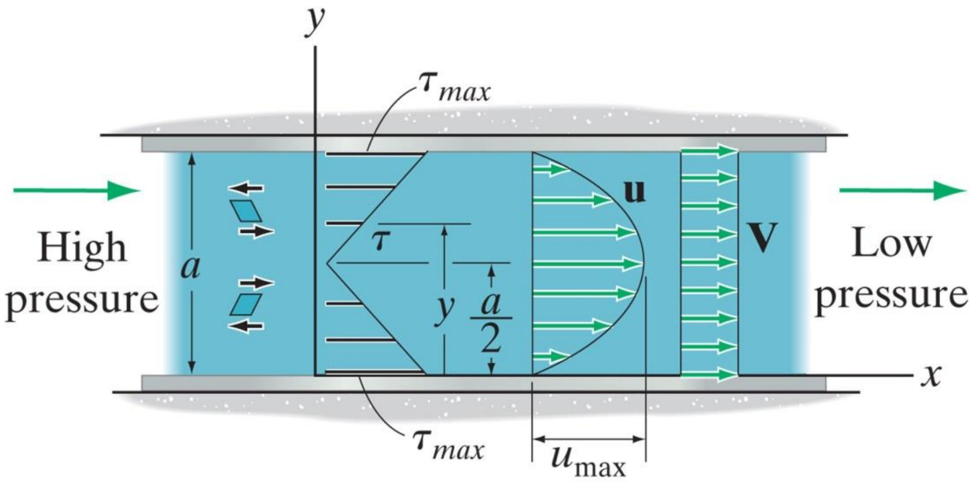
(see discussion in Hibbeler pg. 471)

and

$$-\frac{d}{dx}(P+\gamma h)=\frac{P_1-P_2}{L}+\gamma\frac{h_1-h_2}{L}$$



Steady Laminar Flow – Parallel Plates



Shear-stress distribution and actual and average velocity profiles with *negative* pressure gradient and no motion of the plates.

For U = 0 (no plate movement) and b = plate depth into page, we have

$$Q = \int_A \vec{v} \cdot \hat{n}\, dA = \int_A u\, dA$$

$$= -\frac{ba^3}{12\mu}\frac{d}{dx}\left(P + \gamma h\right)$$

and from Energy Eqn.

$$-\frac{d}{dx}\left(P + \gamma h\right) = \frac{\gamma h_L}{L}$$

Thus,

$$h_L = \frac{12\mu L Q}{\rho g b a^3} = \frac{12\mu L v}{\rho g a^2}$$

Steady Laminar Flow – Darcy Eqn.

Circular Pipe

$$h_L = \frac{128\mu LQ}{\rho g\pi D^4} = \frac{32\mu Lv}{\rho g D^2}$$

$$D_h = \frac{4\left(\pi D^2/4\right)}{\pi D} = D$$

$$h_L = (32)(2)\frac{\mu}{\rho v D}\frac{L}{D}\frac{v^2}{2g}$$

$$h_L = \frac{64}{Re}\frac{L}{D}\frac{v^2}{2g}$$

$$D_h = \frac{4A_f}{P_w}$$

$$h_L = f\frac{L}{D_h}\frac{v^2}{2g}$$

Darcy Eqn.

Parallel Plates (b >> a)

$$h_L = \frac{12\mu LQ}{\rho g b a^3} = \frac{12\mu Lv}{\rho g a^2}$$

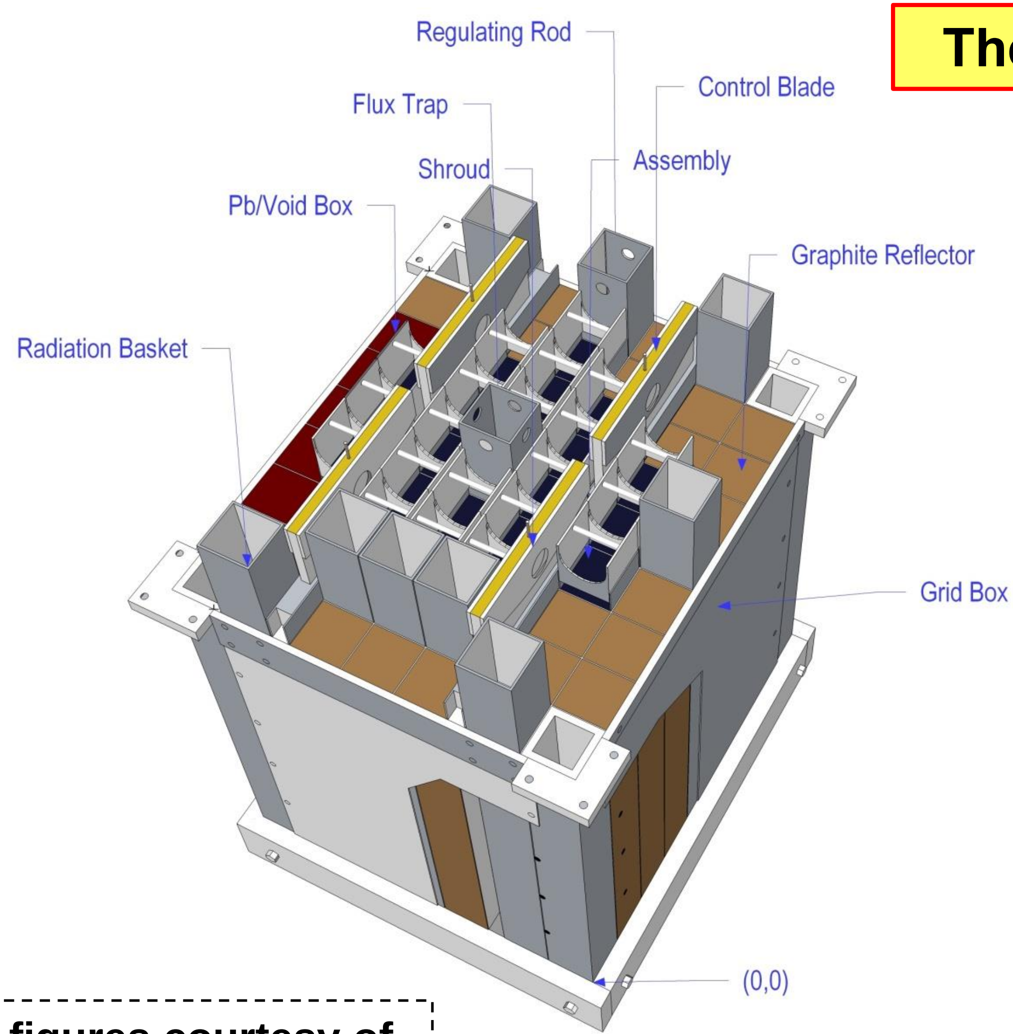
$$D_h = \frac{4ab}{2a+2b} \approx 2a$$

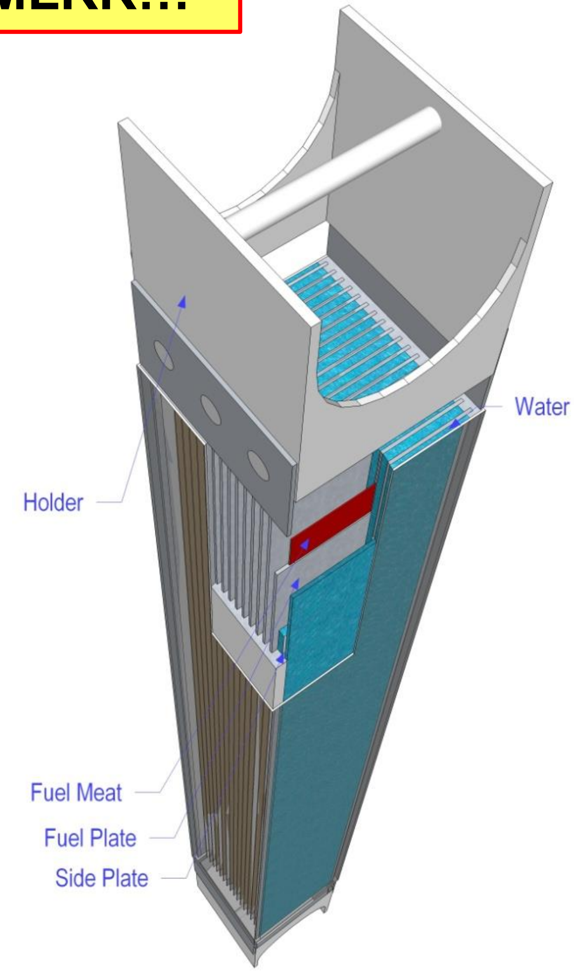
$$h_L = (12)(8)\frac{\mu}{\rho v 2a}\frac{L}{2a}\frac{v^2}{2g}$$

$$h_L = \frac{96}{Re_h}\frac{L}{D_h}\frac{v^2}{2g}$$

Ex. #2 -- ΔP in MTR Fuel

The UMLRR…





figures courtesy of Mr. Odera Dim

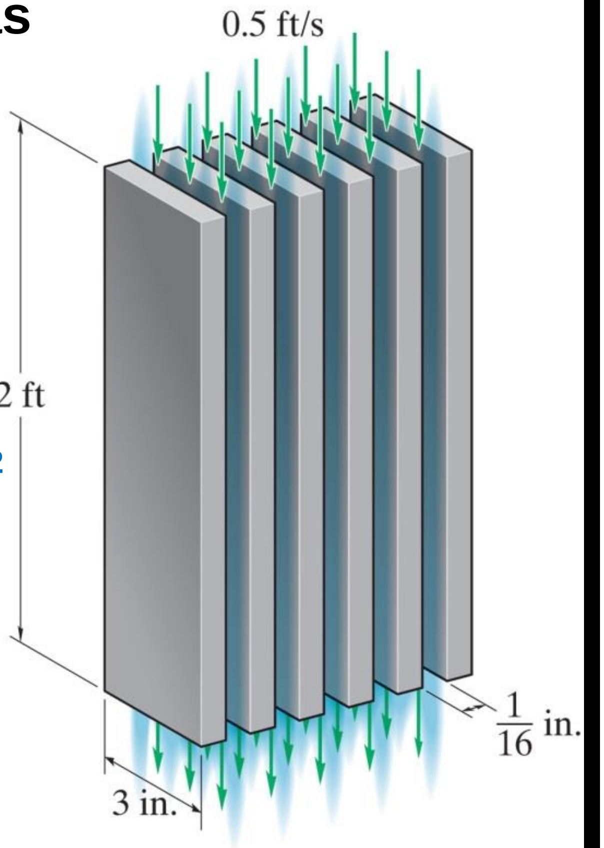
Ex. #2 (continued)

Consider the MTR fuel plate flow geometry as shown.
Water flows downward between the plates.

Neglecting end effects, estimate the ΔP over the 2 ft length of the fuel plates.

water properties:

$\rho = 1.820$ slugs/ft^3 & $\mu = 5.46\times10^{-6}$ lbf-s/ft^2



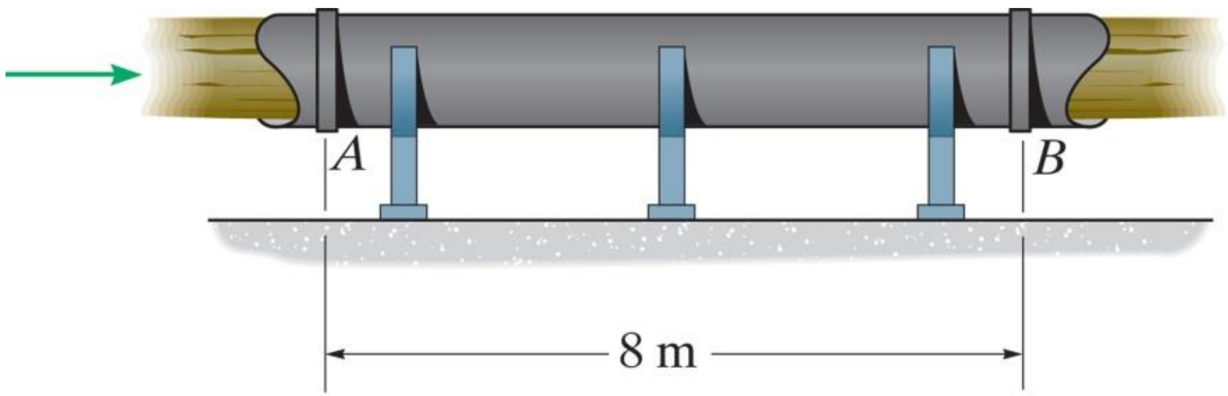
Ex. #3 -- ΔP and head Loss…

Oil has a flow rate Q = 0.004 m³/s through the 0.15 m diameter pipe.

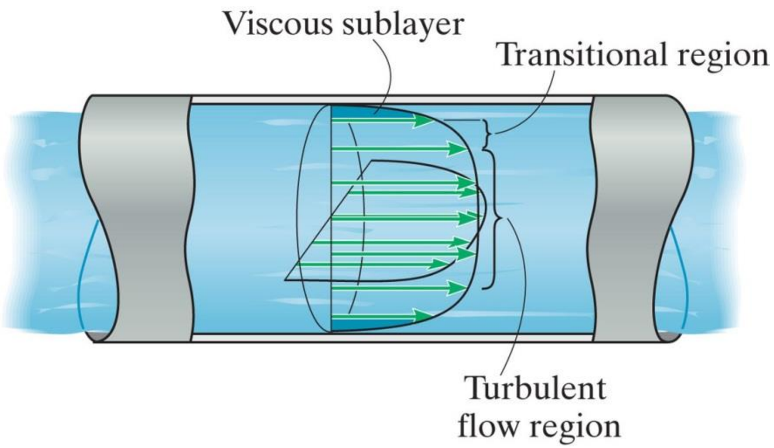
Determine the pressure drop in the 8-m long section and calculate the head loss per meter of pipe.

oil properties:

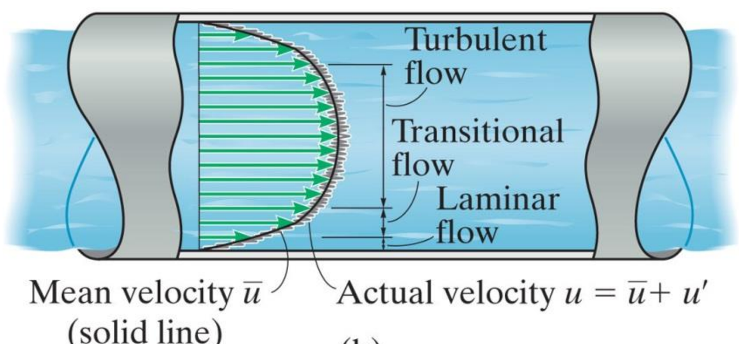
ρ = 900 kg/m³ and μ = 0.370 N-s/m²



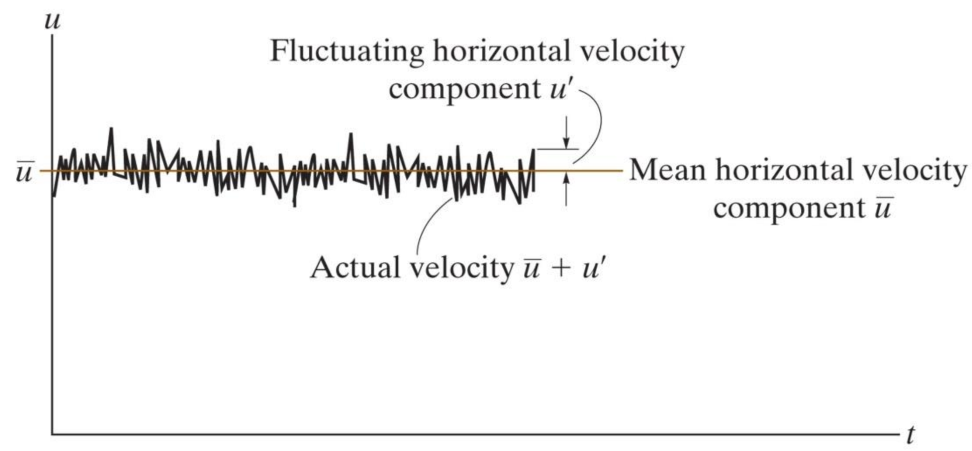
Fully Developed Turbulent Flow



(a)



(b)



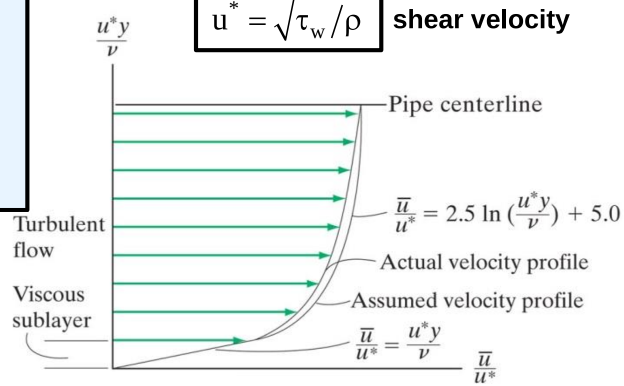
Horizontal velocity components of fluid particles passing through a control volume.

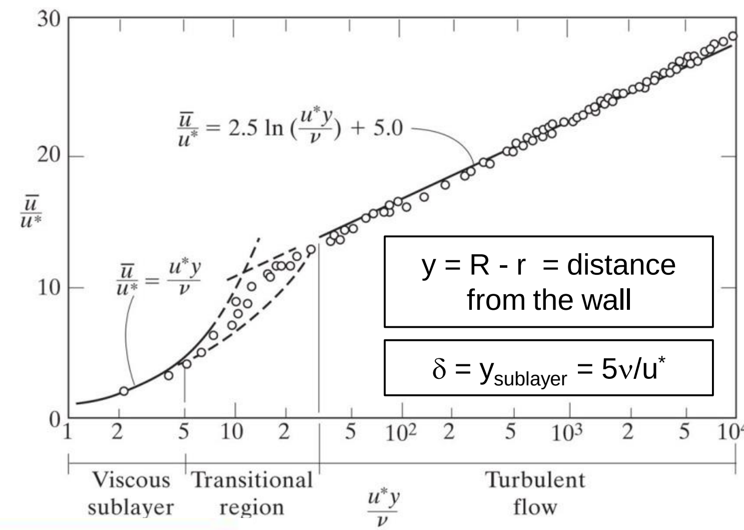
Fully Developed Turbulent Flow

UMASS LOWELL
Learning with Purpose

Dimensionless Velocity fit to Experimental Data

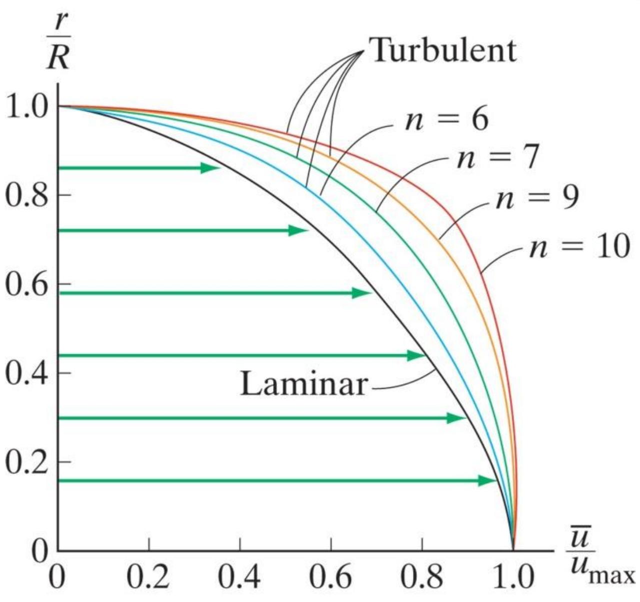
$u^* = \sqrt{\tau_w/\rho}$ **shear velocity**





y = R - r = distance from the wall

$\delta = y_{sublayer} = 5\nu/u^*$



Velocity Profiles

TABLE 9–1

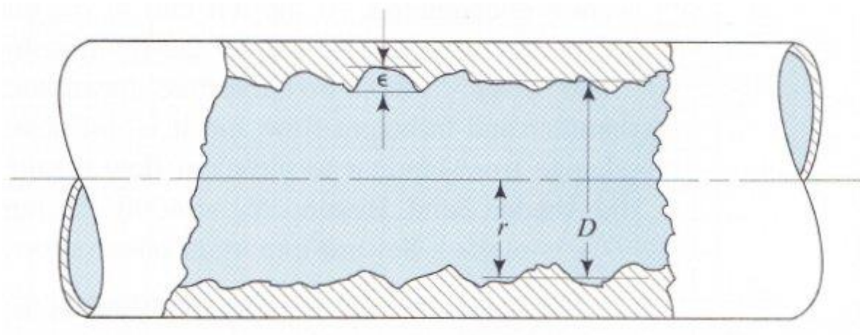
n	Re
6	$4\,(10^3)$
7	$1\,(10^5)$
9	$1\,(10^6)$
10	$3\,(10^6)$

$$\mathbf{u(y) = u_{max}\left(\frac{y}{R}\right)^{1/n}}$$

$$\mathbf{u(r) = u_{max}\left(1 - \frac{r}{R}\right)^{1/n}}$$

Power Law Model

Surface Roughness

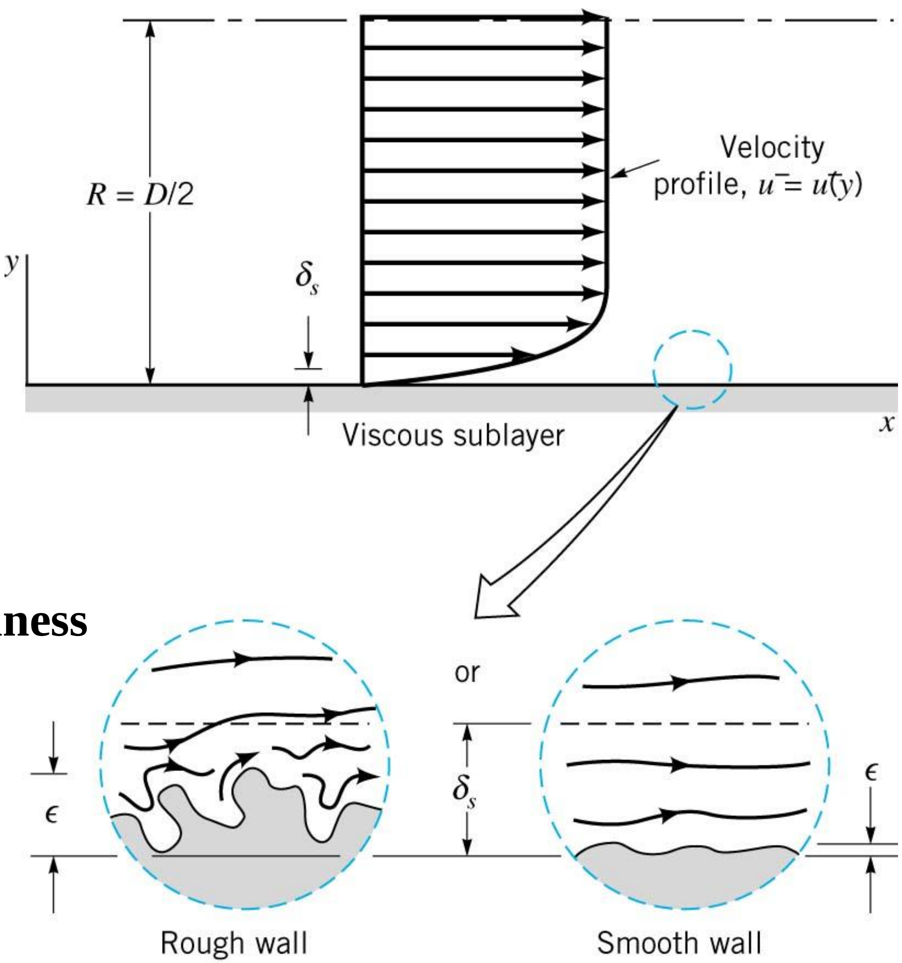


$$h_L = \color{red}{f}\,\frac{L}{D_h}\frac{v^2}{2g}$$

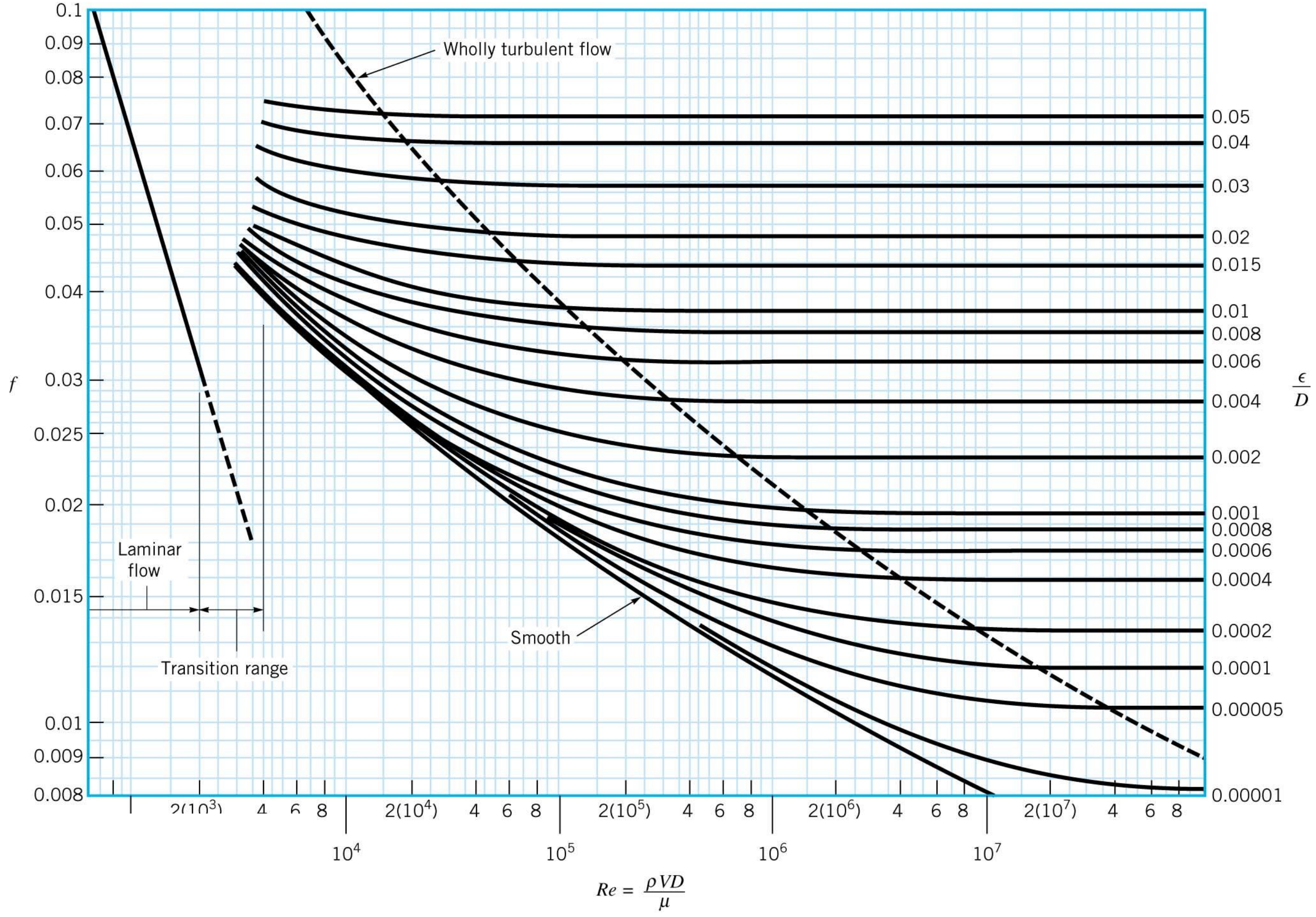
$$f = \phi\left(\frac{\varepsilon}{D}, Re\right)$$

where ε/D = relative roughness

Material	Roughness ϵ (m)	Roughness ϵ (ft)
Glass	Smooth	Smooth
Plastic	3.0×10^{-7}	1.0×10^{-6}
Drawn tubing; copper, brass, steel	1.5×10^{-6}	5.0×10^{-6}
Steel, commercial or welded	4.6×10^{-5}	1.5×10^{-4}
Galvanized iron	1.5×10^{-4}	5.0×10^{-4}
Ductile iron—coated	1.2×10^{-4}	4.0×10^{-4}
Ductile iron—uncoated	2.4×10^{-4}	8.0×10^{-4}
Concrete, well made	1.2×10^{-4}	4.0×10^{-4}
Riveted steel	1.8×10^{-3}	6.0×10^{-3}



The Moody Chart **(preview from Chapter 10)**



Ex. #4 -- Turbulent Flow ...

Water flows in the 2-in diameter smooth pipe with ΔP = 1.5 psi along the 2-ft length.

Determine the shear stress along the wall and at the pipe centerline. Also compute u_{max}.

water properties:

ρ = 62.4 lbm/ft^3 and ν = 16.6×10^{-6} ft^2/s