CHEN.3030 Fluid Mechanics

IV. Flow Rates, Reynolds Transport Theorem, and the Continuity Equation

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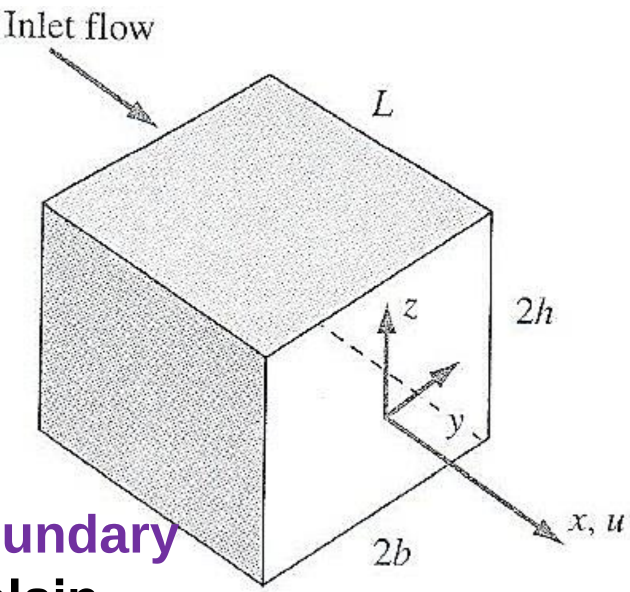
See Chapter 4 (sections 1–4) in your text by Hibbeler

Ex #1: Volume Flow Rates

An incompressible fluid flows steadily through the rectangular duct shown in the sketch. The exit velocity profile is given approximately by

$$u(y,z) = u_{max}\left(1-\frac{y^2}{b^2}\right)\left(1-\frac{z^2}{h^2}\right)$$



a. Does this profile satisfy the correct boundary conditions for viscous fluid flow? Explain…

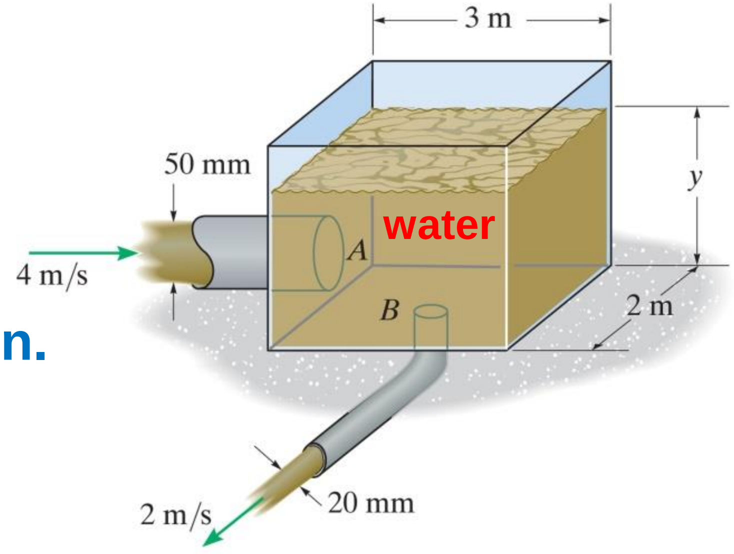
b. Find an analytical expression for the volume flow rate Q at the exit.

c. If the inlet flow rate is 300 ft^3/min, estimate u_{ave} and u_{max} at the exit in m/s for b = h = 10 cm.

Ex #2: Reynolds Transport Theorem

The Reynolds's Transport Theorem is given by

$$\frac{d}{dt}B_{sys} = \frac{d}{dt}\int_{CV} b\rho\, dr^3 + \int_{CS} b\rho\, \vec{v}\cdot\hat{n}\, dA$$



Within the context of the continuity eqn. applied to the flow situation in the diagram, determine the following quantities (also explain your result) :

a. What is b?

b. Determine the value of $\frac{d}{dt}B_{sys}$.

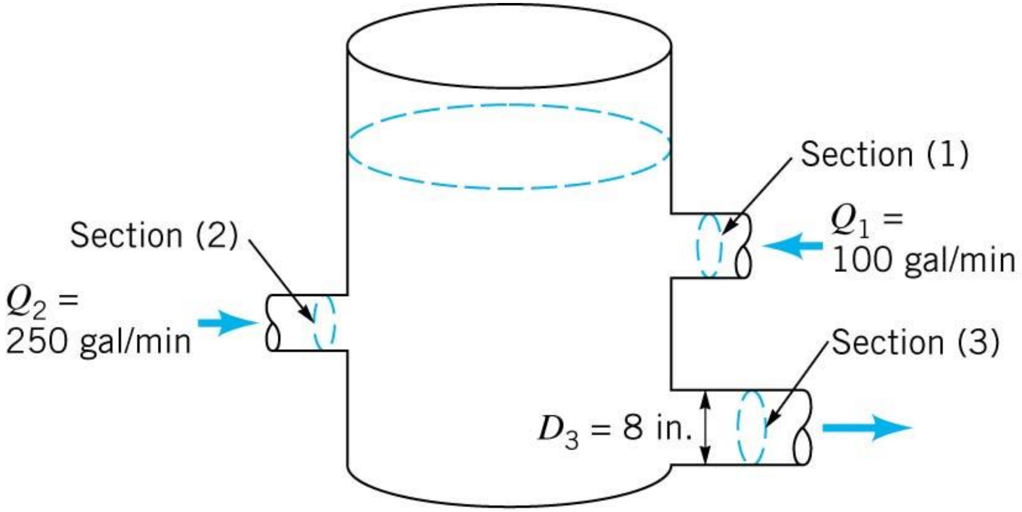
c. Determine the value of the term $\int_{CS} b\rho\, \vec{v}\cdot\hat{n}\, dA$.

d. Determine the term $\frac{d}{dt}\int_{CV} b\rho\, dr^3$.

Ex #3: Steady Incompressible Flow

Water enters a cylindrical tank through two pipes at a rate of 250 gpm and 100 gpm. The level of the tank remains constant.

Determine the average velocity of the flow through the 8″ diameter exit pipe in ft/s.

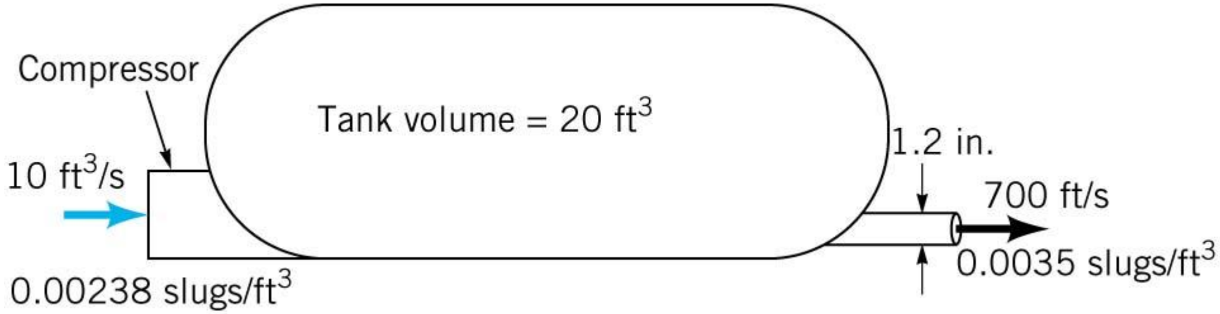


Note: 7.48 gal = 1 ft^3

Ex #4: Unsteady Compressible Flow

Air at standard conditions enters the compressor shown at a rate of 10 ft^3/s. It leaves the tank through the 1.2″ diameter pipe with a uniform speed of 700 ft/s. The inlet and outlet densities and the tank volume are given (as shown in the diagram).

Determine the rate of change of mass of air within the tank in lbm/s.



Note: 1 slug = 32.174 lbm

Ex #5: Non-Uniform Flows

Oil flows in the pipe segment as shown in the sketch.

Determine the maximum velocity, v_{max}, of the oil as it emerges at Point C if the velocity distribution at that point is given by

$$v_c = v_{max}\left(1 - 100r^2\right)$$

where r is in meters measured from the pipe centerline.