Mott 5th Ed

based on this

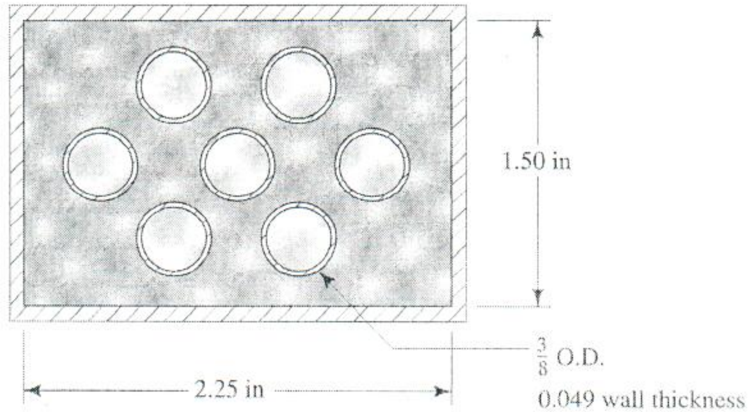
Internal Flow
Laminar Flow in
noncircular channel

1/2

In the figure, ethylene glycol (sg = 1.10 and μ = 0.011 lbm/ft-s) at 77 F flows around the 3/8 inch tubes inside the rectangular channel (i.e. the flow area is the grey area outside the tubes and inside the channel walls).

a. Calculate the volume flow rate of ethylene glycol in gal/min (gpm) required for the flow to have a Reynolds number of 1200.

b. With the conditions of Part a, estimate the energy loss per unit length of channel using Darcy's equation. State any assumptions.



for ethylene glycol with sg = 1.10

$$\rho = 1.10\left(62.4\ \frac{lbm}{ft^3}\right) = 68.5\ lbm/ft^3$$

$$\gamma = 1.10\left(62.4\ \frac{lbf}{ft^3}\right) = 68.5\ lbf/ft^3$$

and $\mu = 0.110\ lbm/ft\text{-}s$ (given)

$$Re = 1200 = \frac{\rho V D_h}{\mu}$$

Thus with a value for D_h we can easily compute V

where $D_h = \dfrac{4A_f}{P_w}$ ← flow area ← wetted perimeter

hydraulic diameter

(a) Geometry

flow area $= A_f =$ area inside rectangle − area of 7 tubes

$$= \frac{(2.25)(1.50)}{144}\ ft^2 - 7\left(\frac{\pi}{4}\right)\left(\frac{3/8}{12}\right)^2 ft^2$$

$$= 0.02344 - 0.00537$$

$$\boxed{A_f = 0.01807\ ft^2}$$

wetted perimeter $= P_w =$ perimeter inside rect. + perimeter of 7 tubes

$$= \frac{2(2.25)}{12} + \frac{2(1.50)}{12} + 7(\pi)\left(\frac{3/8}{12}\right) ft$$

$$= 0.625 + 0.6872$$

$$\boxed{P_w = 1.312\ ft}$$

$$\therefore\ D_h = \frac{4A_f}{P_w} = \frac{4(0.01807)}{1.312} = \boxed{0.0551\text{ ft}}$$

okay, now we can use the Re # expression to find V and Q

$$V = \frac{Re\,\mu}{\rho D_h} = \frac{1200\,(0.110\text{ lbm/ft-s})}{(68.5\text{ lbm/ft}^3)(0.0551\text{ ft})}$$

or $\boxed{V = 3.497\text{ ft/s}}$ $\approx$ $\boxed{V = 3.5\text{ ft/s}}$

and then

$$Q = VA = (3.497\text{ ft/s})(0.01807\text{ ft}^2)$$

$$\boxed{Q = 0.0632\text{ ft}^3/\text{s}}$$ ans

(b) Darcy's Eqn says that

$$h_L = f\,\frac{L}{D_h}\,\frac{V^2}{2g}$$

and for laminar flow $f \approx \frac{64}{Re}$ ← formal for pipe flow but this is also used as an approx relation for non-circular channels

$$\therefore\ \frac{h_L}{L} = f\left(\frac{1}{D}\right)\frac{V^2}{2g}$$

head loss per unit length of channel $= (0.05333)\left(\frac{1}{0.0551\text{ ft}}\right)(0.190)$

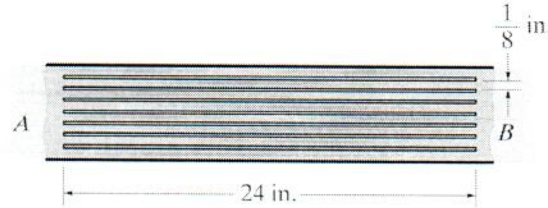
$$f = \frac{64}{Re} = \frac{64}{1200} = 0.05333$$

$$\frac{V^2}{2g} = \frac{(3.497)^2}{2(32.2)} = 0.190\text{ ft}$$

$$= \boxed{0.184\ \frac{\text{ft}}{\text{ft}}}$$ ans

related to...

Air at $T = 80°$ F flows with an average velocity of 20 ft/s through the parallel plates as shown in the sketch. The plates are each 15 in. wide, and the gap between them is 1/8 in. Assuming fully developed flow, determine the pressure difference $\Delta P = P_A - P_B$ (in psi) between the inlet A and the outlet B. Also, for the 8-channel system shown, determine the total discharge.



from App A, air at 80°F: $\rho = 0.00228\ \text{slug}/\text{ft}^3$

$\mu = 0.385 \times 10^{-6}\ \dfrac{\text{lbf-s}}{\text{ft}^2}$

calc Q $\quad a = \frac{1}{8}'' = 0.125'' = 1.0417 \times 10^{-2}\ \text{ft}$

$b = 15'' = 1.25\ \text{ft}$

per channel $\quad A = ab = (1.0417\times10^{-2})(1.25) = 0.01302\ \text{ft}^2$

$Q_{chan} = VA = (20\ \text{ft/s})(0.01302\ \text{ft}^2) = \boxed{0.2604\ \text{ft}^3/\text{s}}$

$\therefore\ Q_{TOT} = 8\,Q_{chan} = \boxed{2.083\ \text{ft}^3/\text{s}}$ ans

calc Re $\quad$ based on $D_h = 2a$ $\quad$ (different from Hibbeler). $\quad$ units ok

$$Re_h = \frac{\rho V 2a}{\mu} = \frac{\left(0.00228\ \frac{\text{slug}}{\text{ft}^3}\right)(20\ \text{ft/s})(1.0417\times10^{-2}\ \text{ft})(2)}{0.385\times10^{-6}\ \frac{\text{lbf-s}}{\text{ft}^2}}$$

$1\text{bf} = \text{slug}\ \frac{\text{ft}}{\text{s}^2}$

$\boxed{Re_h = 2462}$ $\quad$ < 2800 laminar flow

calc ΔP $\quad$ From development in Hibbeler (for laminar flow)

$$Q_{chan} = -\frac{ba^3}{12\mu}\frac{d}{dx}(P + \gamma h)$$

but $\dfrac{dh}{dx} = 0$ horizontal channel $\quad$ and $\quad -\dfrac{dP}{dx} = \dfrac{P_A - P_B}{L} = \dfrac{\Delta P}{L}$

$$\therefore\ \Delta P = \frac{12\mu L}{ba^3}\,Q_{chan}$$

$$= \frac{(12)\left(0.385\times10^{-6}\ \frac{\text{lbf-s}}{\text{ft}^2}\right)(2\ \text{ft})(0.2604\ \text{ft}^3/\text{s})}{(1.25\ \text{ft})(1.0417\times10^{-2}\ \text{ft})^3}$$

$$= \frac{2.406\times10^{-6}}{1.413\times10^{-6}} = \boxed{1.703\ \frac{\text{lbf}}{\text{ft}^2}} = \boxed{0.0118\ \text{psi}}$$

ans

calc ΔP alternate formulation { Energy Eqn + Darcy's eqn

Energy Eqn (A) → (B)

$$\frac{P_A}{\gamma} + \cancel{\frac{V_A^2}{2g}} + \cancel{z_A} + \cancel{h_A} - \cancel{h_R} - h_L = \frac{P_B}{\gamma} + \cancel{\frac{V_B^2}{2g}} + \cancel{z_B}$$

$z_A = z_B$ and $V_A = V_B$ and $h_A = h_R = 0$
no mech devices

$$\therefore \boxed{P_A - P_B = \Delta P = \gamma h_L}$$

← pressure drop is related to head loss in channel

But Darcy Eqn gives

$$h_L = f \frac{L}{D_h} \frac{V^2}{2g}$$

where, for wide parallel plates with $b >> a$, we have

$$f = \frac{96}{Re_h}$$

← from class

$$h_L = \frac{96}{Re_h} \frac{L}{D_h} \frac{V^2}{2g}$$

$$= \left(\frac{96}{2468}\right)\left(\frac{24}{2(1/8)}\right)\left(\frac{[20\ ft/s]^2}{2(32.2\ ft/s^2)}\right)$$

$$h_L = (0.0389)(96)(6.211\ ft) = 23.20\ ft$$

and $\Delta P = \gamma h_L = \rho g h_L$

$$= \left(0.00228\ \frac{slug}{ft^3}\right)\left(32.2\ \frac{ft}{s^2}\right)(23.20\ ft)$$

$$\Delta P = 1.703\ \frac{lbf}{ft^2}$$

$$\Delta P = 0.0118\ psi$$

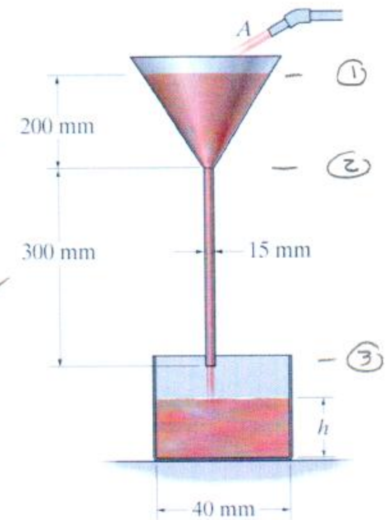
ans

as before
ok

$$\frac{slug}{ft^3} \cdot \frac{ft}{s^2} = \frac{lbf}{ft^2}$$

Oil (sg = 0.92) is poured steadily into the funnel so that the level of 200 mm is maintained. The oil flows through the stem at a steady rate and accumulates in the cylindrical container. If it takes 6 seconds to fill the container to a depth of h = 180 mm, determine the viscosity of the oil.

Hint: Assume laminar flow conditions for this problem, but be sure to verify that this assumption is corect before problem completion. ✓



→ we can compute $Q = \frac{\Delta \forall}{\Delta t}$

$$\Delta \forall = \frac{\pi D^2}{4} h = \pi \frac{(0.04\,m)^2}{4}(0.18\,m)$$

$$= 2.262 \times 10^{-4}\,m^3$$

$$\therefore Q = \frac{2.262 \times 10^{-4}\,m^3}{6\,s} = \boxed{3.770 \times 10^{-5}\,m^3/s}$$

→ The velocity in the stem is given by

$$V = \frac{Q}{A_{stem}} = \frac{3.770 \times 10^{-5}}{1.767 \times 10^{-4}} = \boxed{0.213\,\frac{m}{s}}$$

$$A_{stem} = \pi \frac{(0.015\,m)^2}{4} = 1.767 \times 10^{-4}\,m^2$$

→ The pressure at the top of the stem can be determined via the Bernoulli eqn

$$\cancel{\frac{P_1}{\gamma_o}} + \cancel{\frac{V_1^2}{2g}} + z_1 = \frac{P_2}{\gamma_o} + \frac{V_2^2}{2g} + z_2$$

$$\frac{V_2^2}{2g} = \frac{(0.213\,m/s)^2}{2(9.81\,m/s^2)} = 2.312 \times 10^{-3}\,m$$

pretty small (because of low velocity...)

$$\therefore P_2 = \left[(z_1 - z_2) - \frac{V_2^2}{2g}\right](sg)\gamma_w$$

$$= [0.200 - 0.0023]_m (0.92)\left(9.81\,\frac{kN}{m^3}\right)$$

$$= \boxed{1.784\,kN/m^2}$$

← close to hydrostatic pressure, but not quite

→ Now from the development in the text, for laminar flow in smooth pipe, we have

$$Q = -\frac{\pi R^4}{8\mu}\frac{d}{dx}(p + \gamma h) = -\frac{\pi D^4}{128\mu}\frac{d}{dx}(p + \gamma h)$$

but for vertical pipe, from ② to ③, we have

$$-\frac{d}{dx}(p + \gamma h) = -\left[\frac{P_3 - P_2}{L} + \gamma\frac{z_3 - z_2}{L}\right] = \frac{P_2 - \cancel{P_3}}{L} + \gamma\frac{L}{L} = \frac{P_2}{L} + \gamma$$

$$= \frac{1.784\,kN/m^2}{0.3\,m} + (0.92)\left(9.81\,\frac{kN}{m^3}\right) = \boxed{14.97\,\frac{kN}{m^3}}$$

Thus, solving for μ, gives

$$\mu = \frac{1}{Q}\left[\frac{\pi D^4}{128}\right]\left[-\frac{d}{dx}(p+\gamma h)\right]$$

$$= \left(\frac{1}{3.770\times10^{-5}\,\frac{m^3}{s}}\right)\left(\frac{\pi(0.015m)^4}{128}\right)\left(14.97\times10^3\,\frac{N}{m^3}\right)$$

$$= \boxed{0.493\,\frac{N\text{-}s}{m^2}}$$ ans ← viscosity of oil

note: $\frac{N\text{-}s}{m^2} = \frac{kg\,m/s}{m^2} = \frac{kg}{m\text{-}s}$

Now let's check on the laminar flow assumption

$$Re = \frac{\rho V D}{\mu} = \frac{\left(920\,\frac{kg}{m^3}\right)\left(0.213\,\frac{m}{s}\right)(0.015\,m)}{0.493\,\frac{kg}{m\text{-}s}}$$

$$= \boxed{5.96}$$ OK $Re < 2300$ ∴ laminar flow

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Also, an alternate approach is to use the Energy Eqn and Darcy's Eqn. with $f = \frac{64}{Re}$ for laminar flow

from ② → ③

$$\frac{p_2}{\gamma} + \cancel{\frac{V_2^2}{2g}} + z_2 + \cancel{h_A - h_R} - h_L = \cancel{\frac{p_3}{\gamma}} + \cancel{\frac{V_3^2}{2g}} + z_3$$

free jet; no mech devices; $V_2 = V_3$

$$\therefore h_L = \frac{p_2}{\gamma} + (z_2 - z_3) = \frac{1.784\,kN/m^2}{(0.92)(9.81\,kN/m^3)} + 0.3\,m$$

$$= 0.198 + 0.3 = \boxed{0.498\,m}$$

and $h_L = f\frac{L}{D}\frac{V^2}{2g}$ or $f = \frac{h_L\,D/L}{V^2/2g} = \frac{(0.498m)(15/300)}{2.312\times10^{-3}\,m} = 10.77$

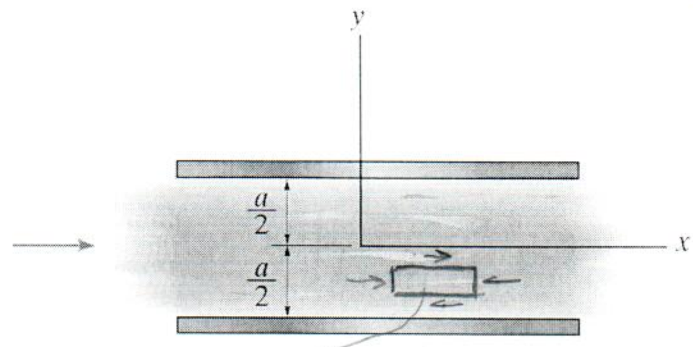
but $\boxed{f = \frac{64}{Re}}$ or $Re = \frac{64}{f} = \frac{64}{10.77} = 5.94$ OK

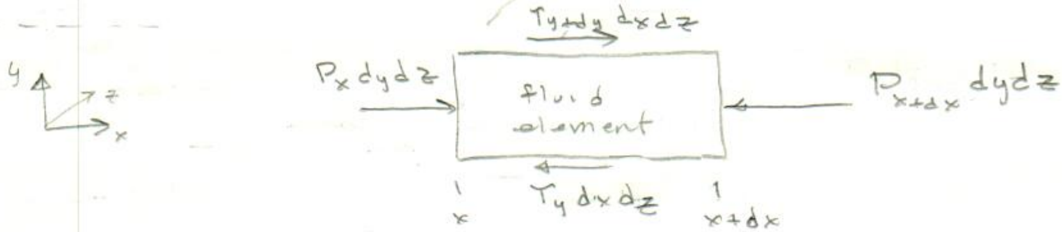
for laminar flow

and finally $\mu = \frac{\rho V D}{Re} = \frac{(920)(0.213)(0.015)}{5.94} = \boxed{0.495\,\frac{N\text{-}s}{m^2}}$

ans

The liquid has laminar flow between the two fixed horizontal plates due to a pressure gradient dp/dx. The width, b, of the plates into the page is large compared to the gap thickness, a. Using the coordinate system shown in the sketch, develop analytical expressions for the shear-stress distribution, τ(y), and the velocity profile, v(y), within the liquid. Assume a Newtonian fluid with viscosity μ. Be formal…





momentum eqn: $\sum F_x = \underbrace{\frac{\partial}{\partial t}\int_{cv} \vec{V}\rho\, d\forall^3}_{0} + \underbrace{\int_{cs} \vec{V}\rho\,\vec{V}\cdot\hat{n}\,dA}_{0}$

steady flow — fully developed flow $V(x,y) = V(y)$ only

$\therefore \sum F_x = 0$

$$\left(P_x\,dy\,dz - P_{x+dx}\,dy\,dz\right) - \left(\tau_y\,dx\,dz - \tau_{y+dy}\,dx\,dz\right) = 0$$

divide everything by $dx\,dy\,dz$

$$-\frac{(P_{x+dx} - P_x)}{dx} + \frac{(\tau_{y+dy} - \tau_y)}{dy} = 0$$

in limit $\boxed{\frac{d\tau}{dy} = \frac{dP}{dx}}$ $\quad \frac{dP}{dx} \neq f(y)$

Let's integrate $\tau(y) = \int \frac{dP}{dx}\,dy = \frac{dP}{dx}\,y + c_1$

$$\therefore \boxed{\tau(y) = \frac{dP}{dx}\,y + c_1}$$

} says that the shear stress is a linear function of y

For Newtonian fluid with laminar flow

$$\boxed{\tau = \mu\frac{du}{dy}}$$

} says that shear stress is proportional to the gradient of velocity profile

Thus

$$\mu \frac{du}{dy} = \frac{dP}{dx} y + c_1$$

$$\boxed{u(y) = \frac{1}{\mu}\left(\frac{dP}{dx}\frac{y^2}{2} + c_1 y + c_2\right)}$$

Now to evaluate the constants, we use the "no-slip" condition at the walls (i.e. at $y = -\frac{a}{2}$ and $y = +\frac{a}{2}$)

$$u\left(-\tfrac{a}{2}\right) = 0 = \frac{1}{\mu}\left(\frac{dP}{dx}\frac{a^2}{2(4)} - c_1\frac{a}{2} + c_2\right) \qquad (1)$$

$$u\left(\tfrac{a}{2}\right) = 0 = \frac{1}{\mu}\left(\frac{dP}{dx}\frac{a^2}{2(4)} + c_1\frac{a}{2} + c_2\right) \qquad 2)$$

adding 1 and 2

$$\frac{dP}{dx}\frac{a^2}{4} + 2c_2 = 0 \quad \Rightarrow \quad c_2 = -\frac{a^2}{8}\frac{dP}{dx}$$

and from 1

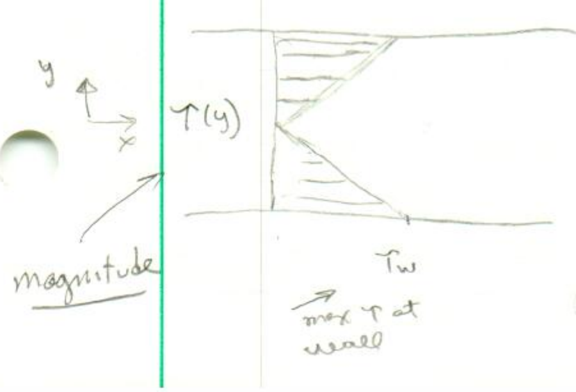
$$\frac{dP}{dx}\frac{a^2}{4} - c_1\frac{a}{2} - \frac{a^2}{4}\frac{dP}{dx} = 0 \quad \Rightarrow \quad c_1 = 0$$

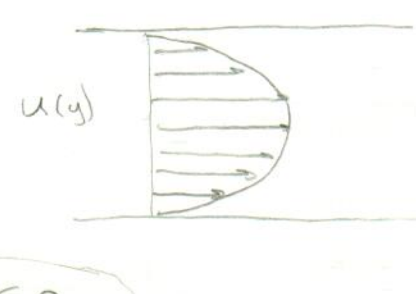
$$\therefore \quad u(y) = \frac{1}{2\mu}\frac{dP}{dx} y^2 - \frac{1}{8\mu}\frac{dP}{dx} a^2$$

or $\boxed{u(y) = \frac{1}{8\mu}\frac{dP}{dx}\left(4y^2 - a^2\right)}$ $\quad -\frac{a}{2} \le y \le \frac{a}{2}$

ans

and $\boxed{\tau(y) = \frac{dP}{dx} y}$ $\quad$ since $c_1 = 0$





for $\frac{dP}{dx} < 0$

A 2-in.-diameter horizontal pipe has a smooth interior surface and transports kerosene at 68°F. If the pressure drops 17 lbf/ft^2 in 15 ft, determine the maximum velocity of the flow. Also determine the thickness of the viscous sublayer. **Hint:** Use the correlation given in eqn. 9–33 from your text.

→ Kerosene properties (68°F):

$\rho = 1.58$ slugs/ft^3 $\qquad \mu = 40.1\times10^{-6}$ lbf-s/ft^2

$\qquad\qquad\qquad\qquad \nu = 25.4\times10^{-6}$ ft^2/s

→ key piece of info: $\dfrac{\Delta P}{L} = \dfrac{17\text{ lbf/ft}^2}{15\text{ ft}} = 1.133\text{ lbf/ft}^3$ ← this is $\dfrac{dP}{dx}$

→ from Hibbeler Text, we have

$$\frac{u}{u^*} = \frac{u^* y}{\nu} \quad \text{in viscous sublayer (eqn. 9-31)}$$

$$\frac{u}{u^*} = 2.5 \ln\left(\frac{u^* y}{\nu}\right) + 5.0 \quad \text{in turbulent region (eqn 9-33)}$$

where $u^* = \sqrt{\tau_w/\rho}$ ← shear velocity

with $\tau(r) = \dfrac{r}{2}\dfrac{d}{dx}(P + \gamma h)$ (eqn 9-10)

Now, for horizontal pipe $\dfrac{dh}{dx} = 0$

$$\therefore \tau(R) = \tau_w = \frac{R}{2}\frac{dP}{dx} = \frac{R}{2}\frac{\Delta P}{L}$$

$$= \frac{1}{2}\left(\frac{1}{12}\text{ ft}\right)\left(1.133\frac{\text{lbf}}{\text{ft}^3}\right)$$

$$= \boxed{0.04722\ \frac{\text{lbf}}{\text{ft}^2}}$$

$1\text{ lbf} = \text{slug}\dfrac{\text{ft}}{\text{s}^2}$

$$u^* = \sqrt{\tau_w/\rho} = \sqrt{\frac{0.04732\text{ lbf/ft}^2}{1.58\text{ slugs/ft}^3}}$$

$$= \boxed{0.1729\text{ ft/s}}$$

Now, with τ_w and u^*, we can compute the max velocity, $u_{max} = u|_{y=R}$ ← centerline

$$u_{max} = u^*\left[2.5 \ln\left(\frac{u^* R}{\nu}\right) + 5.0\right]$$

or $$u_{max} = (0.1729)\left(2.5 \ln\left[\frac{(0.1759)(\frac{1}{12})}{25.4\times10^{-6}}\right] + 5.0\right)$$

$$= (0.1729)\left(2.5 \ln (577.1) + 5.0\right)$$

$$= (0.1729)(20.90) = \boxed{3.613 \text{ ft/s}}$$

ans

Now, from Fig 9-21 in Hibbeler, the thickness of the viscous sublayer is given by

$$y_{sublayer} = \delta$$

$$\frac{u^* \delta}{\nu} = 5$$

or $$\delta = \frac{5\nu}{u^*} = \frac{5(25.4\times10^{-6} \text{ ft}^2/\text{s})}{0.1729 \text{ ft/s}}$$

or $$\delta = \boxed{7.35\times10^{-4} \text{ ft}} = \boxed{8.81 \text{ in}}$$

ans

Note: We do not have a simple expression for the average velocity, but we know from the power law approximation that the velocity profile is given by

$$u(r) = u_{max}\left(1 - \frac{r}{R}\right)^{\frac{1}{n}}$$

and $$u_{ave} = u_{max}\left[\frac{2n^2}{(n+1)(2n+1)}\right]$$

If $n = 7$, for example

$$u_{ave} = 0.817\, u_{max}$$

thus, let's use $0.82\, u_{max} = \boxed{2.96 \text{ ft/s}}$ to estimate the Reynolds #

$$Re = \frac{\rho\, u_{ave} D}{\mu} = \frac{u_{ave} D}{\nu}$$

$$= \frac{(2.96 \text{ ft/s})(\frac{2}{12} \text{ ft})}{25.4\times10^{-6} \text{ ft}^2/\text{s}} = \boxed{1.942\times10^4}$$

much greater than ~4000

so turbulent flow

note – should always check to make sure...

ok

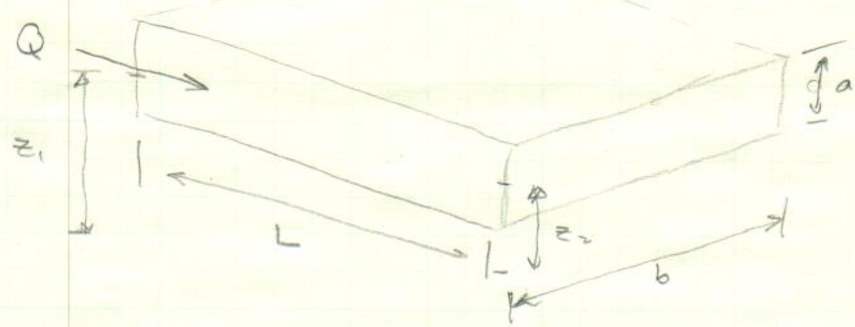
In class, we developed a relationship for the head loss, h_L, for laminar flow in a smooth pipe in terms of the pipe geometry (D and L), the fluid properties (ρ and μ), and the flow rate (Q or v). The result was

$$h_L = \frac{128\mu LQ}{\pi\rho g D^4} = \frac{32\mu L v}{\rho g D^2} \qquad \text{(for laminar flow in a smooth pipe)}$$

Your challenge for this extra credit problem (up to 10 points) is to do a similar development for laminar flow in a smooth rectangular channel of height a and width b. Your starting point should be the general energy equation and the expression for the velocity profile, u(y), for this situation as developed in your text.

Be formal and systematic in your development…

The geometry of interest is as follows:



Energy Eqn

$$\frac{P_1}{\gamma} + \cancel{\frac{v_1^2}{2g}} + z_1 + \cancel{h_A} - \cancel{h_R} - h_L = \frac{P_2}{\gamma} + \cancel{\frac{v_2^2}{2g}} + z_2$$

no mechanical devices

continuity eqn $v_1 = v_2$

$$\therefore \boxed{h_L = \frac{P_1 - P_2}{\gamma} + (z_1 - z_2)} \qquad (1)$$

Laminar Flow Velocity Profile (eqn 9.3 in Hibbeler) with $U = 0$ ← no plate movement

$$\boxed{u(y) = -\frac{1}{2\mu}\left(\frac{dP}{dx} + \gamma\frac{dh}{dx}\right)(ay - y^2)} \qquad (2)$$

but
$$-\left(\frac{dP}{dx} + \gamma\frac{dh}{dx}\right) = \frac{P_1 - P_2}{L} + \gamma\frac{(z_1 - z_2)}{L}$$
$$= \frac{\gamma h_L}{L} \quad \leftarrow \text{from eqn (1)}$$

$$\therefore u(y) = \frac{\gamma h_L}{2\mu L}(ay - y^2)$$

Now

$$Q = \int u\,dA = \int_0^a u(y)\,b\,dy$$

$$= \frac{\gamma h_L b}{2\mu L} \underbrace{\int_0^a (ay - y^2)\,dy}$$

$$\frac{ay^2}{2} - \frac{y^3}{3}\Big|_0^a = \frac{a^3}{2} - \frac{a^3}{3}$$

$$= \frac{3a^3}{6} - \frac{2a^3}{6} = \boxed{\frac{a^3}{6}}$$

$$\therefore \quad Q = \frac{\gamma h_L b a^3}{12\mu L}$$

and solving this for h_L gives

since $Q = VA = Vab$

and $\gamma = \rho g$

$$\boxed{h_L = \frac{12\mu Q L}{\rho g b a^3} = \frac{12\mu L V}{\rho g a^2}}$$

ans

expression for head loss for laminar flow in a smooth rectangular channel

→ same functional dependence as for pipe flow

$h_L \uparrow$ as μ, L, and V increases

$h_L \downarrow$ as channel size (D or a) increases

$\boxed{\text{Note}}$

$$h_L = \frac{12\mu L V}{\rho g a^2} = (12)(8)\left(\frac{\mu}{\rho V 2a}\right)\left(\frac{L}{2a}\right)\left(\frac{V^2}{2g}\right)$$

$$= \frac{96}{Re_h}\frac{L}{D_h}\frac{V^2}{2g} = f\frac{L}{D_h}\frac{V^2}{2g}$$

← Darcy's Eqn

where $\boxed{D_h = \frac{4A_f}{P_w} = 2a}$

$\boxed{Re_h = \frac{\rho V D_h}{\mu}}$ for $b \gg a$

and $\boxed{f = \frac{96}{Re_h}}$ friction factor