CHEN.3030 Fluid Mechanics (Section 202)

Homework Assignment #5 Spring 2017

Flow Rates, the RTT, and the Continuity Equation

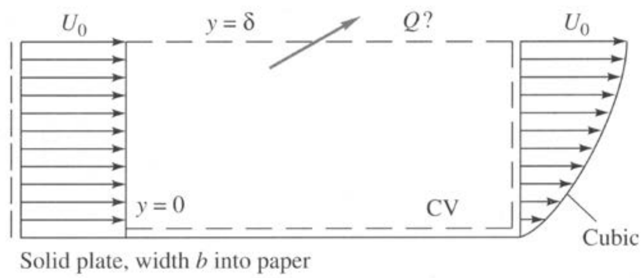
1. An incompressible fluid flows past an impermeable flat plate as shown. The inlet velocity profile is uniform,

$$u = u_o \quad \text{(at inlet)}$$

and it has a cubic polynomial exit profile,

$$u(\eta) = u_o\left(\frac{3\eta - \eta^3}{2}\right) \quad \text{(at exit)}$$

with $\eta = y/\delta$.



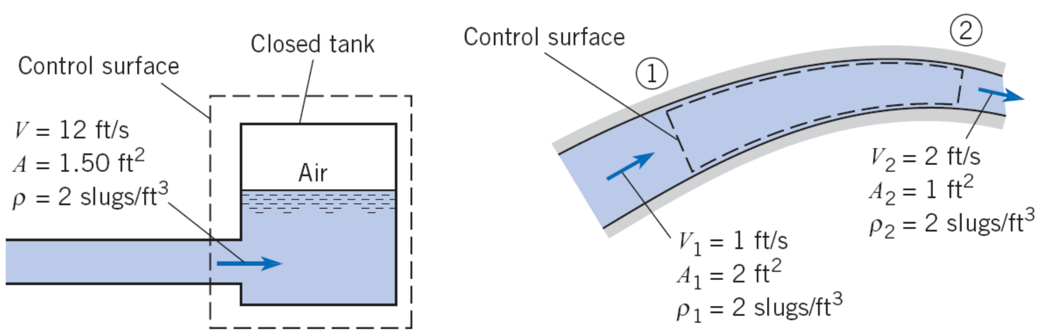
For this situation, **formally develop** an expression for the volume flow rate, Q, across the top surface of the control volume (CV).

Hint: Let L = length of the CV, b = width of the CV into the paper, and δ = height of the CV. Then the volume of the CV is simply $L \times \delta \times b$ since the rectangular parallelepiped has a geometry defined by $0 \le x \le L$, $0 \le y \le \delta$, and $0 \le z \le b$. Also, at x = 0 and x = L, assume that the flow is one-dimensional in the x-direction perpendicular to the y-z plane.

2. The Reynolds's Transport Theorem was developed in class with the following result:

$$\frac{d}{dt}B_{sys} = \frac{d}{dt}\int_{CV} b\rho dr^3 + \int_{CS} b\rho\vec{v}\bullet\hat{n}dA$$

Within the context of the continuity equation applied to the two flow situations (a and b) shown in the diagram below, answer the following questions:



(*a*) (*b*)

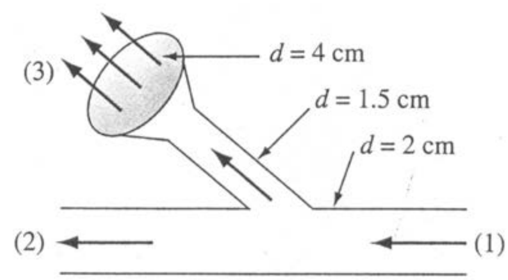
a. What is intensive property b for these problems?

b. Determine $\frac{d}{dt}B_{sys}$. Explain your logic.

c. Determine the value of the last term in the equation, $\int_{CS} b\rho\vec{v}\cdot\hat{n}dA$. Physically what does this mean?

d. Determine the value of $\frac{d}{dt}\int_{CV} b\rho dr^3$. Again, what does this term mean within the context of this problem?

3. Water at 20 C flows steadily through the piping section shown in the diagram, entering section 1 at 20 gpm. The average velocity at section 2 is 2.5 m/s. A portion of the flow is diverted through the showerhead, which contains 100 holes of 1.0 mm diameter. Assuming uniform flow, estimate the exit velocity from the showerhead jets in meters per second.



4. Air flows in a long length of 2.5 cm diameter pipe. At the inlet, the pressure is 200 kPa (abs), the temperature is 150 C, and the average velocity is 10 m/s. At the exit, the pressure and temperature have been reduced by friction and heat loss to 130 kPa (abs) and 120 C, respectively. With this information, determine the exit air velocity.

5. A lake with no outlet is fed by a river with a constant discharge (i.e. flow rate) of 1000 ft^3/s. Water evaporates from the surface at a constant rate of 13 ft^3/s per square mile of surface area. The top surface area of the lake varies with the average depth of the water as follows:

$A = 4.5 + 5.5h$

where h is given in feet and A is in square miles.

a. Under these conditions, what is the equilibrium depth of the lake?

b. Below what river discharge will the lake dry up?

Be sure to explain your logic for both parts to this problem…