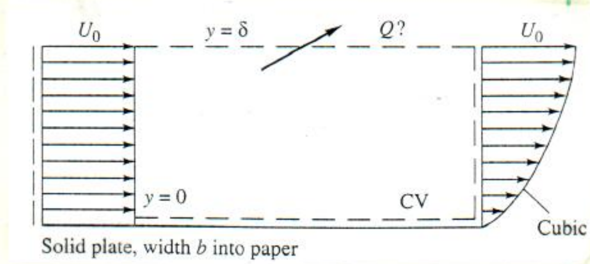
**3.16**

An incompressible fluid flows past an impermeable flat plate as shown. The inlet velocity profile is uniform,

$$u = U_0 \quad \text{(at inlet)}$$




and it has a cubic polynomial exit profile

$$u(\eta) = U_0 \left( \frac{3\eta - \eta^3}{2} \right) \quad \text{(at exit)}$$

with $\eta = y/\delta$

For this problem, develop an expression for the volume flow rate $Q$ across the top surface of the control volume (at $y = \delta$).

Note: Let $L$ = length of the CV, $b$ = width of CV into paper, and $\delta$ = the height of the CV. Then the volume of the CV is simply $Vol = L \delta b$ since the rectangular parallelepiped has a geometry defined by

$$0 \le x \le L \quad , \quad 0 \le y \le \delta \quad , \text{ and } \quad 0 \le z \le b$$

At $x = 0$ and at $x = L$, assume that the flow is one-dimensional in the x-direction perpendicular to the yz plane.

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For incompressible flow, the continuity eqn gives

$$Q_{left} = Q_{in} = Q_{out} = Q_{top} + Q_{right}$$

where $$Q = \int_{cs} \vec{V} \cdot \hat{n} \, dA$$

$$= \int_{cs} V \, dA \quad \text{(for 1-D flows)}$$

At $x = 0$, the flow is 1-D and uniform

$$\therefore \quad Q_{left} = \int_{cs} U_0 \, dA = \int_0^\delta \int_0^b U_0 \, dy \, dz = \boxed{U_0 \delta b}$$

At $x = L$, the flow is also 1-D, but it is non-uniform

$$\therefore \quad Q_{right} = \int_{cs} u(y,z) \, dA = \int_0^\delta \int_0^b u(y) \, dy \, dz$$

or $\quad Q_{right} = b \int_0^{\delta} u(y)\,dy$ $\quad$ } since the velocity is not a function of z

let's change variables

$$\eta = y/\delta \qquad d\eta = \frac{1}{\delta}\,dy$$

$$\text{or } dy = \delta\,d\eta$$

$$\therefore \quad Q_{right} = b\delta \int_0^1 u(\eta)\,d\eta$$

$$= b\delta \frac{u_0}{2} \int_0^1 (3\eta - \eta^3)\,d\eta$$

$$= b\delta \frac{u_0}{2} \left[ \frac{3\eta^2}{2} - \frac{\eta^4}{4} \right]_0^1$$

$$\frac{3}{2} - \frac{1}{4} = \frac{6}{4} - \frac{1}{4} = \frac{5}{4}$$

$$\boxed{Q_{right} = b\delta u_0 \left(\frac{5}{8}\right)}$$

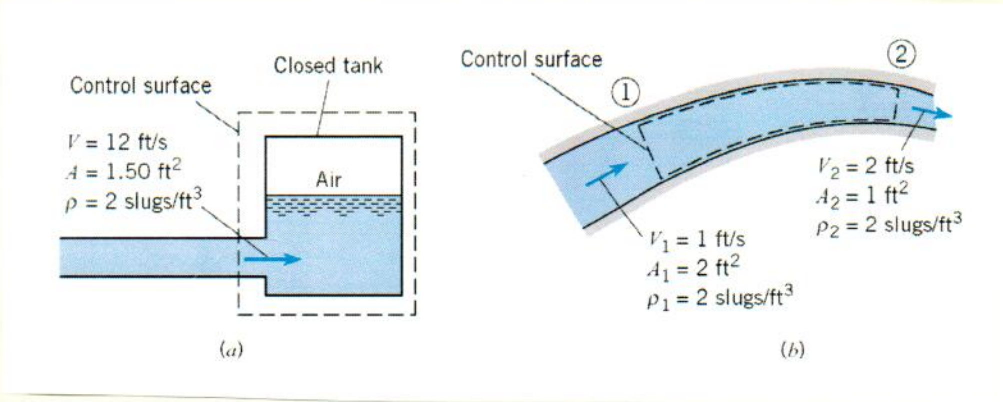
Now, the flow rate out the top is given by

$$Q_{TOP} = Q_{left} - Q_{right}$$

$$= u_0\delta b - u_0\delta b\left(\frac{5}{8}\right)$$

$$\boxed{Q_{TOP} = \frac{3}{8}\,u_0\delta b}$$ ans

For the conditions shown in each of the flow cases (a and b) in the sketch, respond to the following questions and statements concerning the application of Reynold's Transport Theorem to the continuity principle.




a. What is the value of b

(a) and (b) $B = m$ and $b = \frac{dB}{dm} = 1$ — applies to both cases

continuity of mass

b. Determine the value of $dB_{sys}/dt$ — applies to both cases

(a) and (b) since $B_{sys} = m$ Then $\frac{dB_{sys}}{dt} = \frac{dm_{sys}}{dt} = 0$

by defn ⇒ since the mass of a "system" is fixed

c. Determine the value of $\int_{cs} b\rho \vec{v} \cdot \vec{n}\, dA$

$\int_{cs} b\rho \vec{v} \cdot \vec{n}\, dA$ = net mass flow rate out of CV

$= \dot{m}_{out} - \dot{m}_{in}$

(a) $\dot{m}_{in} = \rho A V = \left(2 \frac{slugs}{ft^3}\right)(1.5\, ft^2)(12\, ft/s) = 36$ slugs/s

$\dot{m}_{out} = 0$

$\therefore \int_{cs} b\rho \vec{v} \cdot \vec{n}\, dA = \boxed{-36 \text{ slugs/s}}$ — This is important

(b) $\dot{m}_{in} = \rho A V|_1 = (2)(2)(1) = 4$ slugs/s $\therefore \boxed{\int_{cs} b\rho \vec{v} \cdot \vec{n}\, dA = 0}$

$\dot{m}_{out} = \rho A V|_2 = (2)(1)(2) = 4$ slugs/s

no net mass flow out of CV

ⓓ Determine $\frac{d}{dt}\int_{cv} b\rho \, d\forall$

$$\frac{d}{dt}\int_{cv} b\rho \, d\forall = \text{rate of change of mass within CV} = -\int_{cs} b\rho \vec{v}\cdot\hat{n}\, dA$$

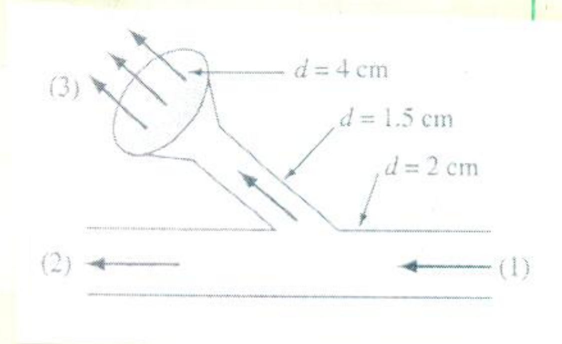
ⓐ $\frac{d}{dt}\int_{cv} b\rho \, d\forall = -(-36\ \text{slugs/s})$

$= +36\ \text{slugs/s}$ } mass is accumulating within the CV (unsteady flow)

ⓑ $\frac{d}{dt}\int_{cv} b\rho \, d\forall = 0$ } no mass accumulation (steady flow)

3.32

Water at 20°C flows steadily through the piping junction in the diagram, entering section 1 at 20 gal/min (0.001262 m³/s).




The average velocity at section 2 is 2.5 m/s. A portion of the flow is diverted through the shower head, which contains 100 holes of 1.0 mm diameter.

Assuming uniform shower flow, estimate the exit velocity from the showerhead jets in m/sec

don't need to give this for open book exam

note: 63.09 $\times 10^{-6}$ m³/s / gpm
$\therefore$ 20 gpm = $1.262 \times 10^{-3}$ m³/s

$$Q_1 = Q_2 + Q_3 \quad \Big\} \text{ continuity eqn for incompressible flow}$$

$$Q_1 = 0.001262 \text{ m}^3/\text{s} = \boxed{1.262 \times 10^{-3} \text{ m}^3/\text{s}} \quad (\text{given})$$

$$Q_2 = A_2 V_2$$
$$= \frac{\pi}{4}(0.02\text{ m})^2 (2.5\text{ m/s}) = 0.0007854 \text{ m}^3/\text{s}$$
$$= \boxed{7.854 \times 10^{-4} \text{ m}^3/\text{s}}$$

$$\therefore Q_3 = Q_1 - Q_2 = \boxed{4.766 \times 10^{-4} \text{ m}^3/\text{s}}$$

Now the flow area at section 3 is

$$A_3 = 100\left[\frac{\pi}{4}(.001\text{ m})^2\right] = 7.854 \times 10^{-5} \text{ m}^2$$

$$\therefore V_3 = \frac{Q_3}{A_3} = \frac{4.766 \times 10^{-4} \text{ m}^3/\text{s}}{7.854 \times 10^{-5} \text{ m}^2} = \boxed{6.068 \text{ m/s}}$$

$$\boxed{V_3 \simeq 6.1 \text{ m/s}}$$

ans

Air flows in a long length of 2.5 cm diameter pipe. At the inlet, the pressure is 200 kPa (abs), the temperature is 150 C, and the average velocity is 10 m/s. At the exit, the pressure and temperature have been reduced by friction and heat loss to 130 kPa (abs) and 120 C, respectively. With this information, determine the exit air velocity.

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The continuity eqn for steady flow says that the mass flow rate is constant.

$$\frac{d}{dT} m_{cv} = \dot{m}_{in} - \dot{m}_{out} = 0 \leftarrow \text{steady flow}$$

$$\therefore \dot{m} = \dot{m}_{in} = \dot{m}_{out}$$

and $\dot{m} = \rho A V$ where $A = A_{in} = A_{out} = \text{constant}$

$$\therefore \rho_{in} V_{in} = \rho_{out} V_{out} \qquad \therefore \boxed{V_{OUT} = \frac{\rho_{in}}{\rho_{out}} V_{in}}$$

Now, assuming air is an ideal gas, we have

$$PV = mRt \quad \text{or} \quad P = \rho RT \quad \text{or} \quad \boxed{\rho = \frac{P}{RT}}$$

$$\therefore \frac{\rho_{in}}{\rho_{out}} = \frac{P_{in}/RT_{in}}{P_{out}/RT_{OUT}} = \frac{P_{in}}{P_{OUT}} \frac{T_{OUT}}{T_{in}} = \left(\frac{200 \text{ kPa}}{130 \text{ kPa}}\right)\left(\frac{393 \text{ K}}{423 \text{ K}}\right)$$

$$\text{or} \quad \frac{\rho_{in}}{\rho_{out}} = 1.429$$

$$\therefore V_{OUT} = \frac{\rho_{in}}{\rho_{out}} V_{in} = (1.429)(10 \text{ m/s}) = \boxed{14.3 \frac{m}{s}}$$

ans

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Some intermediate results (to check student papers)

$$A = \frac{\pi}{4}(0.025 \text{ m}) = 4.909 \times 10^{-4} \text{ m}^2$$

$$\rho_{in} = \frac{P_{in}}{R T_{in}} = \frac{200 \times 10^3 \text{ N/m}^2}{(286.9 \text{ J/kg K})(423 \text{ K})} = 1.648 \frac{kg}{m^3}$$

$\hookrightarrow$ N-m

$$\rho_{out} = \frac{P_{out}}{R t_{out}} = \frac{130 \times 10^3}{(286.9)(393)} = 1.153 \text{ kg/m}^3$$

$$\dot{m}_{in} = \rho A V|_{in} = \left(1.648 \frac{kg}{m^3}\right)(4.909 \times 10^{-4} \text{ m}^2)(10 \text{ m/s}) = 8.09 \times 10^{-3} \frac{kg}{s}$$

$$\dot{m}_{out} = \rho A V|_{out} = (1.153)(4.909 \times 10^{-4})(14.29) = 8.09 \times 10^{-3} \text{ kg/s}$$

OK

Crowe
7th Ed

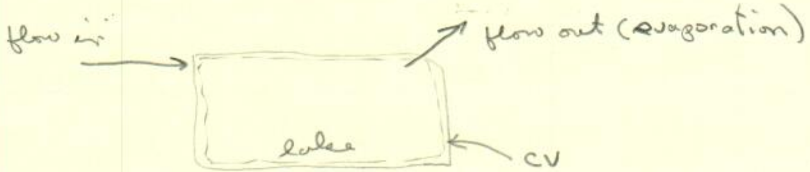
Continuity Eqn

(4.77) A lake with no outlet is fed by a river with a constant flow rate of 1000 $ft^3/s$. Water evaporates from the surface at a constant rate of 13 $ft^3/s$ per square mile of surface area. The top surface area of the lake varies with the average depth of the water as follows

$$A = 4.5 + 5.5h$$

where h is given in feet and A is in square miles

(a.) Under these conditions, what is the equilibrium depth of the lake?




The continuity eqn says that $\frac{dm_{cv}}{dt} = \dot{m}_{in} - \dot{m}_{out}$

in equilibrium $\frac{dm_{cv}}{dt} = 0$ $\therefore$ $\dot{m}_{in} = \dot{m}_{out}$

or $Q_{in} = Q_{out}$ for incompressible flow

$$Q_{in} = 1000\ ft^3/s$$

$$Q_{out} = \left(\frac{13\ ft^3/s}{mi^2}\right)\left(4.5 + 5.5h\right) mi^2$$ with h in feet

$$\therefore \quad 4.5 + 5.5h = \frac{1000}{13} = 76.92$$

$$5.5h = 76.92 - 4.5 = 72.42$$

or $\boxed{h_{eq} \simeq 13.17\ ft}$

(b) Below what river discharge will the lake dry up?

$$\frac{dm_{cv}}{dt} = \rho\left(Q_{in} - Q_{out}\right)$$

can also just set $h = 0$ gives $Q_{in} = Q_{out} = 58.5\ \frac{ft^3}{s}$

if $Q_{in} < Q_{out}$, Then $\frac{dm_{cv}}{dt} < 0$ and the lake will dry out

$$Q_{out} = (13)(4.5 + 5.5h)\ \frac{ft^3}{s} = 58.5 + 71.5h\ ft^3/s$$

for equil $Q_{out} = Q_{in}$ $(58.5 + 71.5h)\ ft^3/s = Q_{in}$

$\therefore$ if $Q_{in} < 58.5\ ft^3/s$ $h < 0 \Rightarrow$ lake drys out