CHEN.3030 Fluid Mechanics

Extra Credit Problem -- Spring 2017

Correction Factors for Non-Uniform Flow in Circular Pipes

Clearly the flow of incompressible fluids within circular pipes represents an important area of study in fluid mechanics. For fully developed steady flows, the axial velocity profile can be written as:

laminar flow: $u(r) = u_o\left[1-\left(\frac{r}{R}\right)^2\right]$ **turbulent flow:** $u(r) = u_o\left(1-\frac{r}{R}\right)^m$

where u_o is the maximum velocity in the center of the pipe and R is the pipe radius. Note that the expression for turbulent flow is an approximation. The value of m usually ranges from 1/9 to 1/5 and, for the current analysis, we will assume a median value of m = 1/7.

Note: The following integral from a standard ***Table of Integrals*** may be useful when computing various quantities requested for the turbulent flow case:

$$\int x(ax+b)^n dx = \frac{1}{a^2(n+2)}(ax+b)^{n+2} - \frac{1}{a^2(n+1)}(ax+b)^{n+1} \quad \text{for } n \neq -1, -2$$

Continuity Equation

A mass balance on a control volume within the pipe leads to a formal definition for the mass flow rate for 1-D incompressible flows:

$$\dot{m} = \rho A v_{ave} = \rho Q = \rho\int_A \vec{v}\cdot\hat{n}\,dA = \rho\int_A u\,dA$$

These relationships also give a formal definition for the average velocity in a 1-D flow situation,

$$v_{ave} = \frac{1}{A}\int_A u\,dA$$

a. Perform the operation implied here for both the laminar and turbulent flow cases to determine the average velocity, v_{ave}, within the pipe in terms of u_o.

b. Evaluate and plot the profiles and average velocity for both laminar and turbulent flow in a circular pipe (with m = 1/7 for the turbulent flow case). Put all these curves on the same plot for easy comparison. Are these profiles as expected?

Energy Equation

In the development of the energy equation for routine practical application, we assumed 1-D **uniform flow** across the inlet and exit regions of the control volume. In particular, the kinetic energy term for the case of 1-D incompressible **uniform flow** from a single exit area gives

$$\int_A\left(\frac{u^2}{2}\right)\rho(\vec{v}\cdot\hat{n})dA = \frac{1}{2}\rho\int_A u^3 dA = \frac{1}{2}v^2\rho vA = \frac{1}{2}\dot{m}v^2 \quad \text{(where v = u for uniform flow)}$$

When the 1-D fluid **flow distribution is not uniform**, one often defines a ***kinetic energy correction factor***, α, with the relationship

$$\frac{1}{2}\rho\int_A u^3 dA = \frac{1}{2}\rho\alpha v_{ave}^3 A = \alpha\left(\frac{1}{2}\dot{m}v_{ave}^2\right)$$

or

$$\alpha = \frac{1}{v_{ave}^3 A}\int_A u^3 dA$$

a. Your job for this part of the problem is to compute the kinetic energy correction factor for the case of laminar and turbulent flow in a circular pipe.

b. Based on your result from Part a, discuss why, in many problems, we simply write the velocity head as $v_{ave}^2/2g$ instead of $\alpha v_{ave}^2/2g$. When is it necessary to include α within the formal energy equation? Explain…

Linear Momentum Equation

Using a similar discussion, a ***momentum flux correction factor***, β, can be defined for 1-D incompressible flow situations, as follows:

$$\int_A u\rho\left(\vec{v}\cdot\hat{n}\right)dA = \rho\int_A u^2 dA = \beta\rho v_{ave}^2 A = \beta\dot{m}v_{ave}$$

or

$$\beta = \frac{1}{v_{ave}^2 A}\int_A u^2 dA$$

a. Compute β for the above laminar and turbulent velocity profiles for flow in a circular pipe. Again, you should evaluate the resultant expression for the turbulent flow case for m = 1/7.

b. Discuss, as above, the need for using this correction factor for practical applications. In particular, when should we use the term $\beta\dot{m}v_{ave}$ instead of $\dot{m}v_{ave}$ within the 1-D linear momentum equation?

Documentation

Documentation for this problem should include a brief description of the problem being addressed, the details of the solution techniques neatly hand written (including the work required to do the integrals), and the key results of your analyses. Also include any program listings (or tabular data if using a spreadsheet) used in generating the summary plots of the velocity profiles. A brief discussion of the results is required for full credit!!!

This Extra Credit HW Assignment will be worth up to **15 extra points** towards your HW grade. Partial credit will be given for partially completed work, but only if significant progress towards a complete solution has been made. Team efforts within 2-person teams are encouraged for this project! Only one project report is needed for each team.