Crowe
7th Ed

Path lines
and
streak line

(4.8) from different perspective

implies $\frac{dV}{ds} = 0$ } no spatial dependence

Consider a uniform flow field with The following time dependent velocity vector

$\vec{V} = 3i - 4j$ $0 < t \leq 5s$

$\vec{V} = 5i$ $t > 5s$

m/s

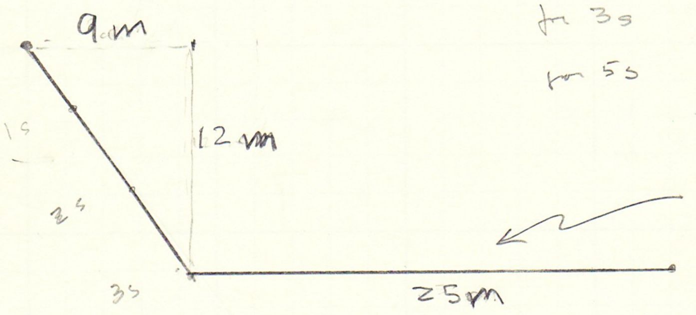
$|V| = \sqrt{u^2 + v^2}$

→ notice That The speed is constant
→ only The flow direction changes with time

ⓐ Draw a pathline at 10s for a single particle emitted at the origin at t = 2s

for 3s $\Delta x = 9$ m $\Delta y = -12$ m

for 5s $\Delta x = 25$ m $\Delta y = 0$

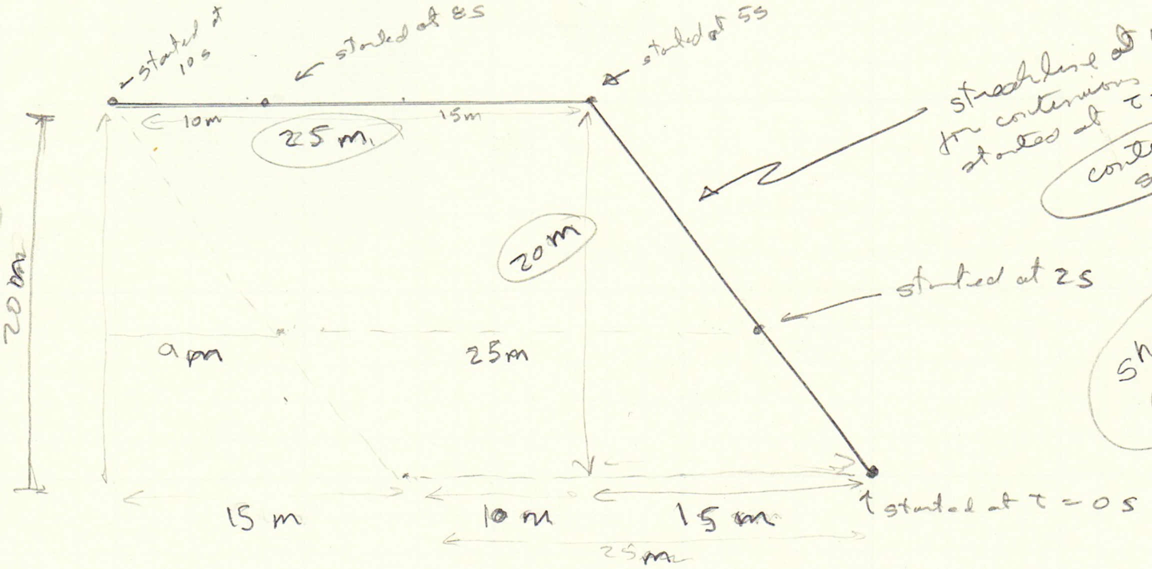


work forwards in time

path line at t=10s for particle emitted at 2sec.

individual particle

ⓑ Draw a streakline at t=10s for a continuous dye stream started at t=0s



streakline at 10s for continuous stream started at t=0s

continuous stream

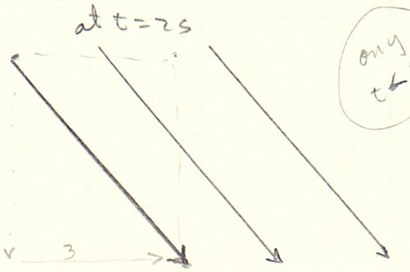
work backwards in time

snapshot at t=10s

do this first

ⓒ Draw streamlines at T=2s and at t=8s

at t=2s



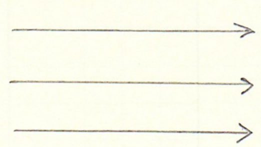
only t≤5s

$\frac{dy}{dx} = \frac{v}{u} = \frac{-4}{3}$

$\tan\theta = -4/3$

$\theta = -53.1°$

at t=8s



only t>5s

$\tan\theta = \frac{dy}{dx} = \frac{v}{u} = \frac{0}{5} = 0$

$\theta = 0$

streamlines for uniform flow field at different times

Hibbeler Ex. 3.5 | also Munson 3E Ex 4.1 | 4.2 | 4.5 | Flow Field Visualization | steady flow #1 | $\frac{1}{2}$

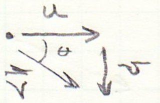
XYFLOW1

Consider the flow field described by

$$\vec{V} = u\hat{i} + v\hat{j} = 2x\,\hat{i} - 2y\,\hat{j}$$

2D steady flow

1. Can plot the velocity vector field at each x, y point



$$\tan\theta = \frac{dy}{dx} = \frac{v}{u} = \underset{\uparrow}{\frac{-y}{x}}$$

This case

→ can plot in Matlab with quiver command

quiver.( x, y, $V_x$, $V_y$ ) ← see XYFLOW1.m
(u) (v)

2. Often it is convenient to plot the direction field

- This is the same as the vector field with unit vectors so that we emphasize the direction of flow only

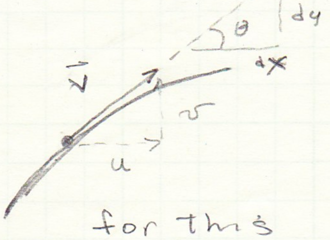
unit vectors $$\frac{\vec{V}}{|\vec{V}|} = \frac{u}{|\vec{V}|}\hat{i} + \frac{v}{|\vec{V}|}\hat{j}$$

where $$|\vec{V}| = \sqrt{u^2 + v^2}$$ ← speed

→ can plot the speed different ways
for example: a contour plot over the direction field

3. Can plot the streamlines over the direction field

velocity vector is everywhere tangent to the streamline — shows the direction of Flow



$$\frac{dy}{dx} = \frac{v}{u}$$ ← y component of $\vec{V}$
← x component of $\vec{V}$

for this case $$\frac{dy}{dx} = \frac{-2y}{2x} = -\frac{y}{x}$$

$$\therefore \frac{dy}{y} = -\frac{dx}{x}$$

$$\ln y = -\ln x + \ln c$$

$$\ln y + \ln x = \ln yx = \ln c$$

or $$\boxed{yx = c} \Rightarrow \boxed{y = \frac{c}{x}}$$

defines the streamlines

stream function
$\psi(x,y) = c$
here $\psi(x,y) = xy$

Note also that the ODE can be easily solved with Matlab's ode23 routine

```
fxy = @(x,y) - y./x                 anonymous function
[x,y] = ode23 (fxy, [xo, xf], yo)
```

do this for several $x_0, y_0$ sets

4. We can also derive and plot the acceleration.

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + \frac{\partial \vec{V}}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial \vec{V}}{\partial y}\frac{\partial y}{\partial t}$$

$$\boxed{\vec{a} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y}}$$

for 2-D case Cartesian geometry

recall we generalized this as

$$\vec{a} = \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \vec{\nabla})\vec{V}$$

here $\vec{V} = u\hat{i} + v\hat{j} = 2x\hat{i} - 2y\hat{j}$

$$\therefore \frac{\partial \vec{V}}{\partial x} = 2i \quad , \quad \frac{\partial \vec{V}}{\partial y} = -2\hat{j} \quad \text{and} \quad \frac{\partial \vec{V}}{\partial t} = 0$$

steady flow

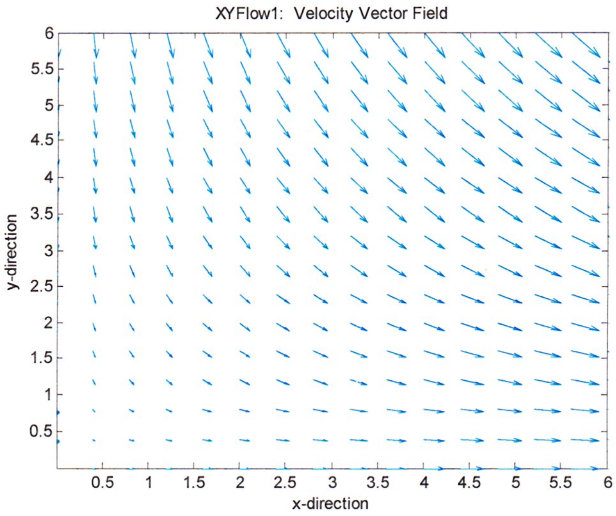
$$\therefore \vec{a} = 2x(2\hat{i}) + (-2y)(-2\hat{j})$$

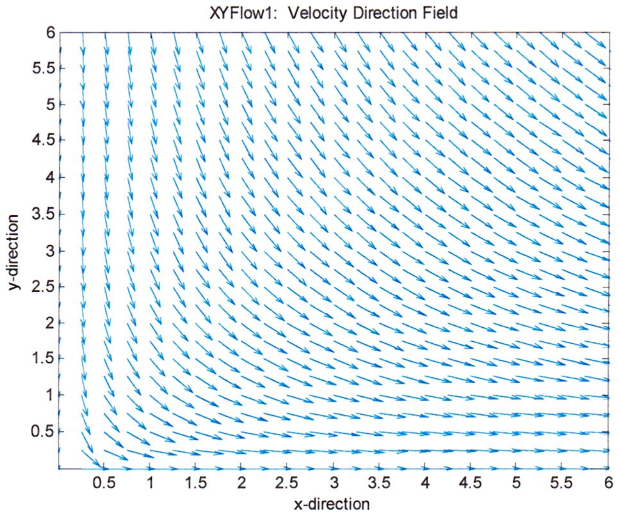
or $\boxed{\vec{a} = 4x\hat{i} + 4y\hat{j}}$

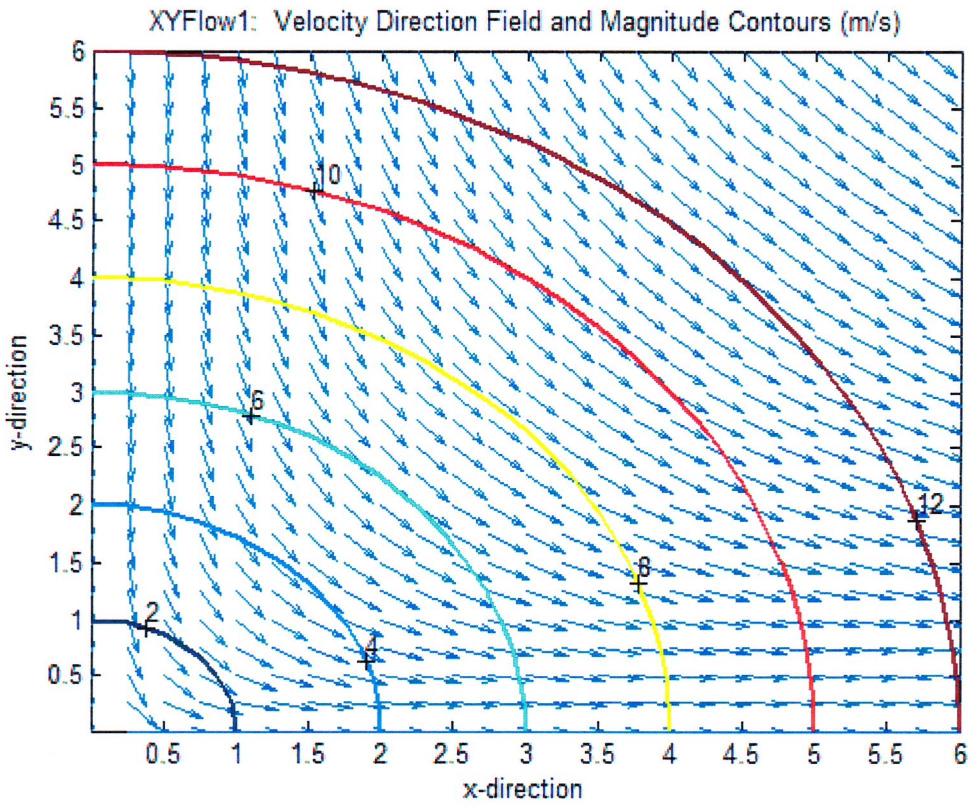
this can be plotted in a similar fashion to the vector velocity field

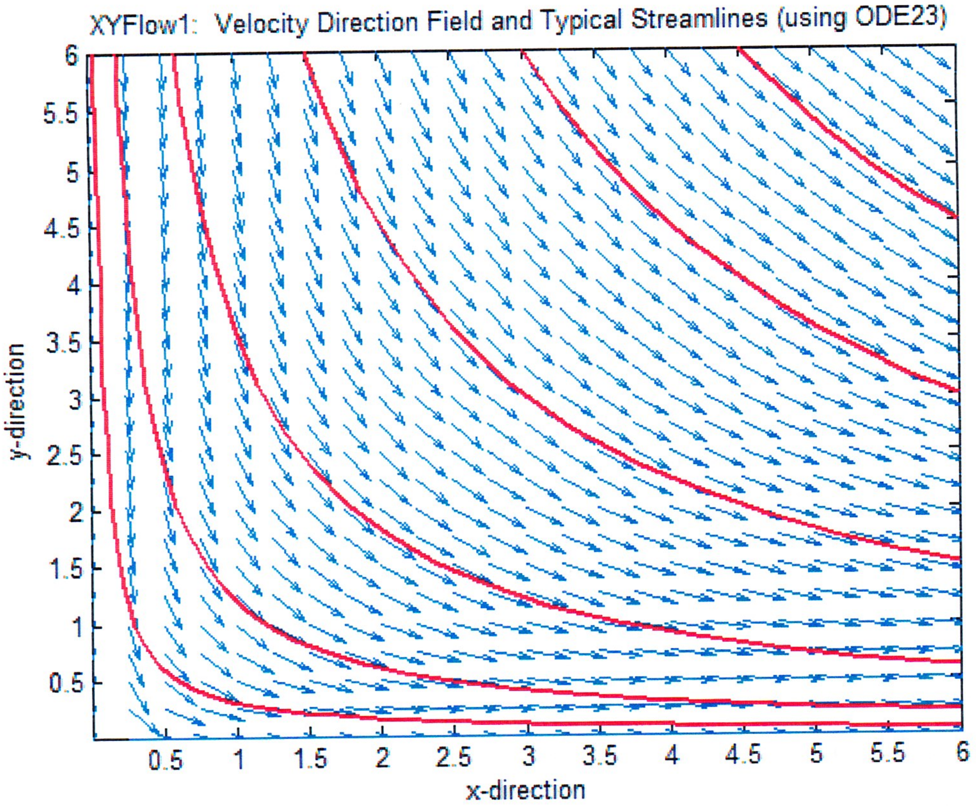
see xyflow1.m

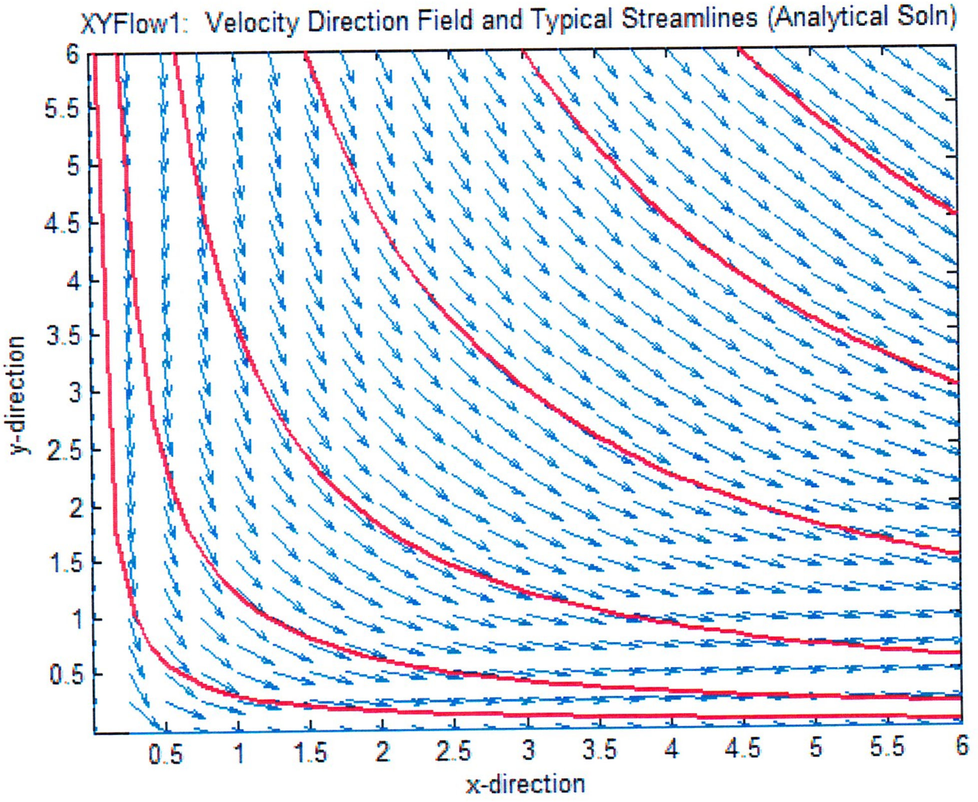
# XYFlow1 – Flow Visualization Example

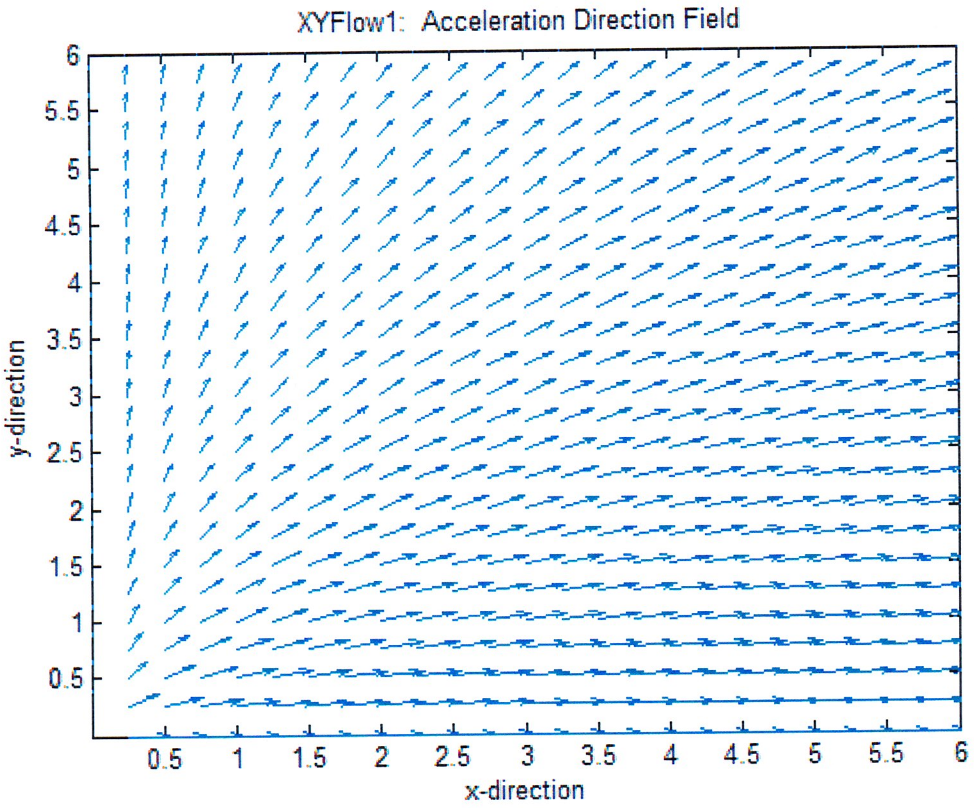


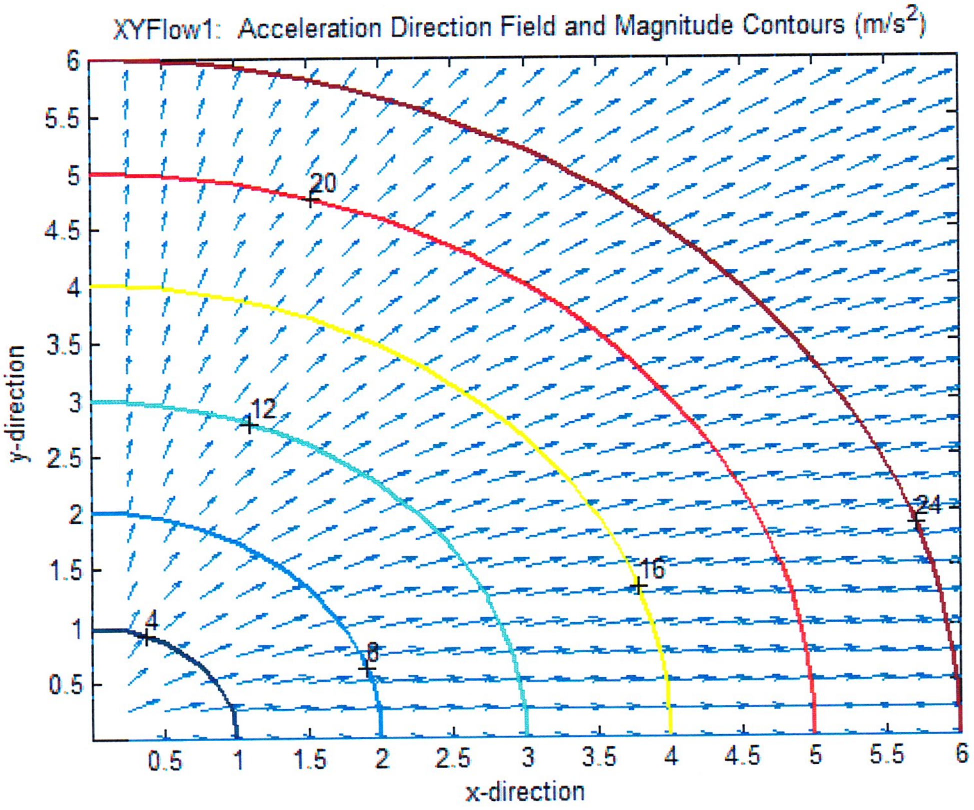





XYFlow1: Velocity Direction Field and Typical Streamlines (using ODE23)
y-direction
x-direction
6
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6


XYFlow1: Velocity Direction Field and Typical Streamlines (Analytical Soln)
y-direction
x-direction
6
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6


XYFlow1: Acceleration Direction Field
y-direction
x-direction
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6


XYFlow1: Acceleration Direction Field and Magnitude Contours (m/s²)
y-direction
x-direction
4
8
12
16
20
24
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6

unsteady-flow1

Consider a 2-D velocity field given by

$$u = 5 \text{ m/s} \quad \text{and} \quad v = -2t \text{ m/s}$$

where t is in seconds. Note that the flow field is uniform (i.e. independent of space) but it is unsteady
↑ Time dependent.

a. Generate a plot that shows the magnitude of the velocity vs Time for $0 < t < 5s$

$$\vec{v} = u\hat{i} + v\hat{j}$$

$$|\vec{v}| = \sqrt{u^2 + v^2}$$

$$= \sqrt{25 + 4t^2} \quad \leftarrow \text{speed vs time}$$

b. Generate a plot that shows the direction of travel vs time.

Note: Since the flow field is uniform in space, all the streamlines will have the same angle $\theta$

However, the streamlines and $\theta$ will be time dependent in a unsteady flow field.

The direction is given by $\tan\theta = \frac{dy}{dx} = \frac{v}{u}$

$$\text{or} \quad \theta(t) = \tan^{-1}\left(\frac{v}{u}\right)$$

$$= \tan^{-1}\left(-\frac{2}{5}t\right) \quad \leftarrow \theta \text{ vs time}$$

c. If a single drop of dye is released at $t=0$ at the origin, plot the path of this fluid particle over 5 seconds. this represents the pathline.

$$u = \frac{dx}{dt} \quad \therefore \int_0^t dx = x(t) = \int_0^t u(t)\,dt$$

← observation Time (upper limit $t$); ← release Time (lower limit $0$)

$$= \int_0^t 5\,dt = 5t\Big|_0^t$$

$$\boxed{x(t) = 5t}$$

integrate from $t_0$ to some t to generate the pathline

$$v = \frac{dy}{dt} \quad \therefore \int_0^t dy = y(t) = \int_0^t v(t)\,dt$$

$$= \int_0^t -2t\,dt = -\frac{2t^2}{2}\Big|_0^t$$

plot y(t) vs x(t)

$$\boxed{y(t) = -t^2}$$

d. If a continuous stream of dye particles is emitted at the origin, what is the spatial distribution of these particles at t = 5 seconds? this represents a streakline.

To generate the streakline at $t_f = 5$ sec, we need to integrate from $\tau$ to $t_f$, where $\tau$ is the time when a particle is released

$$u = \frac{dx}{dt} = 5 \text{ m/s} \qquad \therefore \int_{\underset{\nwarrow \text{ release point}}{x_0}}^{\overset{\leftarrow \text{ position at } t_f}{x_f}} dx = \int_{\underset{\nwarrow \text{ Time of release}}{\tau}}^{\overset{\leftarrow \text{ Time of observation}}{t_f}} u\,dt$$

integrate from $\tau$ to time $t_f$, where $t_f$ is the observation time and $\tau$ is the release time

$$x_f - \cancel{x_0}^{\,0} = 5\Big|_{\tau}^{\tau_f} = 5(t_f - t)$$

$$\boxed{x_f = 5(t_f - t)}$$

$$v = \frac{dy}{d\tau} = -2\tau \text{ m/s} \qquad \therefore \int_{y_0}^{y_f} dy = \int_{\tau}^{t_f} v\,dt$$

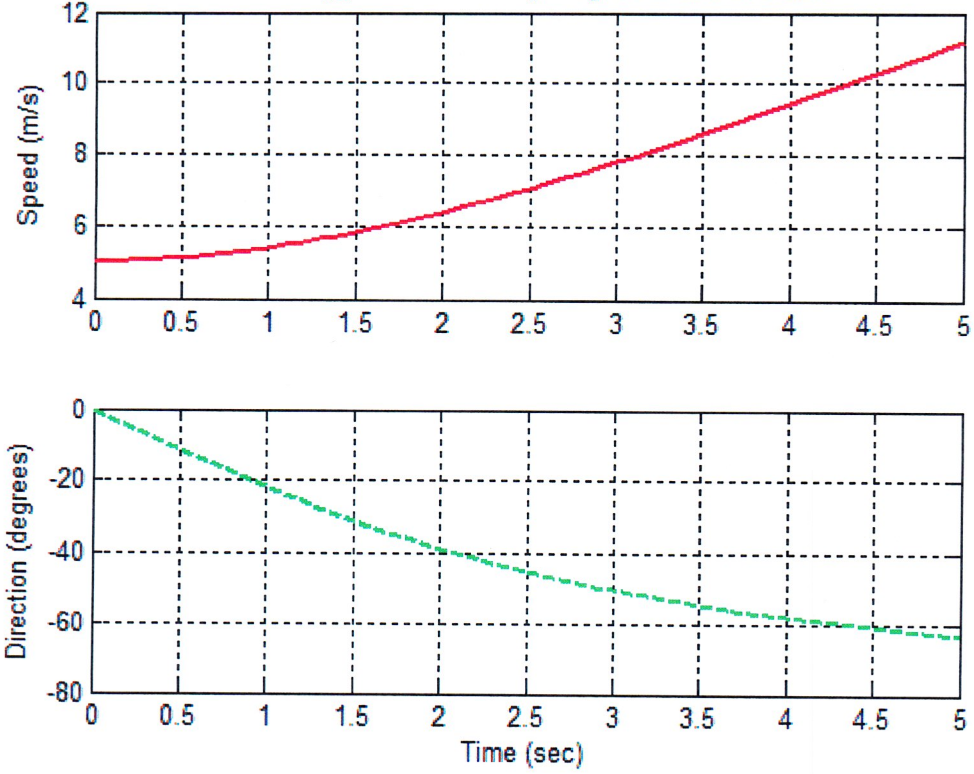
$$y_f - \cancel{y_0}^{\,0} = -\tau^2\Big|_{\tau}^{t_f}$$

$$\boxed{y_f = -(t_f^2 - \tau^2)}$$

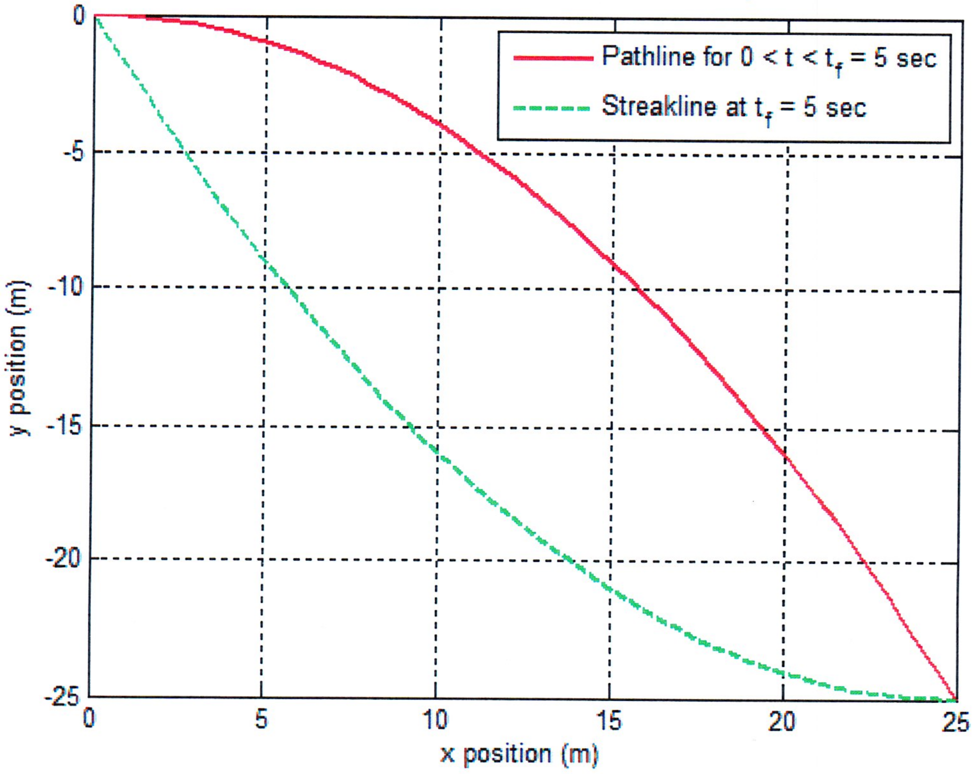
plot $y_f$ vs $x_f$ to get the streakline at $t_f$.

see unsteady-flow1

# Unsteady_Flow1 -- Flow Visualization Example



Unsteady_Flow1: Velocity Magnitude and Direction



Unsteady_Flow1: Pathline and Streakline at $t_f$ = 5 sec