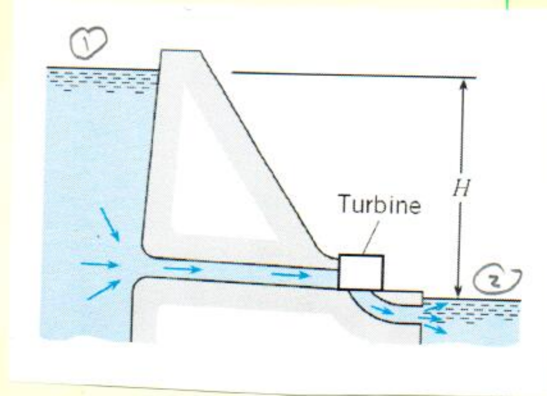
(7.41) Consider the small-scale power system shown in the sketch. The discharge of water through the turbine is 1000 cfs. What power is generated if the turbine efficiency is 85% and the total head loss is 4 ft? The reservoir elevation is H = 100 ft.



— — —

Writing the energy eqn between the two surfaces gives

$$\underbrace{\cancel{\frac{p_1}{\gamma}}}_{\to 0} + \underbrace{\cancel{\frac{V_1^2}{2g}}}_{\to 0} + z_1 + \underbrace{\cancel{h_A}}_{\to 0} - h_R - h_L = \cancel{\frac{p_2}{\gamma}} + \cancel{\frac{V_2^2}{2g}} + z_2$$

free surface   no pump   free surface

$$\therefore \; h_R = z_1 - z_2 - h_L$$

$$= 100\text{ ft} - 4\text{ ft} = 96\text{ ft}$$ (total head removed by the turbine)

$$P_R = h_R \gamma Q = (96\text{ ft})\left(62.4\,\frac{\text{lbf}}{\text{ft}^3}\right)\left(1000\,\frac{\text{ft}^3}{\text{s}}\right)$$

$$= 5.99\times10^6\,\frac{\text{ft-lbf}}{\text{s}} \times \frac{1\text{ hp}}{550\,\frac{\text{ft-lbf}}{\text{s}}} = \boxed{10,892\text{ hp}}$$

$$P_{gen} = \eta P_R = (0.85)(10,892\text{ hp})$$

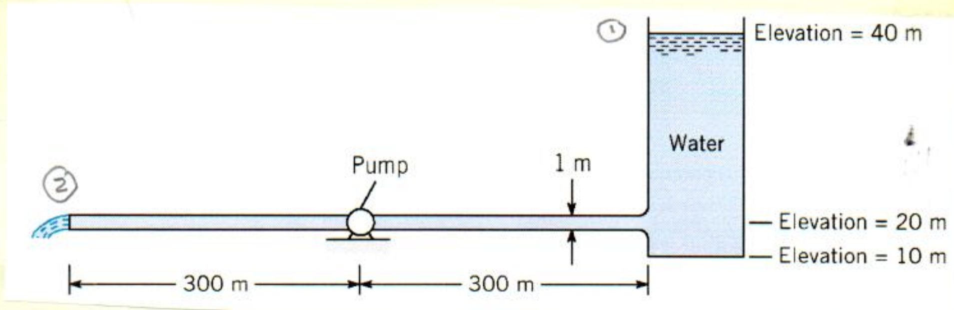
$$= 9258\text{ hp} \Rightarrow \boxed{9260\text{ hp}}$$ ans

Crowe 7th Ed

Energy Eqn

(7.36) Water flows through the horizontal pipe shown at a rate of 7.85 m^3/s. The pipe inside diameter is 1m. The head loss in the pipe was measured to be about $7V^2/2g$, where V is the average fluid velocity.

With this information and the configuration data given on the diagram, determine power delivered by the pump to the fluid to achieve the given flow rate.



The energy eqn from the reservoir surface to the pipe exit is

$$\underbrace{\cancel{\frac{P_1}{\gamma}}}_{\to 0} + \underbrace{\cancel{\frac{v_1^2}{2g}}}_{\to 0} + z_1 + h_A - h_L \underbrace{- \cancel{h_R}}_{\to 0} = \underbrace{\cancel{\frac{P_2}{\gamma}}}_{\to 0} + \frac{V_2^2}{2g} + z_2$$

free surface $\qquad$ free jet

$$h_A = h_L + \frac{V_2^2}{2g} + z_2 - z_1$$

$$= (7+1)\frac{V^2}{2g} + (20-40)\,m$$

$$\boxed{h_A = 8\frac{V^2}{2g} - 20\,m}$$

but $Q = 7.85\,m^3/s$ $\qquad \therefore \boxed{V = \frac{Q}{A} = 10\,m/s}$

$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}\,m^2$

$$\therefore h_A = \frac{8(10\,m/s)^2}{2(9.8\,m/s^2)} - 20\,m = 40.78\,m - 20\,m$$

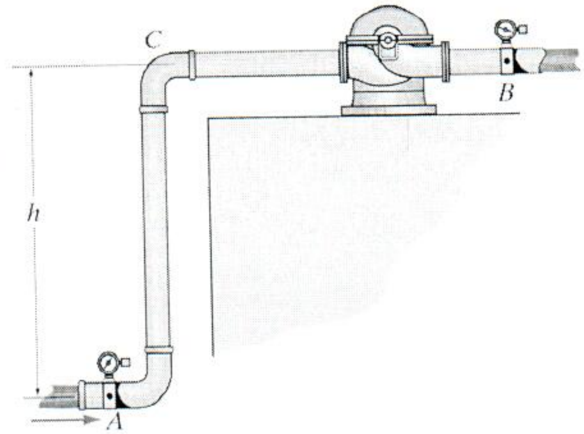
$$= \boxed{20.8\,m}$$

and $P_A = h_A \gamma Q = (20.8)(9.8)(7.85) = \boxed{1600\,kW}$

$(m)\left(\frac{kN}{m^3}\right)\left(\frac{m^3}{s}\right) = \frac{kN\text{-}m}{s} = kW$

$= \boxed{1.6\,MW}$ ans

Consider the situation identified in the diagram where water is drawn into the pump as shown. The pipe inside diameter is 4 inches on the suction side of the pump and 3 inches on the discharge side and the remaining data for the problem are given below:



$P_A = -6$ psi $P_B = 20$ psi

$Q = 4$ ft3/s $h = 5$ ft

Assuming negligible friction loss in the pipe segment ACB,

a. determine the power output (in hp) of the pump needed to achieve the flow rate indicated, and

b. carefully draw and label the energy and hydraulic grade lines for the pipe segment ACB (Note: here "label" means to indicate the head level (in feet) directly on the EGL and HGL diagrams).

(a) write energy eqn between points A and B

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + z_A + h_A - \underbrace{h_R}_{\to 0} - \underbrace{h_L}_{\to 0} = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

$$\boxed{h_A = \frac{P_B - P_A}{\gamma} + \frac{V_B^2 - V_A^2}{2g} + z_B - z_A}$$

$$\frac{P_B - P_A}{\gamma} = \frac{\left[20 - (-6)\ \frac{lbf}{in^2}\right] \times \frac{144\ in^2}{ft^2}}{62.4\ lbf/ft^3} = 46.15 - (-13.85)\ ft = \boxed{60.0\ ft}$$

$$z_B - z_A = \boxed{5\ ft}$$

$$A_B = \frac{\pi}{4}\left(\frac{3}{12} ft\right)^2 = 4.909 \times 10^{-2}\ ft^2$$

$$V_B = \frac{Q}{A_B} = \frac{4\ ft^3/s}{4.909 \times 10^{-2}\ ft^2} = \boxed{81.49\ ft/s}$$

$$A_A = \frac{\pi}{4}\left(\frac{4}{12} ft\right)^2 = 8.727 \times 10^{-2}\ ft^2$$

$$V_A = \frac{Q}{A_A} = \frac{4}{8.727 \times 10^{-2}} = \boxed{45.84\ ft/s}$$

$$\therefore \frac{V_B^2 - V_A^2}{2g} = \frac{81.49^2 - 45.84^2\ ft^2/s^2}{2(32.2)\ ft/s^2}$$

$$= 103.1\ ft - 32.63\ ft$$

$$= \boxed{70.47\ ft}$$

$\therefore h_A = 60\text{ ft} + 70.47\text{ ft} + 5\text{ ft} = \boxed{135.5\text{ ft}}$

$\therefore P_A = \gamma h_A Q$

$= \left(62.4\,\frac{\text{lbf}}{\text{ft}^3}\right)(135.5\text{ ft})\left(4\,\frac{\text{ft}^3}{\text{s}}\right)$

$= 3.382 \times 10^4\,\frac{\text{lbf-ft}}{\text{s}} \times \frac{1\text{ hp}}{550\,\frac{\text{ft-lbf}}{\text{s}}}$

$= \boxed{61.48\text{ hp}}$

$\therefore$ need $\boxed{P_A = 61.5\text{ hp}}$ ans

(b) For the EGL and HGL sketches, we note that

EGL : H_{tot} = Total head $= \frac{P}{\gamma} + \frac{V^2}{2g} + z$

HGL : H_{hyd} = hydraulic head $= \frac{P}{\gamma} + z$

at A $\quad H_{TOT} = \frac{P}{\gamma} + \frac{V^2}{2g} + z = -13.85 + 32.63 + 0$

$= \boxed{18.77\text{ ft}}$

$H_{hyd} = \frac{P}{\gamma} + z = \boxed{-13.85\text{ ft}}$

at C

$H_{TOT} = 18.77\text{ ft}$

$H_{hyd} = -13.85\text{ ft}$

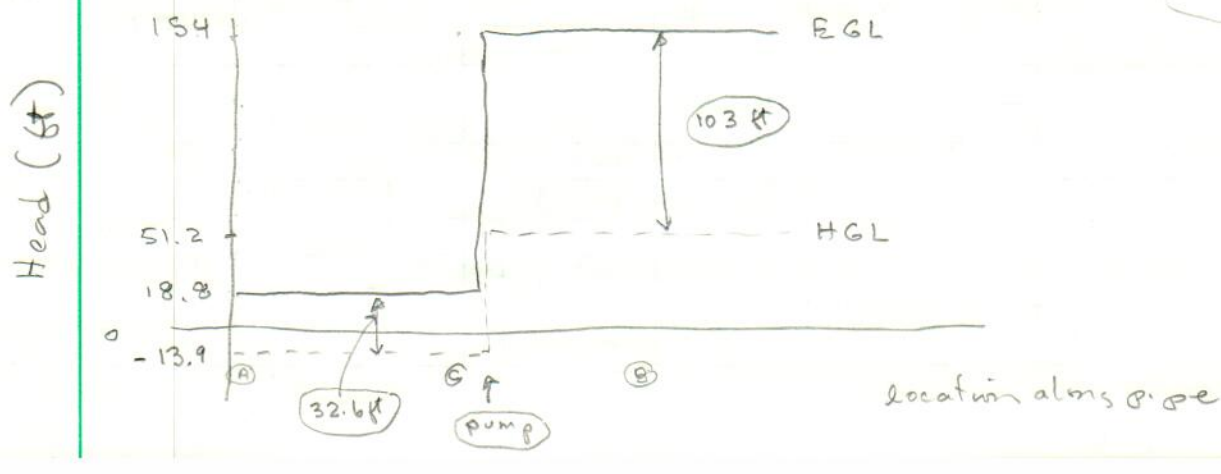
unchanged although $P_A \neq P_C$ and $z_A \neq z_C$

$\frac{P_A}{\gamma} + z_A = \frac{P_C}{\gamma} + z_C$ Bernoulli

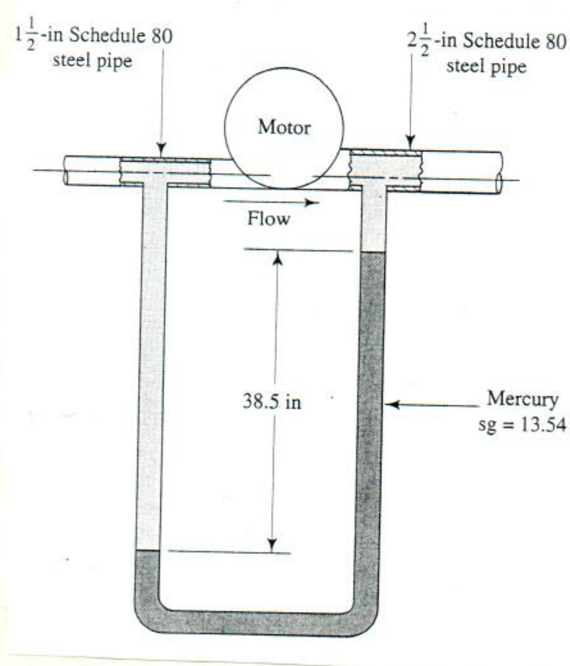
at B $\quad H_{tot} = 46.15 + 103.1 + 5 = \boxed{154.3\text{ ft}}$

$H_{hyd} = \boxed{51.15\text{ ft}}$

note that $\boxed{H_{TOT}\big|_C + h_A = H_{TOT_B}}$ OK



$\boxed{7.44 + 7.45}$



The test setup shown in the diagram measures the pressure difference between the inlet and outlet of the fluid motor. The flow rate of the hydraulic fluid (sg = 0.90) is 135 gal/min.

a. Compute the power removed from the fluid by the motor.

b. If the fluid motor has an efficiency of 78 %, how much power is delivered by the motor?

— — —

The energy eqn from inlet to outlet can be written as

$$\frac{P_1}{\gamma} + \frac{v_1^2}{2g} + \cancel{z_1} + \cancel{h_A} - h_R - \cancel{h_L} = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + \cancel{z_2}$$

$$P_1 - P_2 = (\gamma_{Hg} - \gamma_{oil})\, h = \left(\frac{\gamma_{Hg}}{\gamma_{oil}} - 1\right) \gamma_{oil}\, h$$

or $$\frac{P_1 - P_2}{\gamma_{oil}} = \left(\frac{13.54}{0.90} - 1\right)\left(\frac{38.5}{12}\ ft\right) = (14.04)(3.208\ ft)$$

$$= \boxed{45.06\ ft}$$

$z_1 - z_2 = 0$ same elevation

$h_A = h_L = 0$ no head added and no losses

$Q = A_1 v_1 = A_2 v_2$

$\boxed{\text{Mott App F}}$

Sch 80 $1\frac{1}{2}$" pipe
$A_1 = 0.01227\ ft^2$

Sch 80 $2\frac{1}{2}$" pipe
$A_2 = 0.02944\ ft^2$

$$\therefore v_1 = \frac{Q}{A_1}$$

$$= \frac{(135\ gal/min)\left(\frac{1\ ft^3/s}{449\ gal/min}\right)}{0.01227\ ft^2} \rightarrow \boxed{Q = 0.30\ ft^3/s}$$

$$\boxed{v_1 = 24.50\ ft/s}$$

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{.01227}{.02944}\, 24.5 = \boxed{10.15\ ft/s}$$

now $\dfrac{V_1^2 - V_2^2}{2g} = \dfrac{(24.50)^2 - (10.15)^2 \ ft^2/s^2}{2(32.2 \ ft/s^2)}$

$= \boxed{7.72 \ ft}$

Solving the original energy eqn for h_R give

$h_R = \dfrac{P_1 - P_2}{\gamma} + \dfrac{V_1^2 - V_2^2}{2g}$

$= 45.06 + 7.72 \quad ft$

or $\boxed{h_R = 52.78 \ ft}$

and

$P_R = h_R \gamma Q$

$= (52.78 ft)(0.90)\left(62.4 \dfrac{lbf}{ft^3}\right)(0.30 \ ft^3/s)$

$= \boxed{889.2 \ \dfrac{ft \cdot lbf}{s}} \times \dfrac{1 \ hp}{550 \ \frac{ft \cdot lbf}{s}}$

$= \boxed{1.62 \ hp}$ ans { This is the power removed from the fluid and delivered to the motor

(b) If the motor efficiency is 78%, then only 78% of the input power gets delivered by the motor

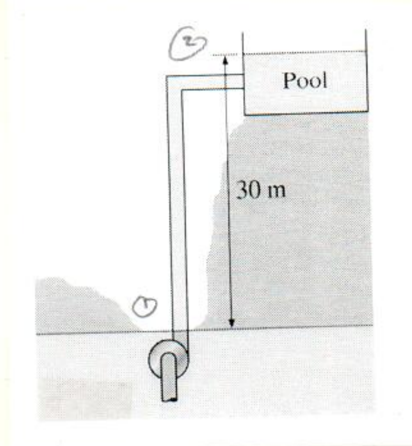
or $P_{out} = \eta P_{in} = \eta P_R$

$= 0.78 (1.62 \ hp)$

$= \boxed{1.26 \ hp}$ ans

Underground water is to be pumped by a 70% efficient 3 KW submerged pump to a pool whose free surface is 30 m above the underground water level. The diameter of the pipe is 7 cm on the intake side and 5 cm on the discharge side.

Assuming negligible head loss in the piping system, and negligible elevation distance between the pump inlet and outlet, determine the maximum flow rate of water and the ΔP across the pump.



→ writing the energy eqn between the surfaces of each reservoir gives

$\alpha_1 = \alpha_2 = 1.0$

$$\underbrace{\cancel{\frac{P_1}{\gamma}}}_{\to 0} + \underbrace{\cancel{\frac{v_1^2}{2g}}}_{\to 0} + z_1 + h_A - \underbrace{\cancel{h_R}}_{\to 0} - h_L = \underbrace{\cancel{\frac{P_2}{\gamma}}}_{\to 0} + \underbrace{\cancel{\frac{v_2^2}{2g}}}_{\to 0} + z_2$$

free surface   free surface

$$\therefore h_A = z_2 - z_1 + h_L$$

→ This gives the max flowrate...

and if we assume h_L is small, then

$$h_A = 30\ \text{m}$$ { head added by the pump is 30 m

Now $P_A\big|_{pump} = h_A \gamma Q$ and $P_A = \eta P_I$ ← power input

power added to fluid

or $P_A = \eta P_I = (0.7)(3\ \text{kW})$

$= 2.1\ \text{kW}$

$\gamma_w = 9.80 \frac{kN}{m^3}$

$$\text{and} \quad Q = \frac{P_A}{h_A \gamma} = \frac{2.1\ \text{kW}}{(30\ \text{m})(9.8\ \text{kN/m}^3)}$$

kW → $\frac{kN\text{-}m}{s}$

$$= \frac{2.1}{(30)(9.8)} = \boxed{7.14 \times 10^{-3}\ m^3/s}$$

{ note that if $h_L > 0$ then this would be lower, since h_A would be higher

units analysis ⇒ Power added $= \dot{W} = h_A \gamma Q$

$$\Rightarrow (m)\left(\frac{N}{m^3}\right)\left(\frac{m^3}{s}\right) = \frac{N\text{-}m}{s} = \frac{J}{s} = \text{Watt}$$

OK

Now, with Q known we can compute the average velocities at the inlet and exit of the pump, $Q = VA$

$$V_{in} = \frac{Q}{A_{in}} = \frac{Q}{\frac{\pi}{4} D_{in}^2} = \frac{4\left(7.14\times10^{-3}\,m^3/s\right)}{\pi\,(0.07m)^2} = \boxed{1.855\,m/s}$$

$$V_{out} = \frac{Q}{A_{out}} = \frac{4\left(7.14\times10^{-3}\right)}{\pi\,(0.05)^2} = \boxed{3.636\,m/s}$$

Now, writing the energy eqn across the pump only

$\alpha_1 = \alpha_2 \simeq 1.0$

$$\frac{P_{in}}{\gamma} + \frac{V_{in}^2}{2g} + \cancel{z_{in}} + h_A - \underbrace{\cancel{h_R}}_{\to 0} - \underbrace{\cancel{h_L}}_{\to 0} = \frac{P_{out}}{\gamma} + \frac{V_{out}^2}{2g} + \cancel{z_{out}}$$

with $z_{out} - z_{in} \approx 0$, this gives

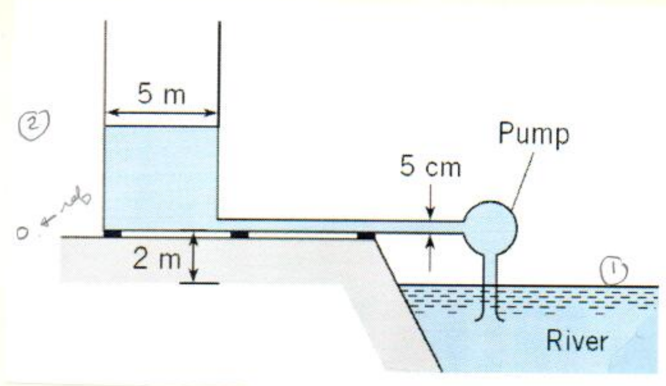
$$\frac{P_{out} - P_{in}}{\gamma} = \frac{\Delta P}{\gamma} = \frac{V_{in}^2 - V_{out}^2}{2g} + h_A$$

$$= \frac{\left(1.855\,m/s\right)^2 - \left(3.636\,m/s\right)^2}{2\left(9.81\,m/s^2\right)} + 30m$$

$$= -0.498\,m + 30\,m = 29.5m$$

$$\therefore\ \Delta P_{pump} = (29.5m)\left(9.81\,\frac{kN}{m^3}\right) = \boxed{\boxed{289.4\,kPa}}$$

A pump is used to fill a tank 5 m in diameter from a river as shown. The water surface in the river is 2 m below the bottom of the tank. The pipe diameter is 5 cm, and the head loss in the pipe is given by $h_L = 10V^2/2g$, where V is the mean velocity in the pipe. The head provided by the pump varies with discharge through the pump as $h_p = 20 - 40000 Q^2$, where Q is given in m^3/s and h_p is in meters.



How long will it take to fill the tank to a depth of 10 m?

$$\frac{dm_{tank}}{dt} = \dot{m}_{in} - \cancel{\dot{m}_{out}}^{\,0}$$

$$\cancel{\rho} A_{tank} \frac{dz_2}{dt} = \cancel{\rho} Q_{in} \qquad \text{where } z_2 = \text{elevation of fluid surface in tank}$$

$$\frac{dz_2}{dt} = \frac{1}{A_{tank}} Q \qquad \text{where } Q = \text{volume flowrate in pipe}$$

Now, from the general energy eqn. at any instant in time, we have

$$\underbrace{\cancel{\frac{p_1}{\gamma}}}_{\to 0} + \underbrace{\cancel{\frac{V_1^2}{2g}}}_{\to 0} + z_1 + h_A - h_L = \underbrace{\cancel{\frac{p_2}{\gamma}}}_{\to 0} + \frac{V_2^2}{2g} + z_2$$

free surface of river

free surface of tank

assuming no minor losses

$z_1 = -2\,m$ given

$z_2(t)$ = quantity of interest

$$h_A = h_p = 20 - 40000 Q^2$$

$$\frac{V_2^2}{2g} = \frac{1}{2g}\left(\frac{dz_2}{dt}\right)^2 = \frac{1}{2g}\left(\frac{1}{A_{tank}}\right)^2 Q^2$$

$$h_L = \frac{10V^2}{2g} = \frac{10}{2gA_p^2} Q^2 = \frac{5}{gA_p^2} Q^2$$

$$\therefore\; -2 + 20 - 40000 Q^2 - \frac{5}{gA_p^2} Q^2 = \frac{1}{2g}\left(\frac{1}{A_{tank}}\right)^2 Q^2 + z_2$$

from continuity eqn

→ This term will be very small

→ let's show this ...

let compute the coeffs

$$A_p = \frac{\pi}{4} D_p^2 = \frac{\pi}{4}(.05m)^2 = 0.001963\ m^2$$

and $$\frac{5}{g A_p^2} = \frac{5}{\left(9.8\frac{m}{s^2}\right)\left(0.001963\ m^2\right)^2} = 132405\ \frac{s^2}{m^5}$$

$$A_{tank} = \frac{\pi}{4} D_{tank}^2 = \frac{\pi}{4}(5m)^2 = 19.63\ m^2$$

and $$\frac{1}{2g A_{tank}^2} = \frac{1}{2(9.8)(19.63)^2} = 1.324\times10^{-4}\ \frac{s^2}{m^5}$$

→ clearly the $\frac{1}{2gA^2_{tank}}$ term is negligible !!!

→ this is equivalent to saying that the $\frac{V_2^2}{2g}$ term ≈ 0

∴ the energy eqn becomes

$$18 - 40000\,Q^2 - 132405\,Q^2 = z_2$$

$$Q^2 = \frac{(18 - z_2)}{172405} \implies Q = \frac{1}{415.2}\sqrt{18-z_2}$$

$$= 2.408\times10^{-3}\sqrt{18-z_2}$$

$$= c\sqrt{18-z_2}$$

Now the continuity eqn becomes

$$\frac{dz_2}{dt} = \frac{c}{A_{tank}}\sqrt{18-z_2}$$

note that c has units of $\frac{m^{5/2}}{s}$

$$\int_0^{10m} (18-z_2)^{-1/2}\,dz_2 = \frac{c}{A_{tank}}\int_0^{\tau} dt$$

let $u = 18 - z_2$
$du = -dz_2$

$$\therefore \int_{18}^{8} -u^{-1/2}\,du = \frac{c}{A_{tank}}\int_0^{\tau} dt$$

$$-2u^{1/2}\Big|_{18}^{8} = \frac{c}{A_{tank}}\,\tau$$

$$\therefore \tau = -\frac{2A_{tank}}{c}\left(\sqrt{8_m} - \sqrt{18_m}\right) = \frac{2(19.63\ m^2)(1.4142\ m^{1/2})}{2.408\times10^{-3}\ m^{5/2}/s}$$

$$= 23057\ sec \times \frac{1\ hr}{3600\ sec} = \boxed{6.4\ hrs}$$

ans