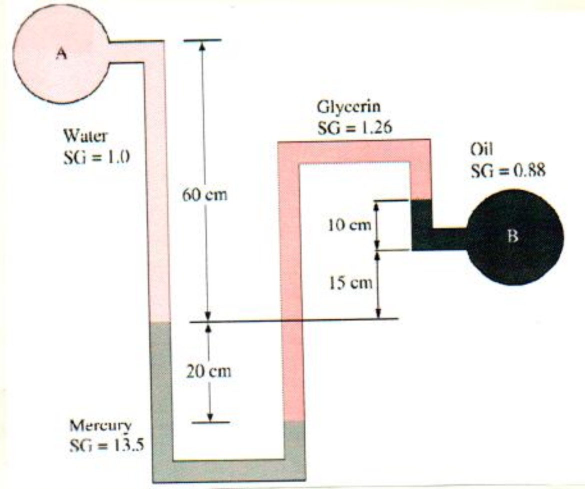
The pressure difference between an oil pipe and a water pipe is measured by the manometer shown in the sketch. For the given fluid heights and specific gravities, calculate the pressure difference $\Delta P = P_B - P_A$.

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Let's write the manometer eqn starting at point A

$$P_A + \gamma_w(0.6m) + 13.5\gamma_w(0.2m) - 1.26\gamma_w(0.45m) + 0.88\gamma_w(.1m) = P_B$$

units of m

$$\therefore P_B - P_A = (0.6 + 2.7 - 0.567 + 0.088)\gamma_w$$

$$= 2.821\gamma_w \qquad \text{but } \gamma_w = \rho_w g$$

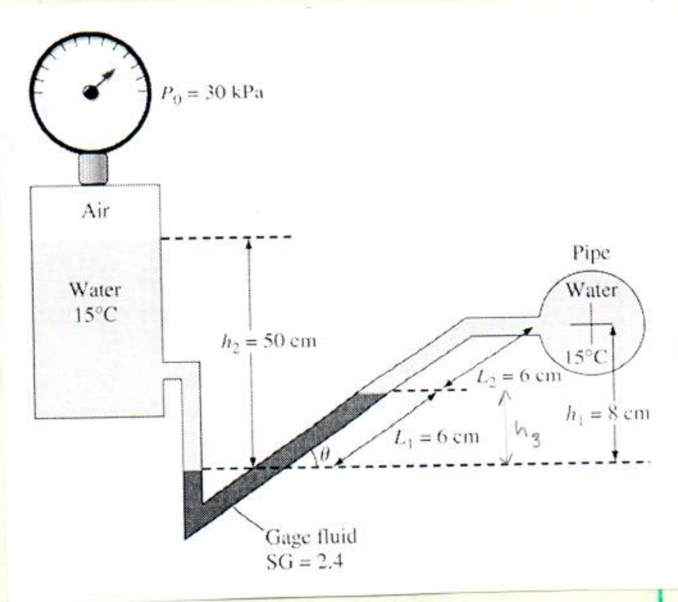
$$= (2.821\,m)\left(9780\,\frac{N}{m^3}\right) \qquad = \left(998\,\frac{kg}{m^3}\right)\left(9.8\,\frac{m}{s^2}\right)$$

$$= 27591\,N/m^2 \qquad = 9780\,N/m^3$$

at 20°C

or $\boxed{\Delta P \simeq 27.6\,kPa}$ ans

The pressure of water flowing through a pipe is measured by an arrangement that involves both a pressure gage and a manometer, as shown. For the values given in the diagram, determine the pressure in the water pipe.




Let's write the manometer eqn starting with the air pressure (gage).

$$P_{air} + \gamma_w h_2 - 2.4\,\gamma_w h_3 - \gamma_w (h_1 - h_3) = P_w$$

but $h_3 = L_1 \sin\theta$ where $\sin\theta = \dfrac{h_1}{L_1 + L_2}$

height of Gage Fluid between dashed lines

also $\rho_w = 1000\ kg/m^3$
$\gamma_w = \rho g$
$= 9.81\ \frac{kN}{m^3}$

Now putting in the values

$$h_3 = \frac{L_1 h_1}{L_1 + L_2} = \frac{(6)(8)}{12} = 4\ cm$$

and

$$P_w = P_{air} + (h_2 - 2.4h_3 - h_1 + h_3)(\gamma_w)$$

$$= 30\ kPa + \underbrace{(50 - 2.4(4) - 8 + 4)}_{36.4}\ cm \times \frac{1\,m}{100\,cm}\left(9.81\ \frac{kN}{m^3}\right)$$

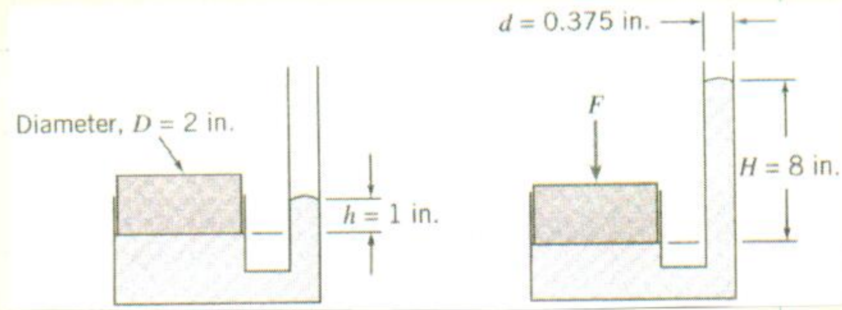
$$= (30 + 3.57)\ kPa$$

$$= \boxed{33.6\ kPa}$$

ans

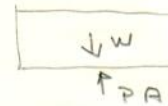
The tube shown is filled with mercury at 20°c.

Calculate the force F, applied to the cylindrical piston.




In left diagram

weight of piston = pressure on bottom * area




$W = PA$

but $P = \gamma_{Hg} h = \left(133 \frac{kN}{m^3}\right)\left(1\,in * \frac{2.54\,cm}{in} \times \frac{1\,m}{100\,cm}\right)$

use SI units here

$= 3.378 \frac{kN}{m^2}$

$\gamma_{Hg} = \rho g = \left(13{,}550 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right) = 133\ kN/m^3$

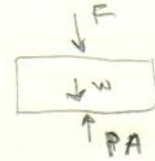
and $A = \frac{\pi}{4} D^2 = \pi\ in^2 = \pi\ in^2 \left(\frac{0.0254\,m}{in}\right)^2$

$= 0.002027\ m^2$

$\therefore\ W = \left(3.378 \times 10^3 \frac{N}{m^2}\right)\left(0.002027\,m^2\right) = \boxed{6.847\ N}$

In right diagram

force + weight = pressure * area




or $F = PA - W$

but $P = \gamma_{Hg} h = \left(133 \frac{kN}{m^3}\right)\left(8\,in * \frac{.0254\,m}{in}\right)$

$= 27.03 \frac{kN}{m^2}$

$\therefore\ F = \left(27.03 \times 10^3 \frac{N}{m^2}\right)\left(.002027\,m^2\right) - 6.847\,N$

$= 54.78 - 6.85$

$= \boxed{47.93\ N} \times \frac{1\ lbf}{4.448\ N} = \boxed{10.78\ lbf}$

SI BG

Note could also have used British units to start with

see next pg

left side

using British units

$$\boxed{\gamma_{Hg} = 847 \frac{lbf}{ft^3}}$$

$$\frac{\rho g}{g_c} = \frac{\left(26.3 \frac{slugs}{ft^3}\right)\left(\frac{32.2\,lbm}{slug}\right)\left(32.2\,ft/s^2\right)}{32.2\,\frac{lbm\text{-}ft/s^2}{lbf}} = 846.9\,lbf/ft^3$$

$$W = PA \quad \text{where } P = \gamma_{Hg}\,h = \left(847 \frac{lbf}{ft^3}\right)\left(\frac{1}{12}\,ft\right)$$

$$= 70.58 \frac{lbf}{ft^2}$$

$$A = \frac{\pi}{4} D^2 = \frac{\pi}{4}\left(\frac{2}{12}\right)^2 ft^2 = 0.02182\,ft^2$$

$$\therefore\ W = \left(70.58 \frac{lbf}{ft^2}\right)\left(0.02182\,ft^2\right)$$

$$= 1.540\,lbf$$

right side diagram

$$F = PA - W \qquad \text{Where } P = (847)\left(\frac{8}{12}\right) = 564.7 \frac{lbf}{ft^2}$$

$$F = (564.7)(.02182) - 1.540$$

$$= 12.32 - 1.54$$

$$= \boxed{10.78\,lbf} \quad \text{or} \quad * \frac{4.448\,N}{lbf} \Rightarrow \boxed{47.95\,N}$$

2K

BG

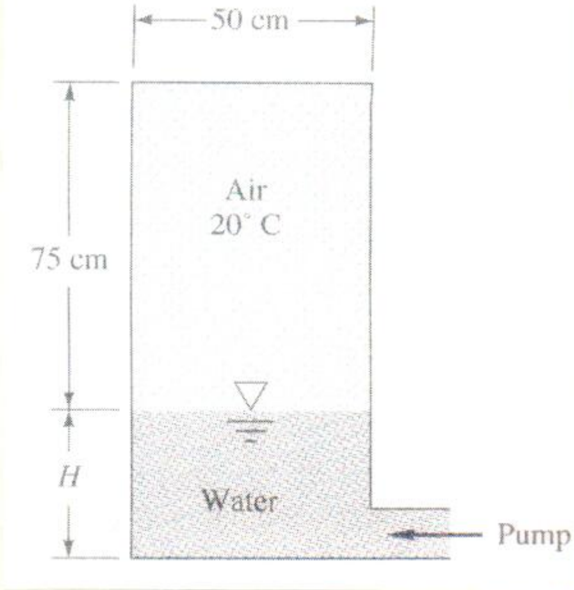
SI

2.47

A cylindrical tank is being filled slowly with water at 20°C. At the instant shown, the absolute air pressure is 110 KPa and H = 35 cm.

The pump is switched off when its exit pressure reaches 175 KPa absolute (This is the pressure at the bottom of the Tank)

For isothermal air compression, estimate the value of H once the pump has stopped.




For the state shown in the diagram, we have

state 1

pressure and volume of air

$$\left.\begin{aligned} P_1 &= 110 \text{ kPa} \\ V_1 &= \frac{\pi}{4} D^2 (L - H_1) = \frac{\pi}{4} (0.5\,m)^2 (0.75\,m) \\ &= 0.2945 \ m^3 \end{aligned}\right.$$

where $L = .75 + .35\,m$
$\boxed{L = 1.1\ m}$

Now for isothermal compression, we have

$$PV = mRT$$

and for constant m and T, this gives

$$P_1 V_1 = P_2 V_2 = \text{const.}$$

Now for state 2 (when pump has stopped), we are given the pressure at the bottom of the water (ie. the exit of the pump).

air pressure

$$\therefore \ P_2 + \gamma_w H_2 = P_{pump}$$

$$\text{but} \quad P_2 = \frac{P_1 V_1}{V_2} = \frac{P_1 \frac{\pi}{4} D^2 (L - H_1)}{\frac{\pi}{4} D^2 (L - H_2)}$$

$$= \frac{P_1 (L - H_1)}{L - H_2}$$

$$\therefore \ P_1 (L - H_1) + \gamma_w H_2 L - \gamma_w H_2^2 = P_{pump} (L - H_2)$$

$$\gamma_w H_2^2 - (P_{pump} + \gamma_w L) H_2 + P_{pump} L - P_1 (L - H_1) = 0$$

This is just a quadratic eqn for $H_2$!!!

Putting in values, we have

$$\gamma_w = 9.81 \frac{kN}{m^3}$$

$$P_{pump} + \gamma_w L = 175 \frac{kN}{m^2} + 9.81 \frac{kN}{m^3}(1.1m)$$

$$= 185.8 \frac{kN}{m^2}$$

$$P_{pump} L - P_1 (L - H_1) = \left(175 \frac{kN}{m^2}\right)(1.1m) - \left(110 \frac{kN}{m^2}\right)(0.75m)$$

$$= 110.0 \frac{kN}{m}$$

note that the units are all consistent ok

$\therefore$ we need to solve

$$9.81 H_2^2 - 185.8 H_2 + 110.0 = 0$$

$$\therefore H_2 = \frac{185.8 \pm \sqrt{(185.8)^2 - 4(9.81)(110.0)}}{2(9.81)}$$

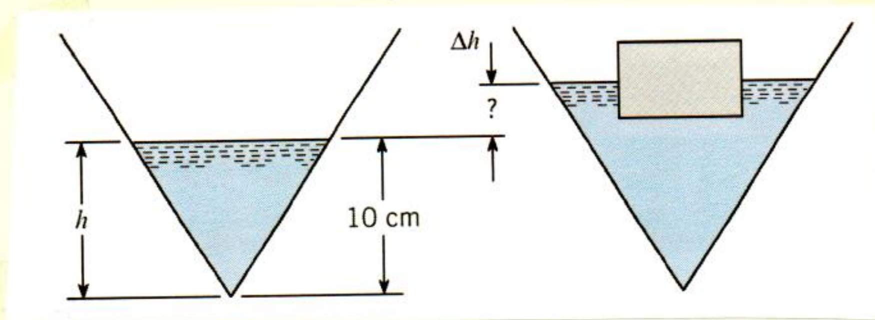
$$= 9.47 \pm 8.86 \text{ m}$$

$$\text{or} \quad H_2 = \begin{cases} 18.33 \text{ m} \\ 0.61 \text{ m} \end{cases}$$

← clearly this is not possible because the total height is only L = 1.1 m

$$\therefore \boxed{H_2 = 0.61 m} \quad \underline{\underline{ans}}$$

(3.114) A 90° inverted cone contains water as shown. The volume of water in the cone is given by $V = \frac{\pi}{3} h^3$. The original depth of the water is 10 cm. A block with a volume of 200 cm³ and a specific gravity of 0.6 is placed in the water. What will be the change (in cm) in water surface height in the cone?




Since the sg of the block is less than 1.0, it will float.

For the block, a FBD gives $F_B = W$

$F_B$

$W$

or $\boxed{\gamma_w V_{Bd}} = sg\, \underbrace{\gamma_w V_B}_{\gamma_B}$

weight of fluid displaced

$\therefore V_{Bd} = sg\, V_B$

$= (0.6)(200\,cm^3)$

$\boxed{V_{Bd} = 120\,cm^3}$ This is the displaced volume

Thus in the final configuration, the volume occupied by the water and the portion of the block below water is given

$$V_{final} = V_{orig} + V_{Bd}$$
$$= \frac{\pi}{3}(10\,cm)^3 + 120\,cm^3$$
$$= 1167\,cm^3$$

With this volume, we can compute the final height

$$V = \frac{\pi}{3} h^3$$
$$h_{final} = \left(\frac{3 V_{final}}{\pi}\right)^{\frac{1}{3}} = \left(\frac{3}{\pi}\, 1167\,cm^3\right)^{\frac{1}{3}} = 10.368\,cm$$

$$\therefore \Delta h = h_{final} - h_{orig} = 0.37\,cm$$

# Buoyancy Effects

The volume and the average density of an irregularly shaped body are to be determined using a spring scale. The body weights 7200 N in air and 4790 N in water. Determine the volume and density of the body. Explain the logic used in your calculations/analyses.

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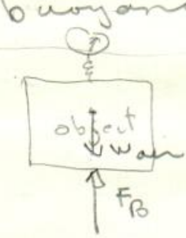
In water the net weight of the object is given by the difference of the object's weight in air and the buoyancy force

water




$$W_{water} = W_{air} - F_B$$

$$\therefore F_B = W_{air} - W_{water}$$

$$= 7200\,N - 4790\,N$$

$$\boxed{F_B = 2410\,N}$$

but the buoyancy force is equal to the weight of the fluid (water) displaced

$$\therefore F_B = \gamma_w V_{object} \leftarrow \text{since it is completely submerged}$$

$$\gamma_{water} = \rho g = \left(998\frac{kg}{m^3}\right)\left(9.81\frac{m}{s^2}\right) = 9790\,N/m^3$$

* at 20°C

$$\text{or } V_{object} = \frac{F_B}{\gamma_{water}}$$

$$= \frac{2410\,N}{9790\,N/m^3}$$

$$\boxed{V_{object} = 0.2462\,m^3}$$

$$\text{and } \rho_{object} = \frac{m_{object}}{V_{object}} = \frac{W_{air}/g}{V_{obj}}$$

$$\boxed{m_{object} = 733.9\,kg}$$

$$= \frac{7200\,kg\cdot m/s^2}{(9.81\,m/s^2)(0.2462\,m^3)} = \boxed{2981\,\frac{kg}{m^3}}$$

$$\therefore \boxed{V_{obj} \approx 0.246\,m^3} \text{ and } \boxed{\rho_{obj} = 2980\,\frac{kg}{m^3}}$$

ans