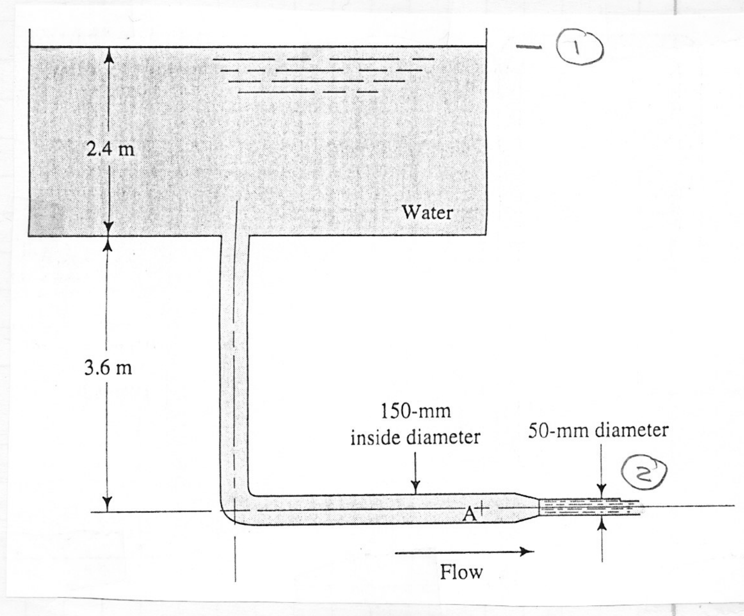
For the system shown compute the following:

a. volume flow rate from the nozzle

b. pressure at pt. A

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(a) Pick pt 1 at the surface and pt 2 at the exit of the nozzle.

Then

$$\cancel{\frac{P_1}{\gamma}}^{0} + \underbrace{\cancel{\frac{V_1^2}{2g}}^{0}}_{\text{small}} + z_1 = \cancel{\frac{P_2}{\gamma}}^{0} + \frac{V_2^2}{2g} + z_2$$

$P_1 = P_2 = 0$ gage and $V_1 << V_2$ because of large area of reservoir

$$\therefore \frac{V_2^2}{2g} = z_1 - z_2$$

or $V_2 = \sqrt{2g(z_1 - z_2)}$

$$= \sqrt{(2)(9.8\,\tfrac{m}{s^2})(6.0\,m)} = \boxed{10.84\ m/s}$$

and $Q_2 = V_2 A_2 = \left(10.84\,\frac{m}{s}\right)\left(\frac{\pi}{4}\right)(0.05\,m)^2$

$$= 2.129 \times 10^{-2}\,\frac{m^3}{s} = \boxed{0.0213\ m^3/s}$$

ans

(b) write the Bernoulli Eqn from pt A to pt 2

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + \cancel{z_A} = \cancel{\frac{P_2}{\gamma}}^{0} + \frac{V_2^2}{2g} + \cancel{z_2}$$

$$P_A = \frac{\gamma}{2g}\left(V_2^2 - V_A^2\right)$$ but

continuity eqn

$$\boxed{V_A A_A = V_2 A_2}$$

$$V_A = \frac{A_2 V_2}{A_A} = \left(\frac{D_2}{D_A}\right)^2 V_2 = \left(\frac{1}{3}\right)^2 V_2$$

$$\boxed{V_A = V_2/9}$$

$$= \frac{\gamma V_2^2}{2g}\left(1 - \frac{1}{81}\right) \leftarrow \frac{80}{81}$$

$$= \frac{40}{81}\left(\frac{9.81\,kN/m^3}{9.8\,m/s^2}\right)\left(10.84\,\frac{m}{s}\right)^2 = \boxed{58.1\ kPa}$$

ans

Note
$P_A \neq \gamma h$
$= (9.81)(6)$
$= 58.9\ kPa$

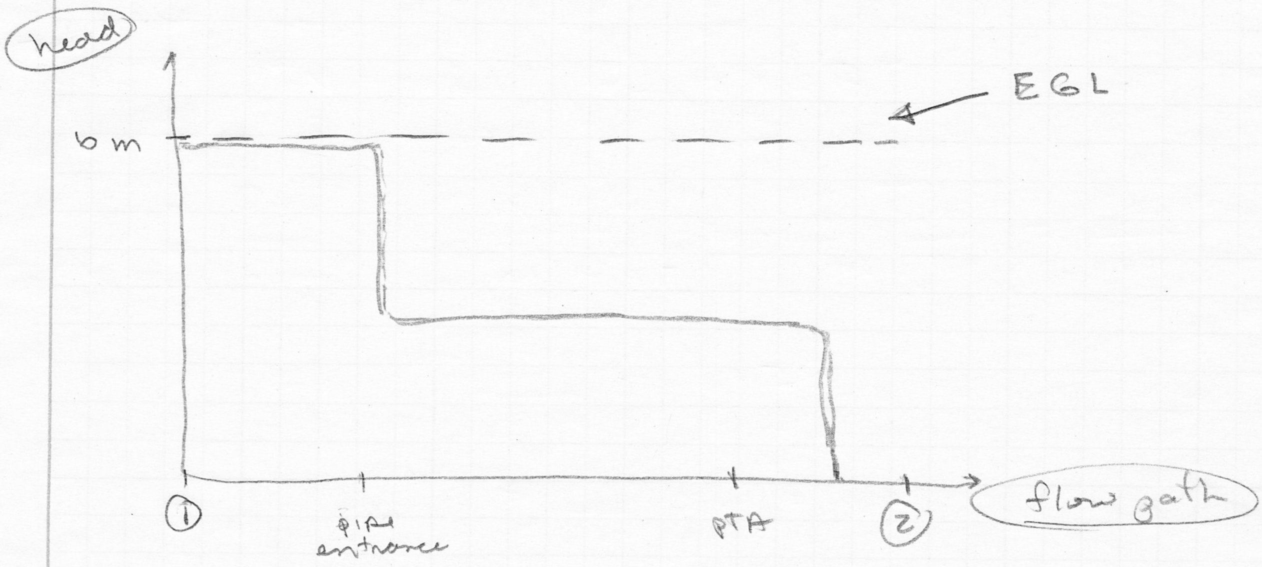
total head $= \frac{P}{\gamma} + \frac{V^2}{2g} + z = H_{Tot}$

hydraulic head $= \frac{P}{\gamma} + z = H_{hyd}$

Energy Grade Line (EGL) – plot of <u>total head</u> along the flow path (a streamline)

Hydraulic Grade Line (HGL) – plot of <u>hydraulic head</u> along flow path

Note for no energy gains / losses (ie. Bernoulli Eqn) the EGL is constant (straight line)




at pt 1 $H_{TOT} = z = 6\,m$ and $H_{hyd} = H_{TOT}$

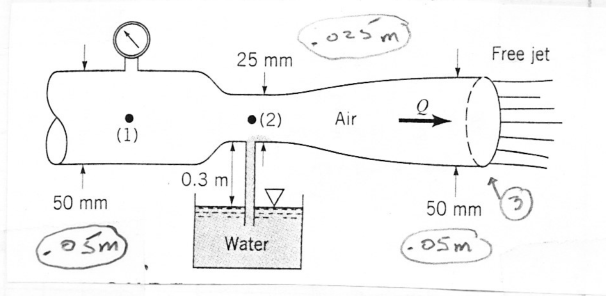
just before pipe $H_{TOT} = \frac{P}{\gamma} + z = 6\,m$ " $H_{hyd} = H_{TOT}$

just inside pipe $H_{tot} = \frac{P}{\gamma} + \frac{V^2}{2g} + z = 6\,m$ $H_{hyd} = \frac{P}{\gamma} + z < H_{TOT}$

pipe transition to horizontal to point A — same but $\frac{P}{\gamma}$ and $z$ are changing — same

at pt 2 $H_{TOT} = \frac{V^2}{2g}$ $H_{hyd} = 0$

(3.51) Air flows through the device shown. If the flowrate is large enough, the pressure within the narrow section will be low enough to draw the water up into the tube. Neglecting compressibility and viscous effect, determine the flowrate Q and the pressure needed at section 1 to draw the water into section 2.

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The pressure at 2 to get the water to rise a distance $h = 0.3\,m$ is simply

$$P_2 = -\gamma_w h \quad \text{(hydrostatic)}$$

$$= -\left(9.81\,\frac{kPa}{m}\right)(0.3\,m) = \boxed{-2.94\ kPa}$$

$P_2 + \gamma_w h = P_{atm} \to 0$ gage

$\therefore P_2 = -\gamma_w h$

Now from the Bernoulli Eqn between pts 2 and 3

$$\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + \cancel{z_2} = \underbrace{\cancel{\frac{P_3}{\gamma}}}_{\to 0 \text{ free jet}} + \frac{V_3^2}{2g} + \cancel{z_3}$$

$$\therefore V_3^2 - V_2^2 = \frac{2g}{\gamma_{air}} P_2$$

Hibbeler air $\rho = 1.23\,kg/m^3$

$\gamma = (1.23)(9.81) = 12.1\,N/m^3 = 12.1\,Pa/m$

but $A_3V_3 = A_2V_2$ or $V_2 = \frac{A_3V_3}{A_2} = 4V_3$

continuity eqn

$(1-16)\,V_3^2$

$$\therefore -15\,V_3^2 = \frac{2g}{\gamma} P_2$$

$$\text{or} \quad V_3 = \sqrt{-\frac{2g}{15\gamma} P_2} = \sqrt{\frac{2(9.81\,m/s^2)(2940\,Pa)}{(15)(12.1\,Pa/m)}}$$

at 15°C

Mott Table E.1 & Munson Table 1.7 Crowe Table A.3

$$= \sqrt{320\,m^2/s^2} = \boxed{17.9\,m/s}$$

$$Q = A_3V_3 = \left(\frac{\pi}{4}\right)(0.05\,m)^2(17.9\,m/s) = \boxed{0.0351\,m^3/s} \text{ ans}$$

Finally to find $P_1$, use Bernoulli Eqn between pts 1 and 3

$$\frac{P_1}{\gamma} + \cancel{\frac{V_1^2}{2g}} + \cancel{z_1} = \frac{P_3}{\gamma} + \cancel{\frac{V_3^2}{2g}} + \cancel{z_3}$$

$z_1 = z_3$

$V_1 = V_3$

$$\therefore \boxed{P_1 = P_3 = 0} \text{ (gage)}$$

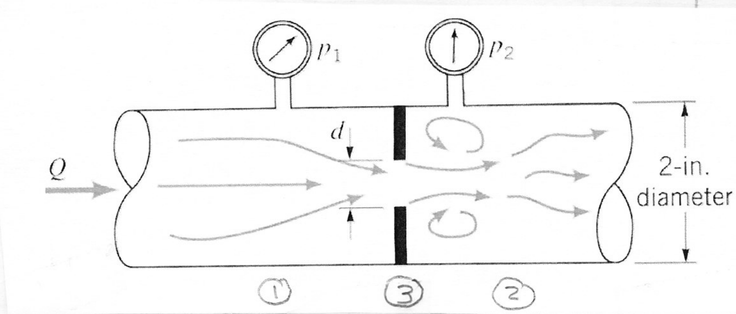
ans

Munson 3E
Prob 3.75

Orifice Meter

Class Example #3

What diameter orifice hole, d, is needed if, under ideal conditions, the flow rate through the orifice meter is to be 30 gpm?




Assume the working fluid is sea water (sg = 1.026) and that $P_1 - P_2 = 2.37$ psi. Also assume that the contraction coefficient is 0.63.

↑ vena contractor

$$\boxed{\frac{A_2}{A_3} = C = 0.63}$$

---

Under ideal conditions (incompressible inviscid flow), the Bernoulli eqn. applies. Since $P_1 - P_2$ is given, let's write the Bernoulli eqn between points 1 and 2:

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + \cancel{z_2}$$

$$\therefore \boxed{V_2^2 - V_1^2 = \frac{2g}{\gamma}(P_1 - P_2)}$$

or

$$\boxed{V_2 = \sqrt{V_1^2 + \frac{2g}{1.026\gamma_w}(P_1 - P_2)}}$$

Let's calc $V_2$:

$$A_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}\left(\frac{2}{12}\right)^2 = 2.182 \times 10^{-2}\ ft^2$$

$$V_1 = \frac{Q}{A_1} = \frac{(30\,gpm)\left(\frac{0.002228\ ft^3/s}{gpm}\right)}{2.182 \times 10^{-2}\ ft^2} = \boxed{3.063\ ft/s}$$

$$V_2 = \sqrt{(3.063)^2 + \frac{2\,(32.2\ ft/s^2)(2.37\ lbf/in^2)(144\ in^2/ft^2)}{1.026\,(62.4\ lbf/ft^3)}}$$

$$= \sqrt{9.382 + 343.29}$$

$$\boxed{V_2 = 18.78\ ft/s}$$

Now $A_2 = \frac{Q}{V_2}$ →

and $A_3 = \frac{\pi}{4} d^2 = \frac{A_2}{C}$ ↗

$$d = \sqrt{\frac{4}{\pi}\,\frac{Q}{C V_2}}$$

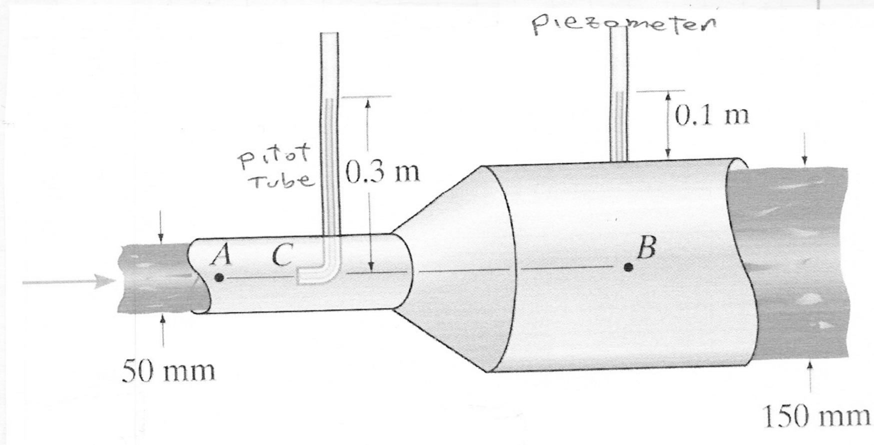
$$= \sqrt{\frac{4}{\pi}\,\frac{30(0.002228)}{0.63(18.78)}}$$

$$d = 0.08481\ ft$$

or $\boxed{d = 1.018\ \text{inches}}$ ans

Hibbeler Prob 5.73 | Bernoulli Eqn | Class Example #4

For the situation shown, determine the volumetric flow rate and the pressure in the pipe at pt A.

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Continuity Eqn

$$Q = V_A A_A = V_B A_B$$

and $V_A = \frac{A_B}{A_A} V_B = 9 V_B$  $\boxed{V_A = 9 V_B}$ or $\boxed{V_A^2 = 81 V_B^2}$ (1)

Bernoulli Eqn A → C

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + \cancel{z_A} = \frac{P_C}{\gamma} + \cancel{\frac{V_C^2}{2g}}^{\,0} + \cancel{z_C}$$

0 stagnation pt.

$$\boxed{\frac{P_A}{\gamma} = \frac{P_C}{\gamma} - \frac{V_A^2}{2g}} \quad (2)$$

also $P_c = \gamma h$

or $\boxed{\frac{P_c}{\gamma} = 0.3\,m}$ (3)

Bernoulli Eqn C → B

$$\underbrace{\cancel{\frac{P_c}{\gamma}}}_{0.3\,m} + \cancel{\frac{V_c^2}{2g}}^{\,0} + \cancel{z_c} = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + \cancel{z_B}$$

$$\boxed{\frac{P_B}{\gamma} = 0.1\,m + 0.075\,m = 0.175\,m} \quad (4)$$

$$\therefore \frac{V_B^2}{2g} = \frac{P_c}{\gamma} - \frac{P_B}{\gamma}$$

$$= 0.3\,m - 0.175\,m = 0.125\,m$$

and $V_B = \sqrt{2(9.81\,m/s^2)(0.125\,m)} = 1.565\,m/s$

$V_A = 9V_B = 14.09\,m/s$

$Q = V_A A_A = (14.09\,m/s)\left(\frac{\pi}{4}\right)(0.05\,m)^2$

$$\boxed{Q = 2.767 \times 10^{-2}\,m^3/s}$$ ans

and, from eqn 2,

$$\frac{P_A}{\gamma} = \frac{P_c}{\gamma} - \frac{V_A^2}{2g} = 0.3\,m - 81(0.125\,m)$$

$$= -9.825\,m$$

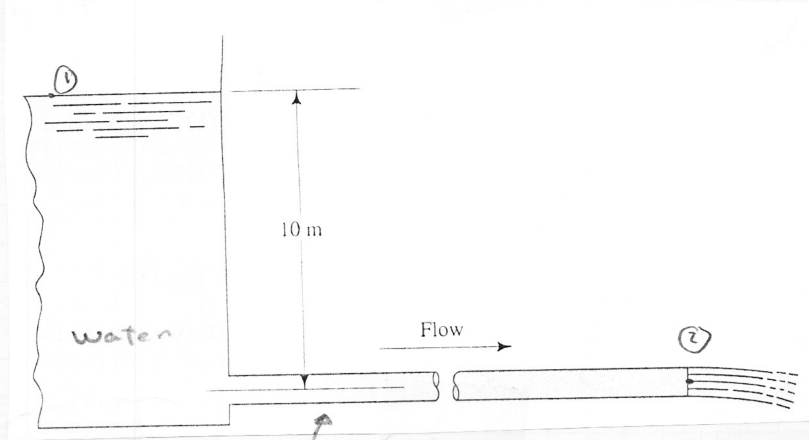
$P_A = (-9.825\,m)\left(9.81\,\frac{kN}{m^3}\right)$

$$\boxed{P_A = -96.4\,kPa}$$ ans

Mott 5E
Prob 7.4

Energy Eqn | Class Example #1

For $Q = 0.085\ m^3/s$, compute $h_L$.




6" sch 40 pipe $(A = 1.865 \times 10^{-2}\ m^2)$

→ Energy Eqn from surface to pipe exit

$$\cancel{\frac{P_1}{\gamma}}_{\to 0} + \cancel{\frac{V_1^2}{2g}}_{\to \approx 0} + z_1 + \cancel{h_A}^{0} - \cancel{h_R}^{0} - h_L = \cancel{\frac{P_2}{\gamma}}_{\to 0} + \frac{V_2^2}{2g} + z_2$$

$$\therefore \boxed{h_L = (z_1 - z_2) - \frac{V_2^2}{2g}}$$

but $Q = VA$ or $V = \frac{Q}{A} = \frac{0.085\ m^3/s}{1.865 \times 10^{-2}\ m^2}$

$$\boxed{V = 4.558\ m/s}$$

$$\therefore h_L = 10\ m - \frac{(4.558\ m/s)^2}{2(9.81\ m/s^2)}$$

$$= 10 - 1.059 = \boxed{8.94\ m}$$ ans

---

Note: IF no losses, $h_L = 0$, and

$$V = \sqrt{2g(z_1 - z_2)}$$

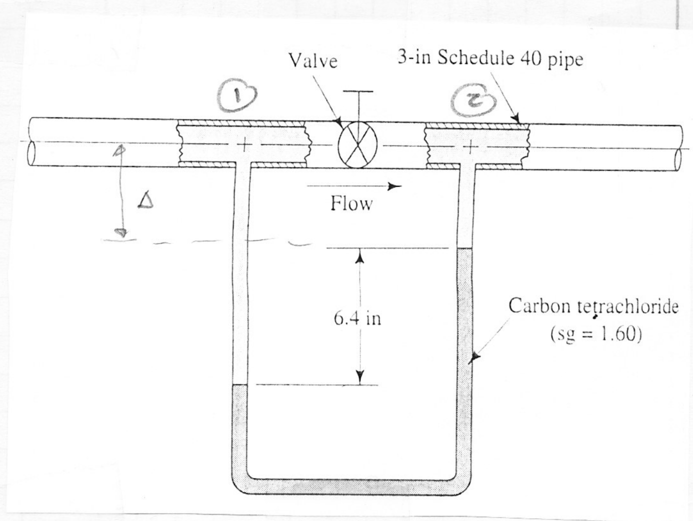
$$= \sqrt{2(9.81\ \tfrac{m}{s^2})(10\ m)} = 14.01\ m/s$$

$$Q = VA = (14.01)(1.865 \times 10^{-2}) = \boxed{0.261\ m^3/s}$$

$$\frac{Q_{\text{no losses}}}{Q_{\text{losses}}} = \frac{0.261}{0.085} \cong \boxed{3.07}$$

therefore, friction losses reduces the flow rate by more than a factor of Three (3)

A test setup to find the energy loss coeff of a valve is shown.




Compute $h_L$ if the measured $Q = 0.10\ ft^3/s$ of water for a given valve position.

For this position, also compute the resistance coeff, K, if $h_L$ is given as

$$h_L = K\frac{v^2}{2g}$$

$$A_{3'' sch\,40} = 0.05132\ ft^2$$

Energy Eqn 1 ⇒ 2

$$\frac{P_1}{\gamma} + \cancel{\frac{V_1^2}{2g}} + \cancel{z_1} + \cancel{h_A}^{\,0} - \cancel{h_R}^{\,0} - h_L = \frac{P_2}{\gamma} + \cancel{\frac{V_2^2}{2g}} + \cancel{z_2}$$

$$z_1 = z_2$$
$$V_1 = V_2$$

$$h_L = \frac{P_1 - P_2}{\gamma}$$

From manometer

$+\gamma_w(\Delta)$    $-\gamma_w(\Delta)$ ← These cancel...

$$P_1 + \gamma_w\left(\frac{6.4}{12}\right) - 1.6\,\gamma_w\left(\frac{6.4}{12}\right) = P_2$$

$$\frac{P_1 - P_2}{\gamma_w} = (1.6 - 1.0)\frac{6.4}{12} = \boxed{0.32\ ft}$$

$$\therefore \boxed{h_L = 0.32\ ft}$$ ans

Also

$$\frac{V^2}{2g} = \frac{Q^2}{2gA^2} = \frac{(0.10\ ft^3/s)^2}{2(32.2\ ft/s^2)(0.05132\ ft^2)^2}$$

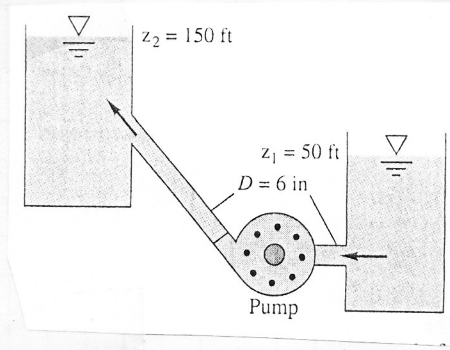
$$\frac{V^2}{2g} = 0.05896\ ft$$

$$\therefore K = \frac{h_L}{\frac{V^2}{2g}} = \frac{0.32\ ft}{0.05896\ ft} = \boxed{5.43}$$ ans

Water is pumped at 1500 gpm from the lower to the upper reservoir.

Pipe friction is given by $h_L = 27 \frac{V^2}{2g}$

If the pump is 75% efficient, what horsepower motor is needed to drive it.

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Energy Eqn.

$$\cancel{\frac{P_1}{\gamma}}^{\to 0} + \cancel{\frac{V_1^2}{2g}}^{\to 0} + z_1 + h_A - \cancel{h_R}^{\to 0} - h_L = \cancel{\frac{P_2}{\gamma}}^{\to 0} + \cancel{\frac{V_2^2}{2g}}^{\to 0} + z_2$$

$$h_A = (z_2 - z_1) + h_L$$

$$h_A = 100\text{ ft} + 27\frac{V^2}{2g}$$

$$A = \frac{\pi}{4}\left(\tfrac{1}{2}\text{ ft}\right)^2$$

$$\boxed{A = 0.1963\text{ ft}^2}$$

but $V = \frac{Q}{A}$

$$= \frac{(1500\text{ gal/min})\left(\frac{1\text{ ft}^3}{7.48\text{ gal}}\right) \times \frac{1\text{ min}}{60\text{ s}}}{0.1963\text{ ft}^2} \Rightarrow \boxed{Q = 3.342\text{ ft}^3/\text{s}}$$

$$= \frac{1500}{(0.1963)(7.48)(60)} = \boxed{17.02\text{ ft/s}}$$

$$\therefore h_A = 100\text{ ft} + \frac{27(17.02\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)}$$

$$= 100 + 121.4 = \boxed{221.4\text{ ft}}$$

and $P_A = h_A \gamma Q$

$$= (221.4\text{ ft})\left(62.4\frac{\text{lbf}}{\text{ft}^3}\right)\left(3.342\frac{\text{ft}^3}{\text{s}}\right)$$

$$= 4.617\times10^4\text{ ft-lbf/s} \times \frac{1\text{ hp}}{550\text{ ft-lbf/s}}$$

$\boxed{P_A = 83.95\text{ hp}}$ ← power added to fluid

$$\eta_{pump} = \frac{P_A}{P_I} \Rightarrow P_I = \frac{P_A}{\eta} = \frac{83.95}{0.75} = \boxed{112\text{ hp}}$$

ans

7.35

Consider the fluid power system shown below. This system shows the flow path for a hydraulic press used to extrude rubber parts.

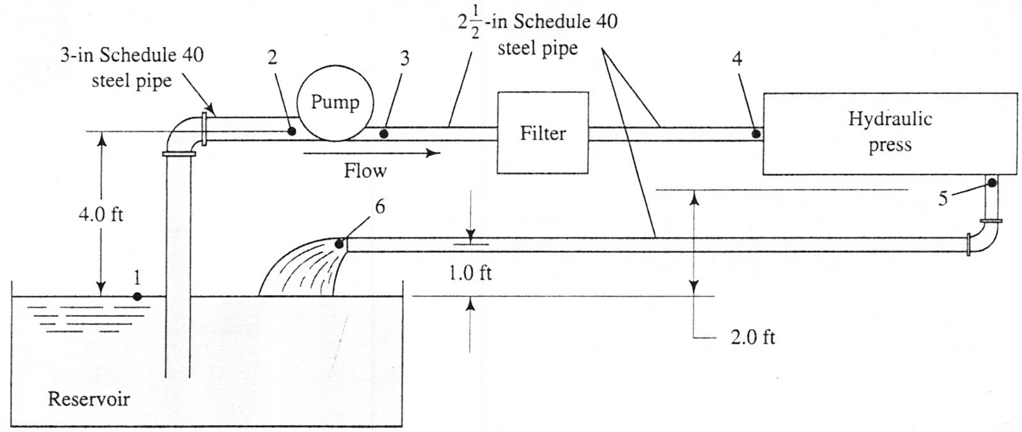


**FIGURE 7.36** Problems 7.35–7.40.

The available data for this system are as follows:

1. The working fluid is oil (SG = 0.93)
2. The volume flow rate is 175 gal/min
3. The input power to the pump is 28.4 hp
4. Pump efficiency is 80 %.
5. Energy losses were measured to be

| | | | | |
|---|---|---|---|---|
| pt 1 | to | pt 2 | = | 2.80 ft-lbf / lbf of fluid |
| 3 | | 4 | = | 28.5 ft-lbf / lbf |
| 5 | | 6 | = | 3.50 ft-lbf / lbf |

Compute the power removed from the fluid by the hydraulic press.

Writing the energy eqn from pts 1 to 6 gives

$$\cancel{\frac{P_1}{\gamma}} + \cancel{\frac{v_1^2}{2g}} + z_1 + h_A - h_R - h_L = \cancel{\frac{P_6}{\gamma}} + \frac{v_6^2}{2g} + z_6$$

→ 0 free surface; → 0 free surface; → 0 free jet

$$\therefore \boxed{h_R = (z_1 - z_6) + h_A - h_L - \frac{v_6^2}{2g}}$$

$$z_1 - z_6 = \boxed{-1.0 \text{ ft}}$$

$$h_L = h_{L\,1\to2} + h_{L\,3\to4} + h_{L\,5\to6}$$

$$= 2.8 + 28.5 + 3.5$$

$$= \boxed{34.8 \text{ ft}}$$

— continued —

$Q = 175 \text{ gal/min}$ and $Q = vA$

$\therefore V_b = \dfrac{Q}{A_b}$ } for $2\frac{1}{2}''$ Sch 40 pipe $A_b = 0.0333 \text{ ft}^2$ App F Mott

$$= \frac{(175 \text{ gal/min})\left(\frac{1 \text{ ft}^3/\text{s}}{449 \text{ gal/min}}\right)}{0.0333 \text{ ft}^2}$$

$$= \frac{0.390 \text{ ft}^3/\text{s}^{\,*\,\leftarrow Q}}{0.0333 \text{ ft}^2} = \boxed{11.70 \text{ ft/s}}$$

and

$$\frac{v_b^2}{2g} = \frac{(11.70 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} = \boxed{2.13 \text{ ft}}$$

Now $P_{I\,pump} = 28.4 \text{ hp}$ and $\eta_{pump} = 0.80$

$\therefore P_A = (\eta \, P_I)_{pump} = 22.72 \text{ hp}$ } power added to fluid

$$= 22.72 \text{ hp} * \frac{550 \text{ ft-lbf/s}}{\text{hp}}$$

$$= \boxed{12\,496 \text{ ft-lbf/s}}$$

and

$$P_A = h_A \gamma Q$$

$$\therefore h_A = \frac{P_A}{\gamma Q} = \frac{12\,496 \text{ ft-lbf/s}}{(0.93)(62.4 \frac{\text{lbf}}{\text{ft}^3})(0.390 \text{ ft}^3/\text{s})}$$

$$= \boxed{552.1 \text{ ft}}$$

Finally, from the above expression for $h_R$

$$h_R = -1.0 + 552.1 - 34.8 - 2.13$$

$$= 514.2 \text{ ft}$$

and

$$P_R = h_R \gamma Q$$

$$= (514.2 \text{ ft})(0.93)(62.4 \frac{\text{lbf}}{\text{ft}^3})(0.390 \frac{\text{ft}^3}{\text{s}})$$

$$= 11638 \frac{\text{ft-lbf}}{\text{s}} * \frac{1 \text{ hp}}{550 \text{ ft-lbf/s}} = \boxed{21.2 \text{ hp}}$$ ans