Consider The following velocity field

$$\vec{V} = (0.5 + 1.2x)\hat{i} - 1.2y\hat{j} \quad ft/s$$

With the flow field given above, determine the time-dependent position of a fluid particle that initially starts at $x(0) = x_o = 0$ and $y(0) = y_o$. That is, find $x(t)$ and $y(t)$ given $(0,y_o)$ as the initial particle position.

Once you have a general result for $x(t)$ and $y(t)$ for a arbitrary y_0 values, numerically evaluate and plot the particle trajectory for five (5) specific values of y_o. -- that is, let $y_o = -8, -3, 0, 3$, and 8 ft. Put all five curves on the same plot. Use $0 \le t \le 2.5$ seconds in your evaluations. Do the y(t) vs. x(t) trajectories behave as expected for a converging duct? Explain…

$$\vec{V} = u\hat{i} + v\hat{j}$$

$$\frac{dx}{dt} = u = 0.5 + 1.2x$$

This is separable

$$\frac{dx}{0.5+1.2x} = dt$$

letting $z = 0.5 + 1.2x$

$dz = 1.2\,dx$

$$\therefore \frac{1}{1.2}\int_{z_o}^{z} \frac{dz}{z} = \int_0^t dt$$

$$\ln\frac{z}{z_o} = 1.2t$$

$$z(t) = z_o e^{1.2t}$$

or

$$0.5 + 1.2x(t) = (0.5 + 1.2x_o)e^{1.2t}$$

since $x_o = 0$, we have

$$\boxed{x(t) = \frac{0.5}{1.2}\left(e^{1.2t} - 1\right)} \text{ ft}$$

$$\frac{dy}{dt} = v = -1.2y$$

This is also separable

$$\frac{dy}{y} = -1.2\,dt$$

$$\int_{y_o}^{y} \frac{dy}{y} = \int_0^t -1.2\,dt$$

$$\ln\frac{y}{y_o} = -1.2t$$

or $$\boxed{y(t) = y_o e^{-1.2t}}$$ ft

Now let's plot these profiles for 5 values of y_o

$x_o = 0$ ft, $y_o = -8, -3, 0, 3, 8$ ft.

(x(t) vs t, y(t) vs t and y(t) vs x(t))

see flow_field2.m

(These are the path lines)

this one shows the flow geometry best...

Consider the following velocity field

$$\vec{V} = (0.5 + 1.2x)\hat{i} - 1.2y\hat{j} \quad ft/s$$

Determine an expression for the acceleration field for fluid particles passing through the duct described above. What is the acceleration vector and magnitude at the point (x,y) = (1,4)?

$$\vec{V} = u\hat{i} + v\hat{j}$$

(2-D problem)

where $\vec{V} = \vec{V}(x, y, t)$

and

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + \frac{\partial \vec{V}}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial \vec{V}}{\partial y}\frac{\partial y}{\partial t}$$

$$= \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y}$$

$$= \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \vec{\nabla})\vec{V}$$

for this case, $\frac{\partial \vec{V}}{\partial t} = 0$ steady flow problem

$$\therefore \vec{a} = u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y}$$

$$= (0.5 + 1.2x)(1.2\hat{i}) + (-1.2y)(-1.2\hat{j})$$

$$\boxed{\vec{a} = (0.6 + 1.44x)\hat{i} + 1.44y\hat{j}}$$ (acceleration vector)

ans

at x = 1 and y = 4 ft, we have

$$\boxed{\vec{a} = 2.04\hat{i} + 5.76\hat{j}} \; ft/s^2 \quad \left\{ \vec{a} \text{ at } 1,4 \right.$$

ans

with magnitude

$$|\vec{a}| = \sqrt{2.04^2 + 5.76^2} = \boxed{6.11 \; ft/s^2} \quad \left\{ |\vec{a}| \text{ at } 1,4 \right.$$

ans

Consider the following velocity field

$$\vec{V} = (0.5 + 1.2x)\,\hat{i} - 1.2y\,\hat{j} \quad ft/s$$

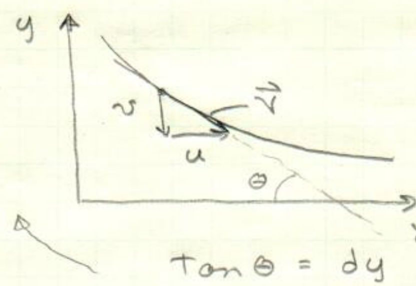
For the above converging duct flow geometry, generate an analytical expression for the flow streamlines. Once you have a general result for the stream function, compute the values of the arbitrary constant for the following x,y pairs; (0,-8), (0,-3), (0,0), (0,3), and (0,8) and plot the five (5) specific streamlines on the same plot over the range $0 \le x \le 8$ ft.

How does this plot compare to the set of pathlines generated in Prob. 1? Explain…

The velocity vector is everywhere tangent to the streamline.

For a 2-D flow field, this is given by

$$\frac{dy}{dx} = \frac{v}{u}$$



$$\tan\theta = \frac{dy}{dx} = \frac{v}{x}$$

For this problem

$u = 0.5 + 1.2x$

$v = -1.2y$

$$\therefore \quad \frac{dy}{dx} = \frac{-1.2y}{0.5 + 1.2x}$$

this is separable

$$\frac{dy}{y} = \frac{-1.2\,dx}{0.5 + 1.2x}$$

$z = 0.5 + 1.2x$

$dz = 1.2\,dx$

$$\int_{y_0}^{y} \frac{dy}{y} = -\int_{z_0}^{z} \frac{dz}{z}$$

$$\ln\frac{y}{y_0} = -\ln\frac{z}{z_0} = -\ln\frac{0.5 + 1.2x}{0.5 + 1.2x_0} = -\ln(1 + 2.4x) \quad \text{for } x_0 = 0$$

$$= \ln\frac{1}{1 + 2.4x}$$

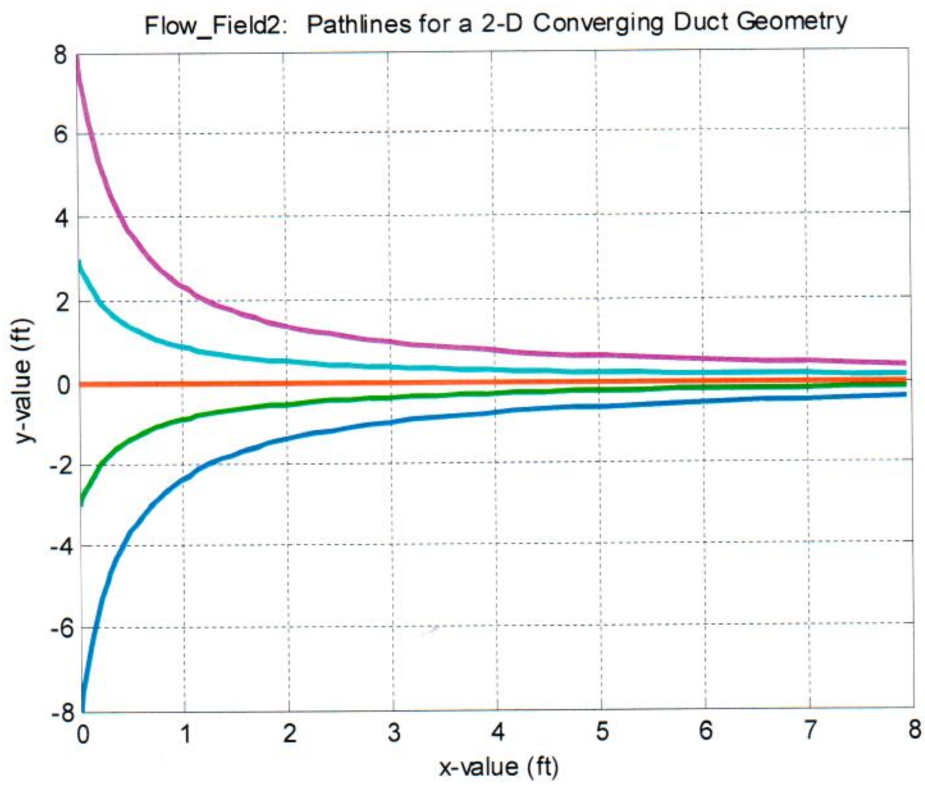
or

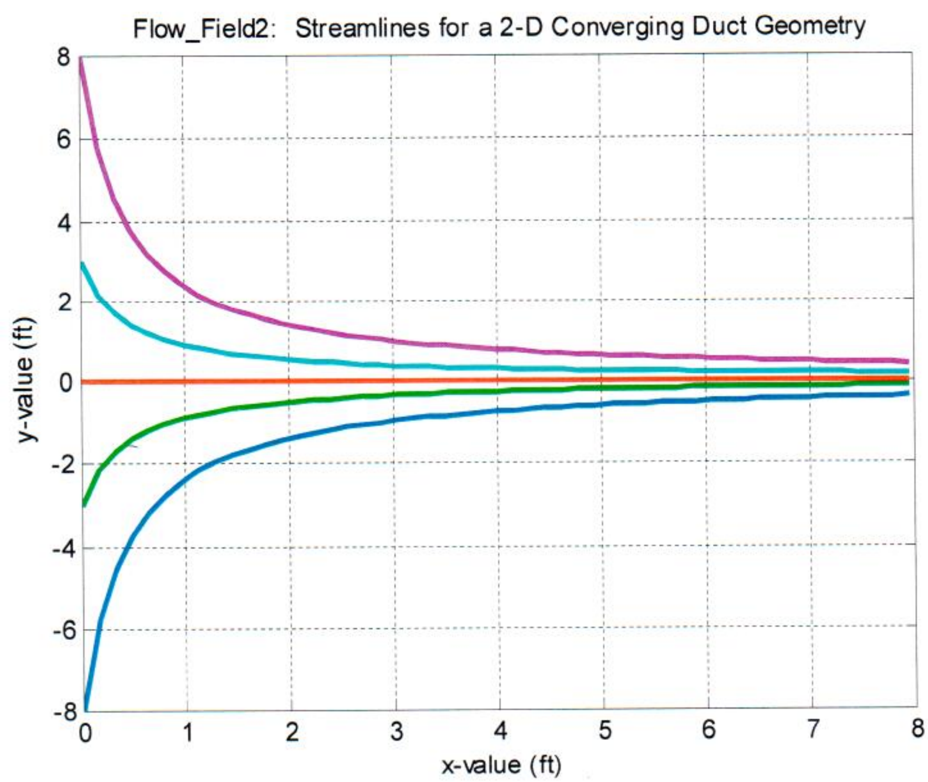
$$y(x) = \frac{y_0}{1 + 2.4x}$$

with different y_0 values, this gives the streamlines for this problem..

streamlines and pathlines are identical ons for steady flow

→ see plot from flow_field2.m


Flow_Field2: Pathlines for a 2-D Converging Duct Geometry
y-value (ft)
x-value (ft)
8
6
4
2
0
-2
-4
-6
-8
0
1
2
3
4
5
6
7
8


Flow_Field2: Streamlines for a 2-D Converging Duct Geometry
y-value (ft)
x-value (ft)
8
6
4
2
0
-2
-4
-6
-8
0
1
2
3
4
5
6
7
8

%

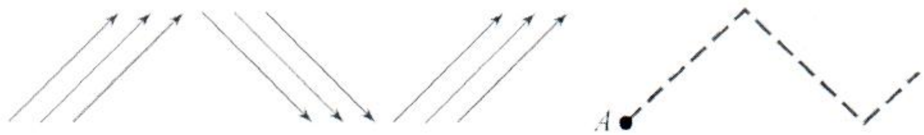
```
%
%   Flow_Field2.M  Sample showing typical stream lines in a converging duct problem
%
%   The velocity field for this problem is given as
%      ->                ^         ^
%       V = (uo+bx)i - (by)j
%   where uo and b are constants.
%
%   The coordinates, x(t) and v(t) are determined by solving dx/dt = u(x,y) = uo+bx
%   and dy/dt = v(x,y) = -by with x(0) = xo = 0  and  y(0) = yo = various values.
%   In addition, the streamlines for this 2-D flow field are given by the solution
%   to dy/dx = v(x,y)/u(x,y) = -by/(uo+bx).
%   The solution to these separable ODEs were derived (with xo = 0) and here we
%   simply want to plot several of these...
%
%   Note that the pathlines and streamlines for a steady flow problem should be
%   identical  --  let's see...
%
%   File prepared by J. R. White, UMass-Lowell (last update: Jan. 2017)
%

     clear all;   close all;   nfig = 0;

%%
%   Part A --  plot several pathlines (use various yo values for xo = 0)
     uo = 0.5;                              % x-directed velocity at x = 0  (ft/s)
     b = 1.2;                               % constant in velocity equation (1/seconds)
     Nt = 51;  t = linspace(0,2.5,Nt);  % time vector for plots
     x = uo*(exp(b*t)-1)/b;                 % x position
     yo = [-8 -3 0 3 8];  Ny = length(yo);  % several values of initial y value
     y = zeros(Nt,Ny);
     for j = 1:Ny                           % y position
       y(:,j) = yo(j)*exp(-b*t);
     end
%
     nfig = nfig+1;   figure(nfig)
     subplot(2,1,1),plot(t,x,'LineWidth',2),grid
     title('Flow\_Field2:  Position vs Time for 2-D Converging Duct Geometry')
     ylabel('x-position (ft)')
     subplot(2,1,2),plot(t,y,'LineWidth',2),grid
     xlabel('time (sec)'),ylabel('y-position (ft)')
     nfig = nfig+1;   figure(nfig)
     plot(x,y,'LineWidth',2),grid
     title('Flow\_Field2:  Pathlines for a 2-D Converging Duct Geometry')
     xlabel('x-value (ft)'),ylabel('y-value (ft)')
%%
%   Part B  --  plot several stream lines
     Nx = 51; xx = linspace(0,x(Nt),Nx)';  % x domain (ft)
     C = yo*uo;  NC = length(C);            % constants for different streamlines
     yy = zeros(Nx,NC);
     for i = 1:NC                           % y position
       yy(:,i) = C(i)./(uo + b*xx);
     end
%
     nfig = nfig+1;   figure(nfig)
     plot(xx,yy,'LineWidth',2),grid
     title('Flow\_Field2:  Streamlines for a 2-D Converging Duct Geometry')
     xlabel('x-value (ft)'),ylabel('y-value (ft)')
%
%   end of problem
```

Flow visualization

A flow field is periodic in that the streamline pattern repeats at definite intervals. For the first second the field is moving upward at 45° to the right and, in the next second, the flow is moving downward at 45° to the right, etc., as shown in the sketch below. The speed of flow is constant at 10 m/s.

After 2.5 s the pathline of a particle released at point A at time zero is also shown in the sketch.

If dye is emitted in a continuous stream from point A starting at time zero, carefully draw the resulting streakline at t = 2.5 s. Explain your plot.



(*a*) Streamlines (*b*) Pathline

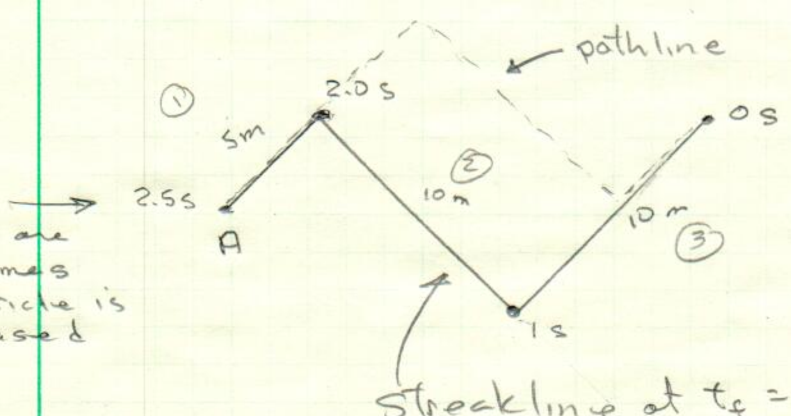
For streaklines, think reverse time.

① from 2 - 2.5 s, the path is upward and to the right at 45° — with $|\vec{V}| = 10$ m/s the particles travels a total of 5 m.

② from 1 - 2 s, particle path is downward to right at 45° ∴ particle travels 10 m in 1 s

③ from 0 - 1 s, path is upward to right at 45° in 1 s the particle travels 10 m

Thus, the streakline at 2.5 s looks as follows:



these are the times a particle is released

t_f = 2.5 s this is the observation time

ans

For a given hypothetical 2-D flow, the space-independent velocity components are given by

$u = -2\ \text{m/s} \qquad v = 0\ \text{m/s} \qquad \text{for} \quad 0 \le t \le 4\ \text{s}$

$u = 1\ \text{m/s} \qquad v = -2.5\ \text{m/s} \qquad \text{for} \quad 4 \le t \le 8\ \text{s}$

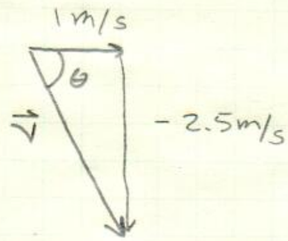
A dye streak was started at a point in the flow field at time t = 0, and the path of a single particle in the fluid was also traced from the same point starting at the same time.

Draw to scale the streakline, pathline of the particle, and streamlines at time t = 8 s for this flow field. Draw and label these curves carefully!

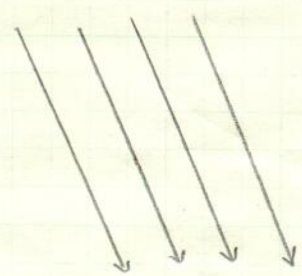
---

streamlines at $t = 8\,s$ $\quad \vec{V} = u\hat{i} + v\hat{j}$

with $u = 1\ \text{m/s}$ and $v = -2.5\ \text{m/s}$



since the velocity vector is parallel to the streamlines, we have



$\tan\theta = \frac{v}{u} = \frac{-2.5}{1}$

$\theta = \tan^{-1}(-2.5)$

$= -68.1°$

pathline for single particle emitted at $x_0 = y_0 = 0$ at $t=0$

(for pathlines work forward in time from the release time)

$\frac{dx}{dt} = u$

$\int_{x_0}^{x} dx = \int_0^t u\,dt$ ← observation time varies; 0 ← release time fixed

(let $x_0 = 0$) $\quad x(t) - x_0 = \int u\,dt$

for $t < 4\,s$ $\quad x(t) = -2\int_0^t dt = -2t$

for $t > 4\,s$ $\quad x(t) = -2\int_0^4 dt + 1\int_4^t dt$

$x(t) = -8 + 1(t-4)$

$\frac{dy}{dt} = v$

$\int_{y_0}^{y} dy = \int_0^t v\,dt$

$y(t) - y_0 = \int_0^t v\,dt$ (let $y_0 = 0$)

$y(t) = 0$

$y(t) = 0\int_0^4 dt - 2.5\int_4^t dt$

$= -2.5(t-4)$

→ see Matlab plots from unsteady-flow-vis2

* These give the position of a single particle at time t that was released at t = 0 at $x_0 = y_0 = 0$

streakline — for continuous stream of particles at time t from $x_0 = y_0 = 0$ with snapshot at $t_f = 8\,s$

(for streaklines work backward in time from observation time)

$$\frac{dx}{dt} = u$$

$$\int_{x_0}^{x_f} dx = \int_{\tau}^{t_f} u\,dt$$

(t_f ← observation time fixed; τ ← release times varies)

(for $x_0 = 0$)

for $\tau > 4\,s$ $\quad x_f(\tau) = 1\int_{\tau}^{t_f} dt = 1(t_f - \tau)$

for $\tau < 4\,s$ $\quad x_f(\tau) = -2\int_{\tau}^{4} dt + 1\int_{4}^{t_f} dt$

$$= -2(4-\tau) + (t_f - 4)$$

$$\frac{dy}{dt} = v$$

$$\int_{y_0}^{y_f} dy = \int_{\tau}^{t_f} v\,dt$$

(for $y_0 = 0$)

$$y_f(\tau) = -2.5\int_{\tau}^{t_f} dt$$

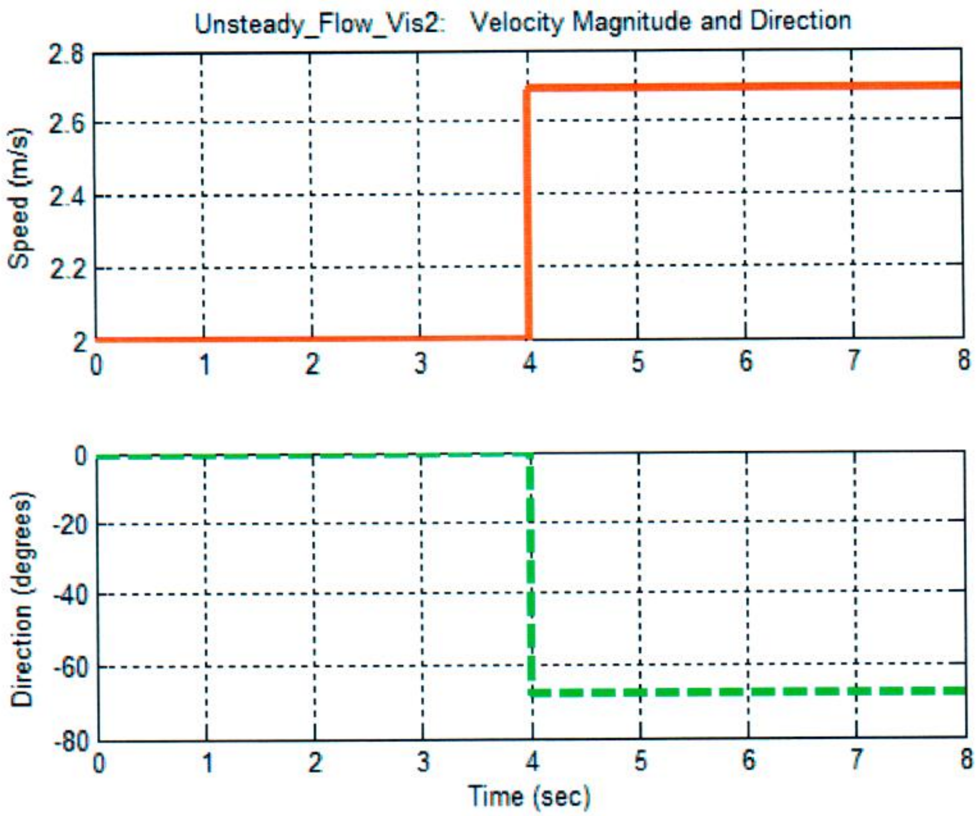
$$= -2.5(t_f - \tau)$$

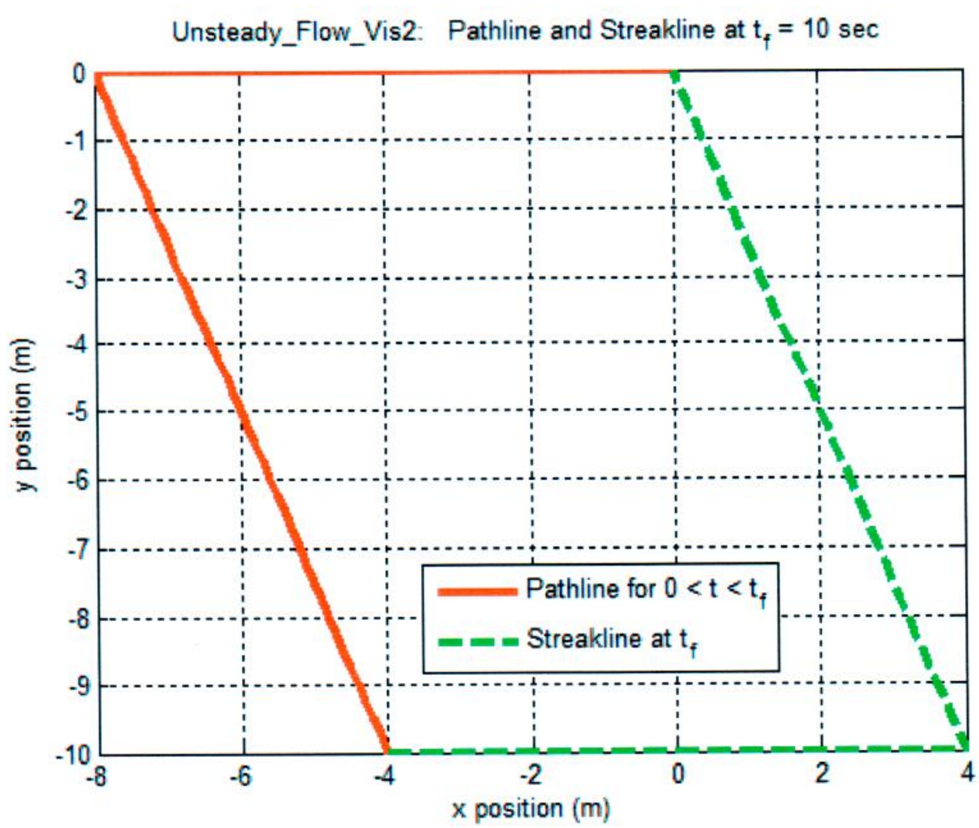
$$y_f(\tau) = 0\int_{\tau}^{4} dt - 2.5\int_{4}^{t_f} dt$$

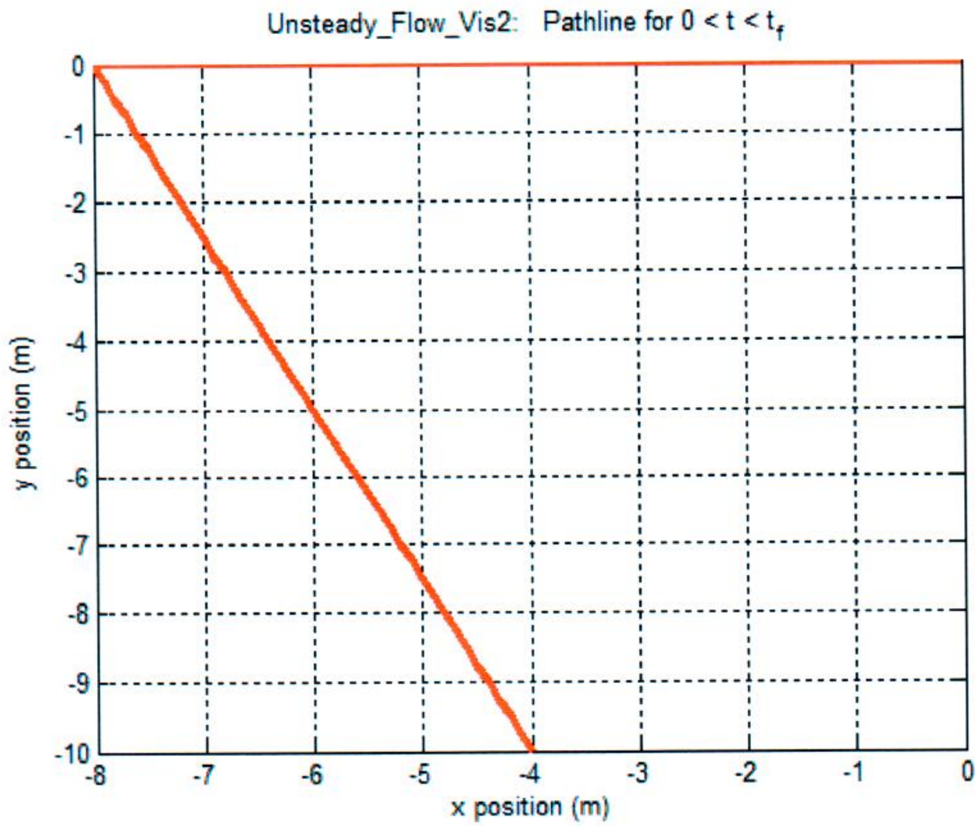
$$= -2.5(t_f - 4)$$

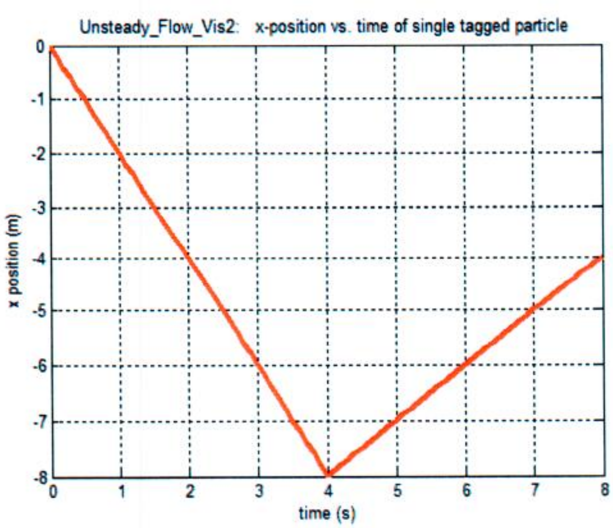
→ see matlab plots from unsteady_flow_vs2

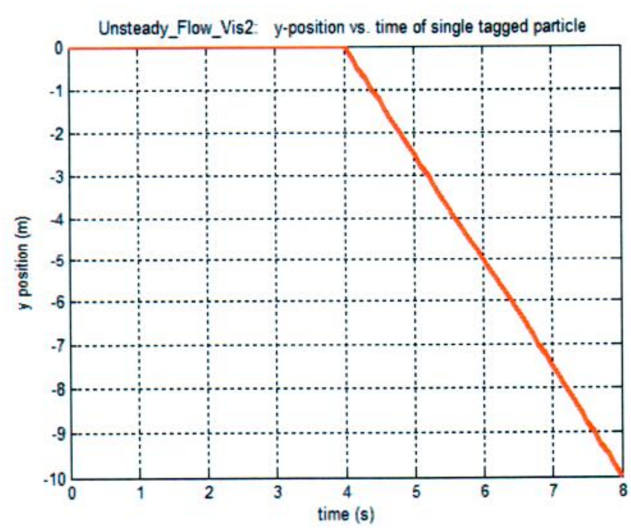
* these give the location of a continuous stream of particles at t_f that were emitted at $x_0 = y_0 = 0$ starting at $t = 0$.

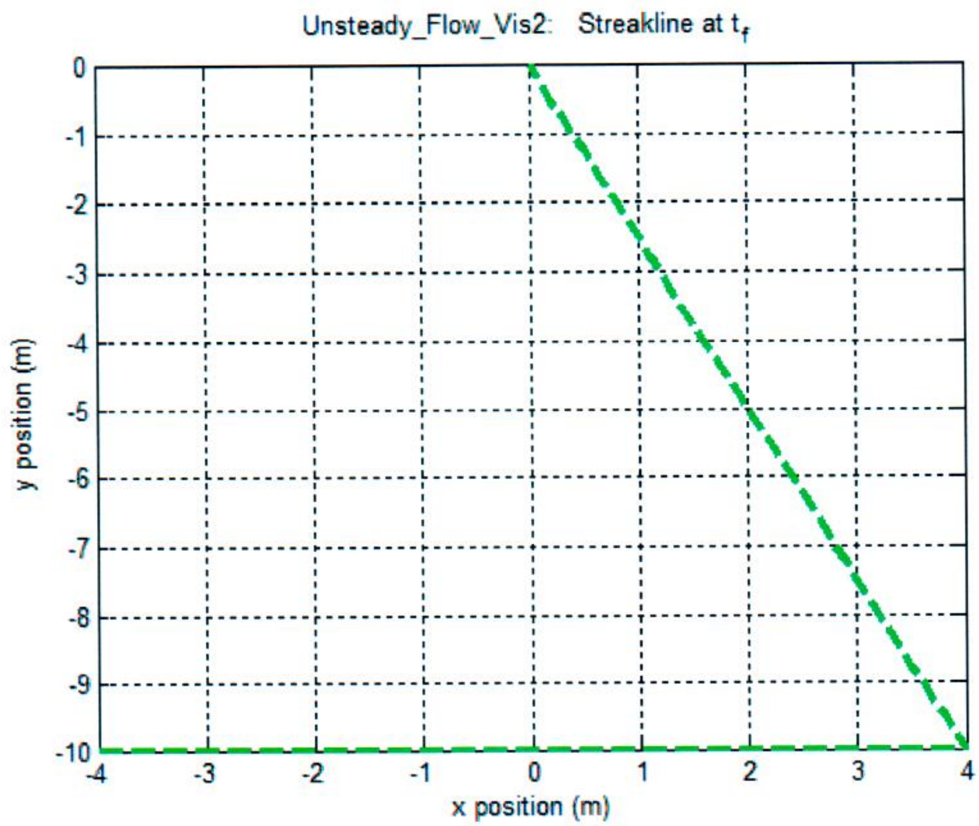

Unsteady_Flow_Vis2: Velocity Magnitude and Direction
Speed (m/s)
2.8
2.6
2.4
2.2
2
0 1 2 3 4 5 6 7 8
Direction (degrees)
0
-20
-40
-60
-80
0 1 2 3 4 5 6 7 8
Time (sec)

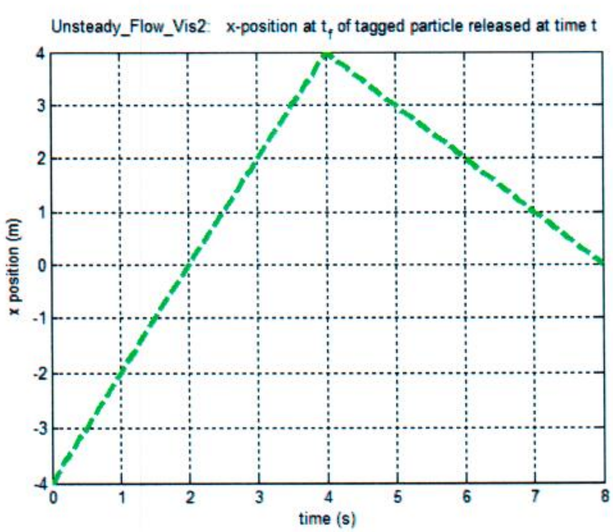

Unsteady_Flow_Vis2: Pathline and Streakline at t_f = 10 sec
y position (m)
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
-8 -6 -4 -2 0 2 4
x position (m)
Pathline for 0 < t < t_f
Streakline at t_f

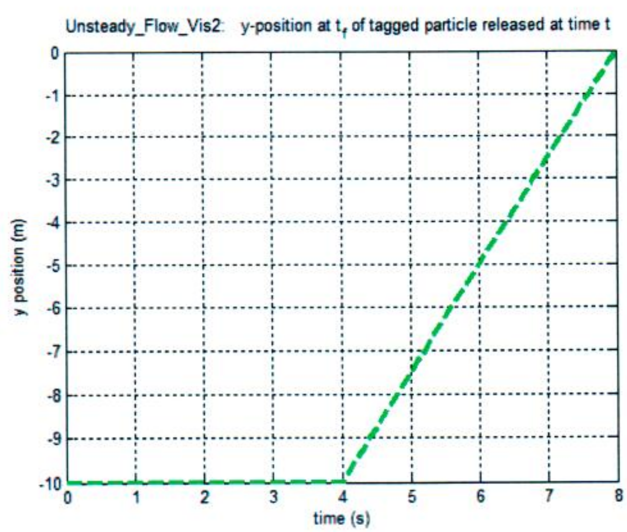

Unsteady_Flow_Vis2: Pathline for 0 < t < t_f
y position (m)
x position (m)
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
-8
-7
-6
-5
-4
-3
-2
-1
0


Unsteady_Flow_Vis2: x-position vs. time of single tagged particle
x position (m)
time (s)


Unsteady_Flow_Vis2: y-position vs. time of single tagged particle
y position (m)
time (s)


Unsteady_Flow_Vis2: Streakline at t_f
y position (m)
x position (m)
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
-4
-3
-2
-1
0
1
2
3
4


Unsteady_Flow_Vis2: x-position at t_f of tagged particle released at time t
x position (m)
time (s)


Unsteady_Flow_Vis2: y-position at t_f of tagged particle released at time t
y position (m)
time (s)

```
%
%   UNSTEADY_FLOW_VIS2.M  Plots some functions associated with an unsteady flow problem
%
%   This problem is based on Prob. 4.7 in Crowe 7E (with different values).
%
%   This case is very simple, since the flow field is spatially constant. However,
%   it does vary with time -- thus, this is an unsteady flow problem.  The time-
%   dependent velocity profile is defined by the components of the 2-D flow field:
%       for t < 4 s     u = -2 m/s    and      v = 0 m/s
%       for t > 4 s     u = 1 m/s     and      v = -2.5 m/s
%
%   With this time-dependent (but spatially independent) flow field, we will compute
%   the following:
%      1. Magnitude and direction of the velocity vector versus time.
%      2. Pathline for a particle starting at the origin  --  this is the path
%         followed if a single droplet of dye released at t = 0 at the origin.
%      3. Streakline at tf  --  this is the spatial distribution at tf of a
%         continuous stream of dye particles emitted at the origin starting at t = 0.
%
%   File prepared by J. R. White, UMass-Lowell (last update: Jan. 2017)
%

      clear all;   close all;   nfig = 0;
%
%   define the velocity vector field and plot magnitude and direction vs time
      tt = 4;   t1 = linspace(0,tt,41);
      u1 = -2*ones(size(t1));  v1 = zeros(size(t1));
      tf = 8;  t2 = linspace(tt,tf,41);
      u2 = 1*ones(size(t2));  v2 = -2.5*ones(size(t2));
      t = [t1 t2];  u = [u1 u2];  v = [v1 v2];
      vmag = sqrt(u.*u + v.*v);  vdir = atan(v./u)*180/pi;

      nfig = nfig+1;   figure(nfig)
      subplot(2,1,1),plot(t,vmag,'r-','LineWidth',3),grid
      title('Unsteady\_Flow\_Vis2:   Velocity Magnitude and Direction')
      ylabel('Speed (m/s)')
      subplot(2,1,2),plot(t,vdir,'g--','LineWidth',3),grid
      xlabel('Time (sec)'),ylabel('Direction (degrees)')
%
%   compute the path of a single particle (a pathline) starting at t=0 out to t=tf
      x1 = -2*t1;   x2 = -8 + 1*(t2-tt);   x_path = [x1 x2];
      y1 = 0*t1;    y2 = -2.5*(t2-tt);     y_path = [y1 y2];
%
%   compute the distribution of points at tf sec made by a continuous stream
%   of particles emitted at t (a streakline)
      x1 = -2*(tt-t1) + 1*(tf-tt)*ones(size(t1)); x2 = 1*(tf-t2); x_streak = [x1 x2];
      y1 = -2.5*(tf-tt)*ones(size(t1));   y2 = -2.5*(tf-t2);   y_streak = [y1 y2];
%
%   plot both the pathline and streakline at tf
      nfig = nfig+1;   figure(nfig)
      plot(x_path,y_path,'r-',x_streak,y_streak,'g--','LineWidth',3),grid
      title('Unsteady\_Flow\_Vis2:   Pathline and Streakline at t_f = 10 sec')
      xlabel('x position (m)'),ylabel('y position (m)')
      legend('Pathline for 0 < t < t_f ','Streakline at t_f')
%
%   add some more plots to help in the visualization of the PATHLINE
      nfig = nfig+1;   figure(nfig)
      plot(x_path,y_path,'r-','LineWidth',3),grid
      title('Unsteady\_Flow\_Vis2:   Pathline for 0 < t < t_f')
      xlabel('x position (m)'),ylabel('y position (m)')
%
      nfig = nfig+1;   figure(nfig)
      plot(t,x_path,'r-','LineWidth',3),grid
```

```
      title('Unsteady\_Flow\_Vis2:   x-position vs. time of single tagged particle')
      xlabel('time (s)'),ylabel('x position (m)')

      nfig = nfig+1;    figure(nfig)
      plot(t,y_path,'r-','LineWidth',3),grid
      title('Unsteady\_Flow\_Vis2:   y-position vs. time of single tagged particle')
      xlabel('time (s)'),ylabel('y position (m)')
%
%   add some more plots to help in the visualization of the STREAKLINE
      nfig = nfig+1;    figure(nfig)
      plot(x_streak,y_streak,'g--','LineWidth',3),grid
      title('Unsteady\_Flow\_Vis2:   Streakline at t_f')
      xlabel('x position (m)'),ylabel('y position (m)')
%
      nfig = nfig+1;    figure(nfig)
      plot(t,x_streak,'g--','LineWidth',3),grid
      title('Unsteady\_Flow\_Vis2:   x-position at t_f of tagged particle released at time t')
      xlabel('time (s)'),ylabel('x position (m)')
%
      nfig = nfig+1;    figure(nfig)
      plot(t,y_streak,'g--','LineWidth',3),grid
      title('Unsteady\_Flow\_Vis2:   y-position at t_f of tagged particle released at time t')
      xlabel('time (s)'),ylabel('y position (m)')
%
%   end of demo
```