CHEN.3030 Fluid Mechanics (Section 202)

Homework Assignment #4 Spring 2017

Fluid Kinematics

An approximate velocity field for steady, incompressible 2-D flow through a particular converging duct is given by

$$\vec{v} = (0.5 + 1.2x)\hat{i} - 1.2y\hat{j}$$

where the units for v are in ft/s. Note that this equation ignores viscous effects along the walls, but it is a reasonable approximation throughout the majority of the flow field.

This system will be used for Problems 1 – 3.

1. With the flow field given above, determine the time-dependent position of a fluid particle that initially starts at $x(0) = x_o = 0$ and $y(0) = y_o$. That is, find x(t) and y(t) given $(0,y_o)$ as the initial particle position.

Once you have a general result for x(t) and y(t) for a arbitrary y_o values, numerically evaluate and plot the particle trajectory for five (5) specific values of y_o. -- that is, let y_o = -8, -3, 0, 3, and 8 ft. Put all five curves on the same plot. Use $0 \le t \le 2.5$ seconds in your evaluations. Do the y(t) vs. x(t) trajectories behave as expected for a converging duct? Explain…

2. Determine an expression for the acceleration field for fluid particles passing through the duct described above. What are the acceleration vector and magnitude at the point (x,y) = (1,4)?

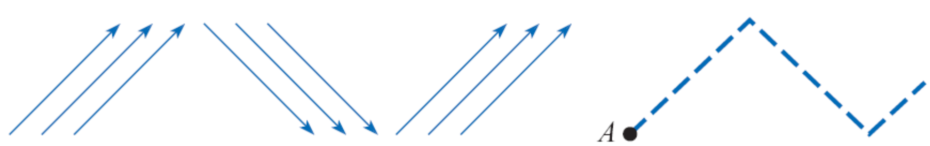
3. For the above converging duct flow geometry, generate an analytical expression for the flow streamlines. Once you have a general result for the stream function, compute the values of the arbitrary constant for the following x,y pairs; (0,-8), (0,-3), (0,0), (0,3), and (0,8) and plot the five (5) specific streamlines on the same plot over the range $0 \le x \le 8$ ft.

How does this plot compare to the set of pathlines generated in Prob. 1? Explain…

4. A flow field is periodic in that the streamline pattern repeats at definite intervals. For the first second the field is moving upward at 45^o to the right and, in the next second, the flow is moving downward at 45^o to the right, etc., as shown in the sketch below. The speed of flow is constant at 10 m/s.

After 2.5 s the pathline of a particle released at point A at time zero is also shown in the sketch.

If dye is emitted in a continuous stream from point A starting at time zero, carefully draw the resulting streakline at t = 2.5 s. Explain your plot.



(*a*) Streamlines (*b*) Pathline

5. For a given hypothetical 2-D flow, the space-independent velocity components are given by

$$u = -2\ \text{m/s} \qquad v = 0\ \text{m/s} \qquad \text{for} \quad 0 \le t \le 4\ \text{s}$$

$$u = 1\ \text{m/s} \qquad v = -2.5\ \text{m/s} \qquad \text{for} \quad 4 \le t \le 8\ \text{s}$$

A dye streak was started at a point in the flow field at time t = 0, and the path of a single particle in the fluid was also traced from the same point starting at the same time.

Draw to scale the streakline, pathline of the particle, and streamlines at time t = 8 s for this flow field. Draw and label these curves carefully!