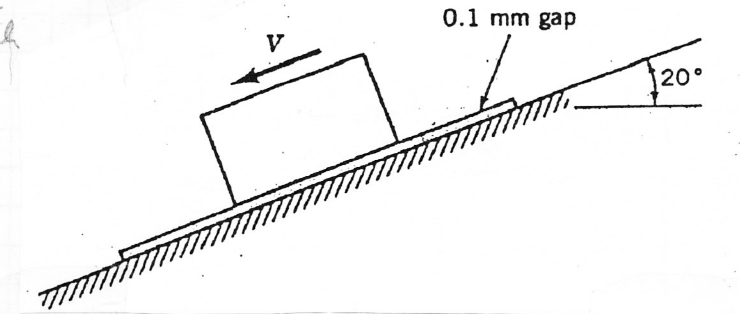
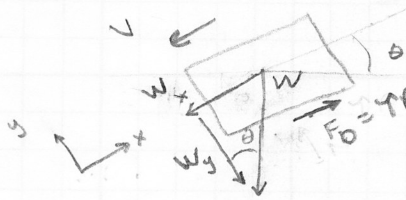
(1.56) A 10 kg block slides down a smooth inclined surface as shown. Determine the terminal velocity of the block if the 0.1 mm gap between the block and the surface contains oil with $\mu = 0.38\ N\text{-}s/m^2$. Assume that the velocity profile in the gap is linear, and the area of the block in contact with the oil is $0.2\,m^2$.



A FBD for this system gives



at equilibrium $\sum F_x = 0$

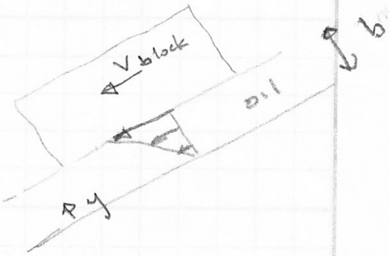
$\therefore\ \tau A - W_x = 0$

where $W_x = W \sin\theta$

$\therefore$ we have $\tau A = W \sin\theta$

or $\tau = \dfrac{W \sin\theta}{A} = \dfrac{(10\,kg)(9.81\,m/s^2)(\sin 20^\circ)}{0.2\,m^2}$

$$\boxed{\tau = 167.8\ N/m^2}$$



But we also know that

$\tau = \mu \dfrac{dV(y)}{dy}$ Newtonian fluid

where $V(y)$ is the velocity profile in the small gap between the inclined plane and the block. The no-slip condition forces $V(0) = 0$ and $V(b) = V_{block}$ (b ← gap thickness). Thus, if we assume a linear profile,

$$\boxed{V(y) = V_{block}\,\frac{y}{b}}$$

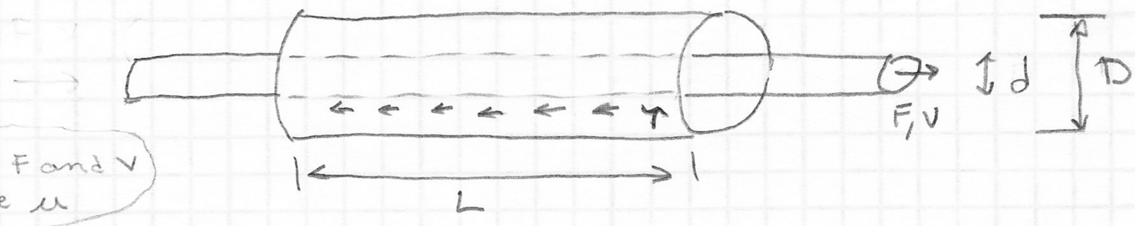
and $\dfrac{dV(y)}{dy} = \dfrac{V_{block}}{b}$

$\therefore\ V_{block} = \dfrac{b\,\tau}{\mu} = \dfrac{(1\times10^{-4}\,m)(167.8\,N/m^2)}{0.38\,N\text{-}s/m^2} = \boxed{0.0442\ m/s}$

ans

A Simple Viscometer

Consider a horizontal shaft of diameter d being pulled along the axial centerline of a bearing sleeve of diameter D. The clearance is filled with the fluid of interest.



Measure F and V to give μ

At equilibrium, the force needed to pull the rod through the sleeve at a constant velocity, V, is exactly balanced by the viscous friction along the sides of the shaft

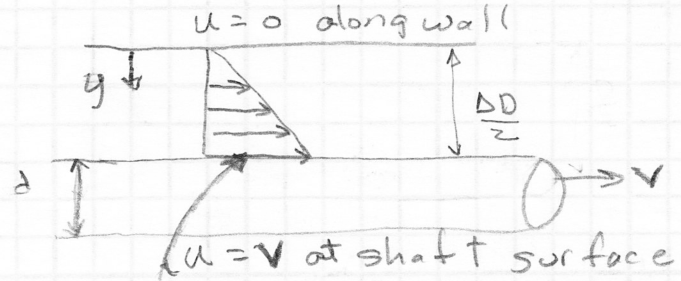
Newtonian fluid

$$\therefore\ F = F_D = \tau A = \mu \frac{du}{dy} (\pi d L)$$

where du/dy is the velocity gradient within the fluid in the gap region.

$\Delta D = D - d$

For a very small gap, $\Delta D/2$, the velocity profile will be nearly linear



no slip condition

$$\therefore\ \frac{du}{dy} = \frac{V - 0}{\Delta D/2} = \frac{2V}{\Delta D}$$

$$\therefore\ F = \mu \left(\frac{2V}{\Delta D}\right) \pi d L$$

$u(y) = \frac{Vy}{b} = \frac{2Vy}{\Delta D}$ (b: gap thickness)

$u(y) = \frac{2Vy}{D-d}$

$$\text{or}\quad \boxed{\mu = \frac{F\,\Delta D}{2\pi d L V}}$$

where $\Delta D = D - d$

measure F and V for a given geometry and one can determine μ of the fluid

Given that $d = 6\,cm$, $L = 40\,cm$ $\Delta D = 0.02\,cm$

What is the kinematic viscosity of the test fluid (sg = 0.88) if the measured steady state velocity was 0.4 m/s for an applied force of 800 N.

Putting in the values (and units) gives:

$$\mu = \frac{F \Delta D}{2\pi D L V} = \frac{(800\,N)(0.02\,cm)}{2\pi(6\,cm)(0.4\,m)(0.4\,m/s)}$$

$$= 2.653\ \frac{N\text{-}s}{m^2}$$

$$\boxed{\mu = 2.65\ Pa\text{-}s}$$

This is a very viscous fluid!!! (like cornsyrup or honey)

and

$$\nu = \frac{\mu}{\rho} = \frac{\mu}{sg\,\rho_w} = \frac{2.653\left(kg\frac{m}{s^2}\right)\left(\frac{s}{m^2}\right)}{(0.88)(1000\,kg/m^3)}$$

$$\boxed{\nu = 3.015\times10^{-3}\,\frac{m^2}{s}} \times \frac{1\,St}{10^{-4}\,m^2/s}$$

or $\boxed{\nu = 30.2\ St}$