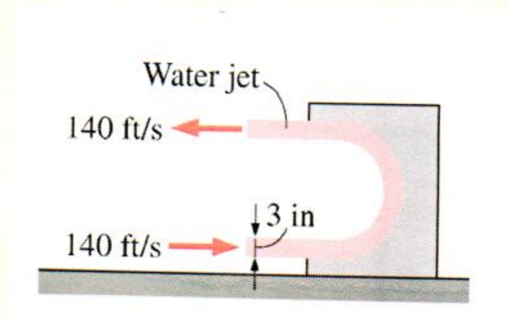
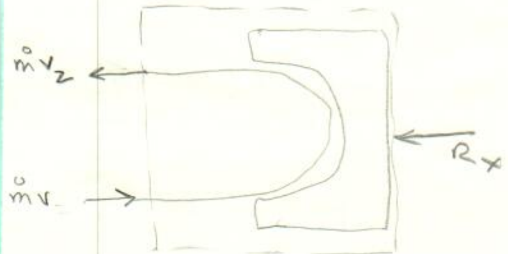
Cengel Prob 6-34

Momentum Eqn

A 3 inch diameter horizontal water jet having a velocity of 140 ft/s strikes a curved plate which deflects 180° at the same speed. Ignoring friction losses, determine the force required to hold the plate in place against the water stream.





$$-R_x = \dot{m}\vec{v}_2 - \dot{m}\vec{v}_1$$

$$R_x = \dot{m}(\vec{V}_1 - \vec{V}_2)$$

but $\vec{v}_2$ is in the negative direction

$$\therefore \; R_x = \dot{m}(v_1 + v_2)$$ where these are velocity magnitudes (speed)

$$\dot{m} = \rho A V = \left(\frac{62.4\ \text{lbm}}{\text{ft}^3}\right)\left(\frac{\pi}{4}\right)\left(\frac{3}{12}\ \text{ft}\right)^2\left(140\ \frac{\text{ft}}{\text{s}}\right)$$

$$\boxed{\dot{m} = 428.8\ \frac{\text{lbm}}{\text{s}}}$$

$$\therefore \; R_x = \dot{m}(v_1 + v_2)$$

$$= \left(428.8\ \frac{\text{lbm}}{\text{s}}\right)(2)\left(140\ \frac{\text{ft}}{\text{s}}\right) \times \left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\ \text{ft}/\text{s}^2}\right)$$

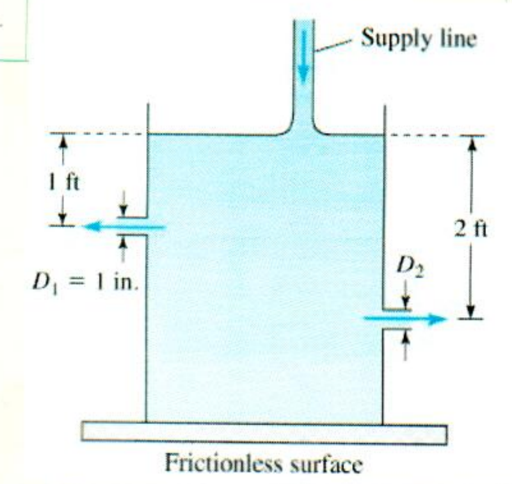
g_c

$$= 3728.9\ \text{lbf}$$

$$\boxed{R_x = 3729\ \text{lbf}}$$ ans

Shaughnessy
7.110

Momentum Eqn



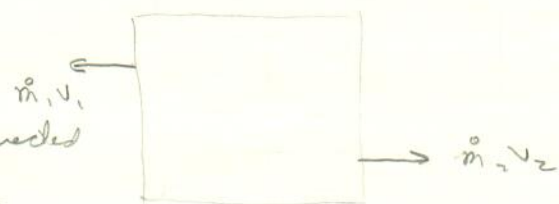
The large tank shown is resting on a frictionless surface. The volume flowrate through the supply line is adjusted so that the water level in the tank remains constant.

The water surface and the exit nozzles are open to the atmosphere.

With the information given in the diagram, and neglecting friction losses, determine the diameter D_2 needed so that the tank remains motionless. Explain the logic used in your analysis.

A momentum balance in the x-direction gives

$$\sum F_x = \sum_{out} \dot{m}\vec{V}_x - \sum_{in} \dot{m}\vec{V}_x$$



but, here there are no x-directed momentum flow into the CV and we desired $\sum F_x = 0$ ← for tank to remain motionless

$$\therefore \sum_{out} \dot{m}\vec{V}_x = 0 = \dot{m}_1\vec{V}_1 + \dot{m}_2\vec{V}_2$$

but $\vec{V}_1 = -|\vec{V}_1| = -V_1$ $\quad\therefore\ \dot{m}_1 V_1 = \dot{m}_2 V_2$

$$\rho A_1 V_1^2 = \rho A_2 V_2^2$$

or $\boxed{A_2 = \frac{V_1^2}{V_2^2} A_1}$

Now, from the Bernoulli eqn (assuming no friction losses)

surface to exit #1

$$\frac{\cancel{P_0}}{\gamma} + \frac{V_0^2}{2g} + z_1 = \frac{\cancel{P_1}}{\gamma} + \frac{V_1^2}{2g} + z_2 \qquad V_1^2 = 2g(z_0 - z_1) = 2gh_1$$

free surface (→ 0, → 0), free jet (→ 0)

surface to exit #2

similar → $\quad V_2^2 = 2g(z_0 - z_2) = 2gh_2$

$$\therefore A_2 = \frac{h_1}{h_2} A_1$$

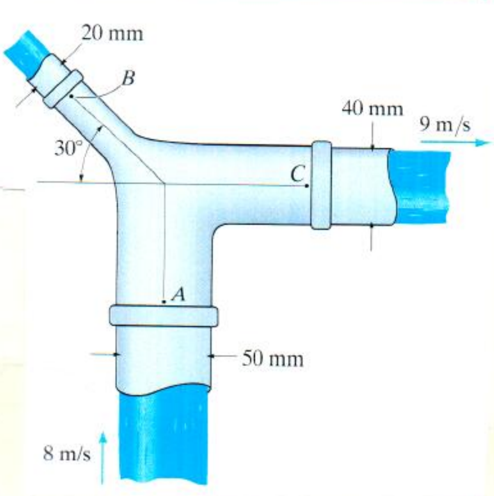
$$D_2^2 = \frac{h_1}{h_2} D_1^2 \Rightarrow \frac{1}{2}(1\,in^2)$$

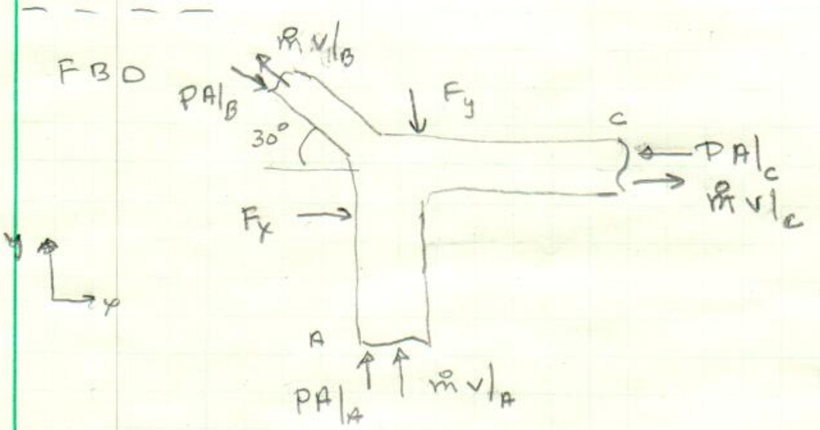
$$\therefore \boxed{D_2 = \sqrt{\tfrac{1}{2}\,in^2} = 0.707\ in}$$

note $V_1 = \sqrt{2gh_1}$
$= \sqrt{2(32.2\ ft/s^2)(1\ ft)}$
$= 8.02\ ft/s$

$V_2 = \sqrt{2gh_2}$ ← 2 ft
$= 11.35\ ft/s$

6–19. Water enters A with a velocity of 8 m/s and pressure of 70 kPa. If the velocity at C is 9 m/s, determine the horizontal and vertical components of the resultant force that must act on the transition to hold it in place. Neglect the size of the transition.





→ continuity eqn (assume outflow at B)

$$Q_A = Q_B + Q_C \qquad Q_A = vA = \left(8\tfrac{m}{s}\right)\tfrac{\pi}{4}(0.05m)^2$$

$$= (8)(1.963\times10^{-3}) = \boxed{1.571\times10^{-2}\ \tfrac{m^3}{s}}$$

or $Q_B = Q_A - Q_C$

$$= (1.571\times10^{-2}) - (1.131\times10^{-2}) \qquad Q_C = \left(9\tfrac{m}{s}\right)\left(\tfrac{\pi}{4}\right)(0.04m)^2$$

$$Q_B = \boxed{4.40\times10^{-3}\ m^3/s} \qquad = (9)(1.257\times10^{-3}) = \boxed{1.131\times10^{-2}\ \tfrac{m^3}{s}}$$

$$V_B = \frac{Q}{A} = \frac{4.40\times10^{-3}}{\frac{\pi}{4}(0.02)^2}$$

$$= \frac{4.40\times10^{-3}}{3.142\times10^{-4}} = \boxed{14.0\ m/s}$$

also

$$PA|_A = (70\times10^3)(1.963\times10^{-3}) = \boxed{137.4\ N}$$

→ Bernoulli Eqn A → B

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + \cancel{z_A} = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + \cancel{z_B}$$

$$P_B = P_A + \frac{\rho}{2}\left(V_A^2 - V_B^2\right)$$

$$= 70kPa + 500\frac{kg}{m^3}(64 - 196)\frac{m^2}{s^2} \qquad \boxed{\frac{kg\text{-}m/s^2}{m^2} = \frac{N}{m^2} = Pa}$$

$$= (70 - 66)\,kPa$$

$$\boxed{P_B = 4\ kPa} \Rightarrow PA|_B = \left(4\times10^3\tfrac{N}{m^2}\right)(3.142\times10^{-4}\ m^2)$$

$$\boxed{PA|_B = 1.26\ N}$$

→ Bernoulli Eqn A → C

$$P_C = P_A + \frac{\rho}{2}\left(V_A^2 - V_C^2\right)$$

$$= 70kPa + 500(64 - 81)\,Pa \qquad (PA)|_C = (61.5\times10^3)\times(1.257\times10^{-3})$$

$$= (70 - 8.5)\,kPa \qquad \boxed{P_C = 61.5\ kPa} \qquad = \boxed{77.3\ N}$$

$$= 61.5\ kPa$$

→ momentum eqn (x-dir)

$$\sum F = F_x - PA|_c + PA|_B \cos 30 = \dot{m}\vec{v}|_c + \dot{m}\vec{v}\cos 30|_B$$

$$F_x = PA|_c - PA|_B \cos 30 + \rho Q_c v_c + \rho Q_B(-v_B \cos 30)$$

$$= 77.3 - 1.26\cos 30 + (1000)(1.31\times10^{-2})(9) - 1000(4.4\times10^{-3})\times(14\cos 30°)$$

$$= 77.3 - 1.1 + 101.8 - 53.3 \text{ N}$$

$$\boxed{F_x = 124.7 \text{ N}}$$

ans

→ momentum eqn (y-dir) (ignore weight of fluid + pipe transition)

$$\sum F = -F_y + PA|_A - PA|_B \sin 30 = \dot{m}\vec{v}|_B \sin 30 - \dot{m}\vec{v}|_A$$

$$\therefore F_y = PA|_A - PA_B \sin 30° - \rho Q v|_B \sin 30 + \rho Q v|_A$$

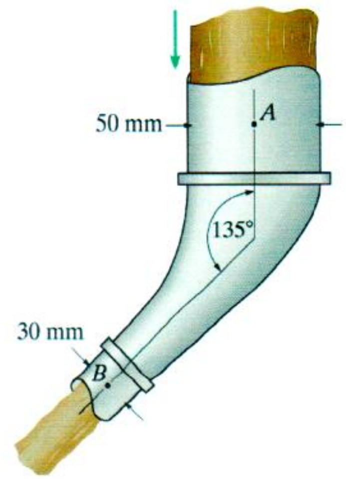
$$= 137.4 - 1.26\sin 30 - (1000)(4.4\times10^{-3})(14)\sin 30° + (1000)(1.571\times10^{-2})(8)$$

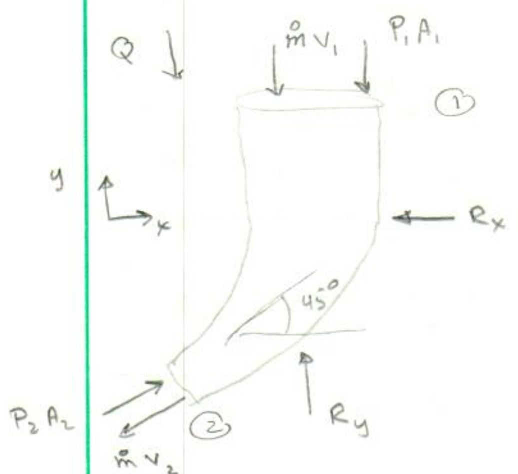
$$= 137.4 - 0.6 - 30.8 + 125.7 \text{ N}$$

$$\boxed{F_y = 231.7 \text{ N}}$$

Crude oil flows through the horizontal tapered 45° elbow at $0.025\ m^3/s$. If the pressure at Point A is 450 kPa, determine the horizontal and vertical components of the resultant force that the oil exerts on the elbow.

Neglect the size of the elbow.





steady flow momentum balance give

sum of forces = net momentum flow out of CV

$$\sum \vec{F} = \dot{m}\vec{V}_2 - \dot{m}\vec{V}_1$$

This is a vector eqn

reaction and pressures forces

in x-direction

V_2 in neg x-direction

$$P_2A_2 \cos 45 - R_x = -\dot{m}|V_2| \cos 45$$

$$\therefore \boxed{R_x = P_2A_2 \cos 45 + \dot{m}|V_2| \cos 45} \qquad (1)$$

in y-direction

V_2 in neg y dir $\qquad$ V_1 in neg y dir

$$P_2A_2 \sin 45 + R_y - P_1A_1 = -\dot{m}|V_2| \sin 45 - (-\dot{m}|V_1|)$$

$$\therefore \boxed{R_y = P_1A_1 - P_2A_2 \sin 45 + \dot{m}|V_1| - \dot{m}|V_2| \sin 45} \qquad (2)$$

Now to find all the needed elements

$$V_1 = \frac{Q}{A_1} = \frac{0.025\ m^3/s}{\frac{\pi}{4}(.05\ m)^2} = \frac{0.025}{1.963 \times 10^{-3}} = 12.73\ m/s$$

$$v_2 = \frac{Q}{A_2} = \frac{0.025}{\frac{\pi}{4}(0.03)} = \frac{0.025}{7.069\times10^{-4}} = \boxed{35.37\ m/s}$$

Bernoulli from pt 1 → pt 2

$$\frac{P_1}{\gamma} + \frac{v_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + \cancel{z_2} \qquad z_1 = z_2$$

$$P_2 = P_1 + \frac{\rho \cancel{g}(v_1^2 - v_2^2)}{2\cancel{g}}$$

ρ crude oil App A in Hibbeler

$$= 450\text{ kPa} + \frac{(880\ kg/m^3)(12.73^2 - 35.37^2)\ m^2/s^2}{2}$$

$$P_2 = (450 - 479.2)\text{kPa} = \boxed{-29.2\text{ kPa}}$$ ← partial vacuum

$$\mathring{m} = \rho Q = \left(880\ \frac{kg}{m^3}\right)\left(0.025\ m^3/s\right) = \boxed{22\ kg/s}$$

Okay, now to compute the desired forces

$$R_x = P_2 A_2 \cos 45 + \mathring{m}|v_2|\cos 45$$

$$= \left(-29.2\times10^3 \frac{N}{m^2}\right)\left(7.069\times10^{-4}\ m^2\right)(0.707)$$

$$+ \left(22\ \frac{kg}{s}\right)\left(35.37\ m/s\right)(0.707)$$

$$= -14.59\ N + 550.1\ N$$

$$\boxed{R_x = 535.5\ N}$$ ans

$$R_y = P_1 A_1 - P_2 A_2 \sin 45 + \mathring{m}|v_1| - \mathring{m}|v_2|\sin 45$$

$$= (450\times10^3)(1.962\times10^{-3}) - (-29.2\times10^3)(7.069\times10^{-4})(0.707)$$

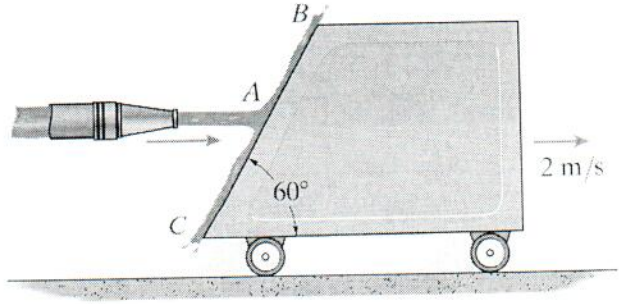
$$+ (22)(12.73) - (22)(35.37)(0.707)$$

$$= 882.9 + 14.59 + 280.1 - 550.1$$

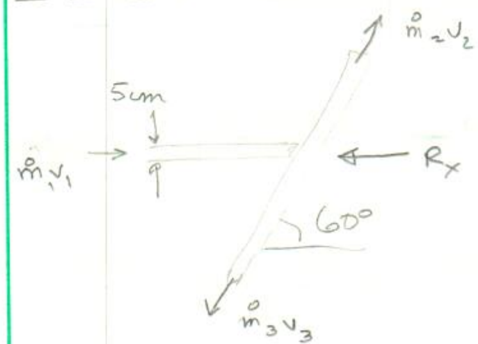
$$\boxed{R_y = 627.5\ N}$$ ans

based on...

Flow from the water stream strikes the inclined surface of the cart. Determine the power produced by the stream if, due to rolling friction, the cart moves to the right with a constant velocity of 2 m/s. The discharge from the 5 cm diameter nozzle is 0.04 m^3/s. Assume that one-third of the discharge flows down the incline, and two-thirds flows up the incline.



Hint: Power is simply F×v when the force F and speed v are in the same direction.



→ all free jets ∴ No pressure forces

→ however the cv is moving to the right at constant vel. $\boxed{V_{cs} = 2\ m/s}$

→ also given
$\dot{m}_2 = \frac{2}{3}\dot{m}_1$ and $\dot{m}_3 = \frac{1}{3}\dot{m}_1$

→ Also from Bernoulli ① → ②

$$\cancel{\frac{P_1}{\gamma}} + \frac{V_1^2}{2g} + \cancel{z_1} = \cancel{\frac{P_2}{\gamma}} + \frac{V_2^2}{2g} + \cancel{z_2}$$

$z_1 = z_2$
$P_1 = P_2$

$\therefore V_1 = V_2$

and likewise $V_1 = V_3$

$\therefore V_1 = V_2 = V_3$

note that $A_1 \neq A_2 \neq A_3$

→ $V_A = \dfrac{Q}{\frac{\pi}{4} D_A^2} = \dfrac{0.04\ m^3/s}{\frac{\pi}{4}(0.05m)^2} = \dfrac{0.04}{1.963\times10^{-3}} = 20.37\ m/s$

$V_r = V_A - V_{cs} = 20.37 - 2 = 18.37\ m/s$

$\dot{m}_r = \rho A V_r = \left(1000\ \frac{kg}{m^3}\right)\left(1.963\times10^{-3}\ m^2\right)\left(18.37\ m/s\right)$

$= 36.06\ kg/s$

$\therefore \dot{m}_{1r} = 36.06\ kg/s$ $\quad \dot{m}_{2r} = \frac{2}{3}\dot{m}_{1r} = 24.04\ kg/s$

$\dot{m}_{3r} = \frac{1}{3}\dot{m}_{1r} = 12.02\ kg/s$

Now the steady flow momentum balance eqn gives

$$\sum F_x = \text{net x-directed momentum out of CV}$$

-x-direction

$$-R_x = \left(\dot{m}_{2r}|V_r|\cos 60\right) + \left(-\dot{m}_{3r}|V_r|\cos 60\right) - \dot{m}_{1r}|V_r|$$

or

$$R_x = \dot{m}_{1r}|V_r| + \dot{m}_{3r}|V_r|\cos 60 - \dot{m}_{2r}|V_r|\cos 60$$

$$= \underset{kg/s}{(36.06)}\underset{m/s}{(18.37)} + (12.02)(18.37)(0.5) - (24.04)(18.37)(0.5)$$

$$= 662.4 + 110.4 - 220.8$$

$$\boxed{R_x = 551.7\ N}$$

for equilibrium, the force the stream exerts on the cart is 551.7 N. Thus, since the cart is moving at 2 m/s, the power is

$$\text{Power} = \text{force} \times \text{vel.}$$

$$= (551.7\ N)\left(2\ \frac{m}{s}\right)$$

$$= \boxed{1103.4\ W}$$

ans