(1.48) The Reynolds number is a dimensionless quantity that plays an important role in characterizing fluid flow phenomena. Within the context of viscous flow through pipes, the Reynolds number, Re, is defined as

$$Re = \frac{\rho V D}{\mu} = \frac{V D}{\nu}$$

where ρ is the fluid density, V is the mean fluid velocity, D is the pipe diameter, and μ is the fluid viscosity.

a. Compute Re for water flowing through a 3 mm diameter tube, if the mean velocity is 3 m/s and the fluid temperature is 30°C.

---

for water at 30°C (from Table A.1 in Mott)

also Crowe Table A.5 | Hibbelen App A

$\rho = 996\ kg/m^3 \qquad \mu = 8.00 \times 10^{-4}\ N\text{-}s/m^2$

$$\therefore Re = \frac{(996\ kg/m^3)(3\ m/s)(3 \times 10^{-3}\ m)}{8.00 \times 10^{-4}\ N\text{-}s/m^2}$$

note $N\text{-}s = \frac{kg\text{-}m}{s}$ ok

Turbulent flow

$\boxed{Re_{H_2O} = 11205}$ (dimensionless) ans

b. Recompute Re for air as the conducting fluid in Part a. Assume standard atmospheric pressure.

---

for air at 30°C (from table E.1 in Mott)

$\rho = 1.164\ kg/m^3 \qquad \mu = 1.86 \times 10^{-5}\ N\text{-}s/m^2$

Crowe Table A.3

$$\therefore Re = \frac{(1.164)(3)(3 \times 10^{-3})}{1.86 \times 10^{-5}}$$

$\rho = 1.17\ kg/m^3$

$\Rightarrow Re = 566$

close enough

laminar flow

$\boxed{Re_{air} = 563}$ ans (dimensionless)

---

important quantity

could also use the kinematic viscosity $\boxed{\nu = \frac{\mu}{\rho}}$

$\nu_w = 0.804 \times 10^{-6}\ m^2/s \quad | \quad \nu_{air} = 16.0 \times 10^{-6}\ m^2/s$

factor of 20 — ok

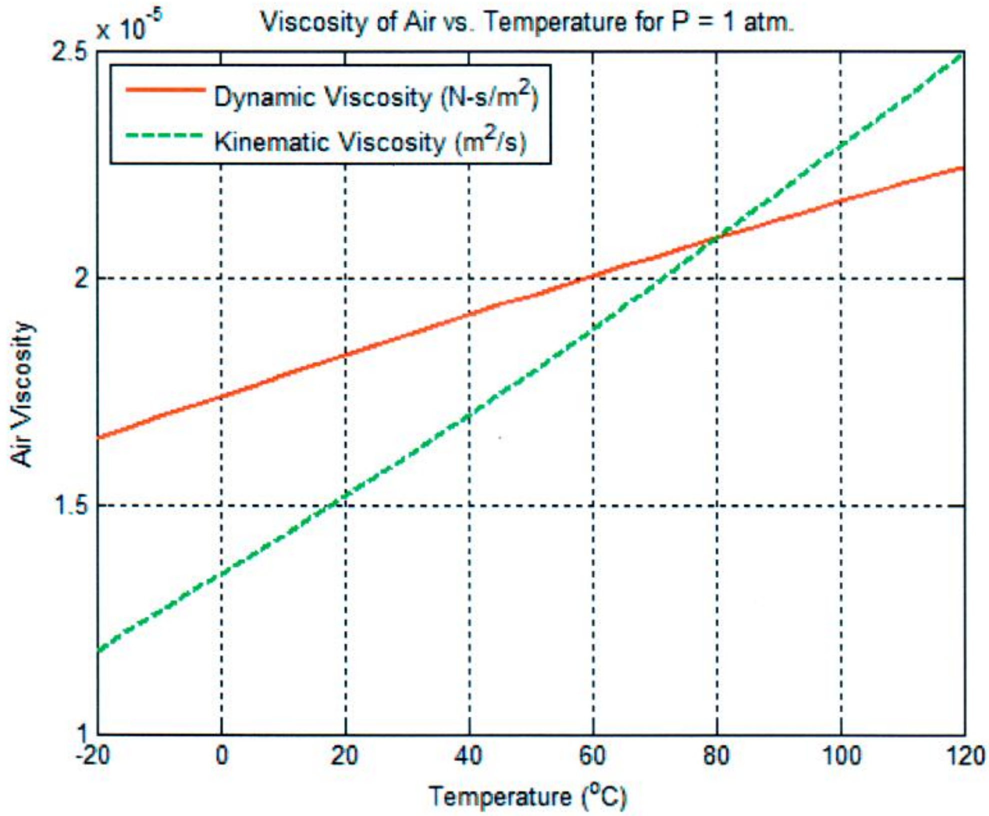
Temperature: $T\ (^oK) = T\ (^oC) + 273$

Sutherland Eqn.: $\mu(T) = \dfrac{a_1 T^{3/2}}{T + a_2}$ with $a_1 = 1.357 \times 10^{-6}$ N-s/m^2-K$^{1/2}$ and $a_2 = 78.84$ K

Ideal Gas Law: $P = \rho R T$ or $\rho = \dfrac{P}{RT}$

where $P = 101.3 \times 10^3$ N/m^2 and $R = 286.9$ N-m/kg-K for air

Kinematic Viscosity: $\nu = \dfrac{\mu}{\rho}$



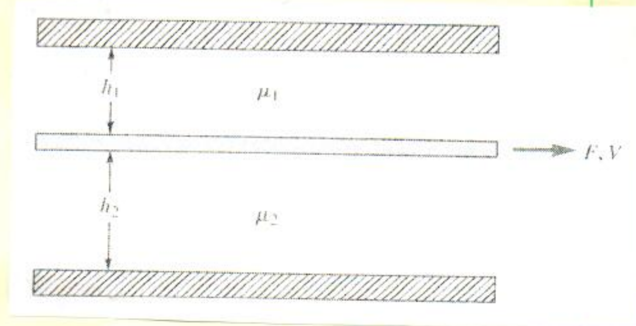
Validation Check:

Temperature (^{o}C)	Dynamic Viscosity (N-s/m$^{2)}$		Kinematic Viscosity (m^2/s)	
	Sutherland Eqn.	Hibbeler Text	Sutherland Eqn.	Hibbeler Text
0	17.4×10^{-6}	17.2×10^{-6}	13.5×10^{-6}	13.3×10^{-6}
100	21.6×10^{-6}	21.7×10^{-6}	22.9×10^{-6}	23.0×10^{-6}

```
%
%   VISCOSITY_AIR.M  Evaluate and plot the viscosity of air vs temperature
%
%   This file evaluates and plots the dynamic and kinematic viscosity of air versus
%   temperature for atmospheric pressure.
%
%   File prepared by J. R. White, UMass-Lowell (Jan. 2017)
%

     clear all;   close all;   nfig = 0;
%
%   define problem parameters
     a1 = 1.357e-6;          % constant in Sutherland's eqn. for air (N-s/m^2-K^(1/2))
     a2 = 78.84;             % constant in Sutherland's eqn. for air (K)
     R = 286.5;              % gas constant for air (N-m/kg-K)
     P = 101.3e3;            % atmospheric pressure (N/m^2)
%
%   set temperature range
     Tc = linspace(-20,120,29);  Tk = Tc + 273;
%
%   compute quantities of interest
     mu = a1*Tk.^(3/2)./(Tk + a2);  % dyn. viscosity via Sutherland's eqn (N-s/m^2)
     rho = P./(R*Tk);                % density via ideal gas law (kg/m^3)
     nu = mu./rho;                   % kinematic viscosity (m^2/s)
%
%   now plot mu(T) and nu(T)
     nfig = nfig+1;   figure(nfig)
     plot(Tc,mu,'r-',Tc,nu,'g--','LineWidth',2),grid
     title('Viscosity of Air vs. Temperature for P = 1 atm.')
     xlabel('Temperature (^oC)'),ylabel('Air Viscosity')
     legend('Dynamic Viscosity (N-s/m^2)','Kinematic Viscosity (m^2/s)', ...
            'Location','NorthWest')
%
%   end of problem
```

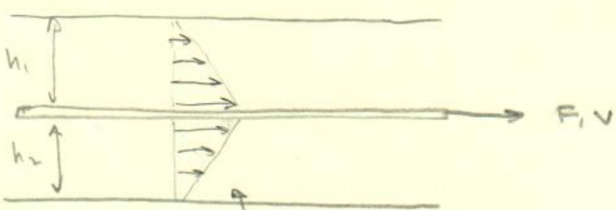
(1.48) A thin plate is separated from two fixed plates by very viscous liquids, μ_1 and μ_2, respectively (see sketch). The plate spacings, h_1 and h_2 may be unequal. The contact area between the center plate and each fluid is A.



(a) Assuming a linear velocity distribution in each fluid, derive an expression for the force, F, required to pull the plate at velocity, V.

— — —

The basic geometry is as follows



→ linear velocity profiles with velocity V at central plate and zero velocity at fixed plate

→ note that the slopes may be different since the gap widths, h_1 and h_2, can differ.

At equilibrium (for const V), the force F is balanced by the friction force along the two sides of the central plate.

$$F = \tau_1 A + \tau_2 A$$

$$= \mu_1 \frac{dV_1}{dy} A + \mu_2 \frac{dV_2}{dy} A$$

but because the velocity profile is linear

$$\frac{dv_i}{dy} = \frac{\Delta V}{\Delta y} = \frac{V}{h_i}$$

$$= \mu_1 \frac{V}{h_1} A + \mu_2 \frac{V}{h_2} A$$

$$\therefore \quad \boxed{F = VA\left(\frac{\mu_1}{h_1} + \frac{\mu_2}{h_2}\right)}$$

(b) For a specific situation

$h_1 = 6\ mm$ and $h_2 = 4\ mm$

and $\mu_1 = 0.04\ \frac{N\text{-}s}{m^2}$ $\mu_2 = 0.08\ N\text{-}s/m^2$

and plate area $= A = 1.2\ m^2$

determine the force needed to move the plate with $v = 1\ cm/s$

$$F = vA\left(\frac{\mu_1}{h_1} + \frac{\mu_2}{h_2}\right)$$

or

$$F = \left(0.01\frac{\cancel{m}}{\cancel{s}}\right)\left(1.2\cancel{m^2}\right)\left(\frac{0.04\ \frac{N\text{-}\cancel{s}}{\cancel{m^2}}}{0.006\ \cancel{m}} + \frac{0.08}{0.004}\right)$$

$$= 0.012\ (6.667 + 20)\ N$$

$$= \underbrace{\boxed{0.320\ N}}_{ans} \times \frac{1\ lbf}{4.448\ N} = 0.0719\ lbf$$

pretty small force

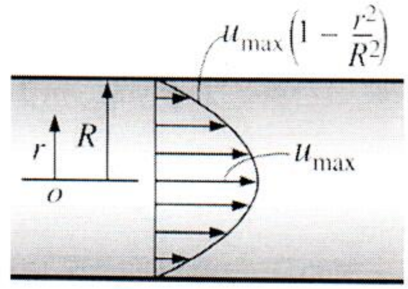
Note

This is a small force. The force needed is proportional to the speed, v, and to the surface area, A. Thus, if these increase considerably, then much larger forces are needed to overcome the friction force.

Cengal
Prob 2.53

Friction Drag for Laminar Pipe Flow

In regions far from the entrance, fluid flow through a circular pipe is one-dimensional, and the velocity profile for laminar flow is given by $u(r) = u_{max}(1 - r^2/R^2)$, where R is the radius of the pipe, r is the radial distance from the center, and u_{max} is the maximum flow velocity, which occurs at the center of the pipe (see sketch).



a. Obtain a relation for the drag force applied by the fluid on a section of pipe of length L.

b. Determine the value of the drag force for water flow at 20 C with R = 0.08 m, L = 15 m, u_{max} = 3 m/s and μ = 0.0010 N-s/m^2.

(a) Since water is a Newtonian fluid

$$\tau_w = -\mu \left.\frac{du}{dr}\right|_{r=R}$$ ← shear stress at wall

(negative sign indicates that the shear stress is opposite to direction of u)

and

$$F_D = \tau_w (\text{wall area}) = \tau_w (\underbrace{2\pi R L}_{\text{surface area of cylinder}})$$

$$\therefore \boxed{F_D = -2\pi R L \mu \left.\frac{du}{dr}\right|_{r=R}}$$ ← desired drag force

Now given that, for laminar flow, we have

$$u(r) = u_{max}\left(1 - \frac{r^2}{R^2}\right)$$

$$\left.\frac{du}{dr}\right|_{r=R} = u_{max}\left.\left(-\frac{2r}{R^2}\right)\right|_{r=R} = -\frac{2u_{max}}{R}$$

$$\therefore \boxed{F_D = 4\pi L \mu u_{max}}$$

(b) For the parameters given,

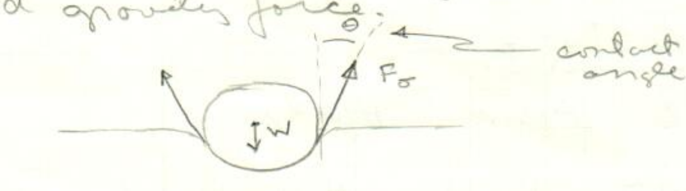
$$F_D = 4\pi (15\,m)\left(0.0010\,\frac{N\text{-}s}{m^2}\right)(3\,m/s)$$

$$\boxed{F_D = 0.565\ N} \times \frac{1\,lbf}{4.448\,N} = \boxed{0.127\ lbf}$$

↑ ans

relatively small but important...

1.69 (a) A solid cylindrical needle of diameter D, length L, and density ρ may float on a liquid of surface tension σ. Neglecting buoyancy effects, derive a formula for the maximum diameter needle able to float on the liquid. Hint: simply compare the upward surface tension force with the downward gravity force.



$$W = \text{weight of needle} = \rho \underbrace{\frac{\pi}{4} D^2 L}_{\text{volume}} g = \underset{\text{mass}}{m} g$$

$$F_\sigma = 2 \sigma L \quad \leftarrow \underline{\text{assume } L >> D}$$

two sides — force per unit length — length of needle

$\therefore$ comparing the vertical components

$$2 \sigma L \cos\theta \geq \rho \frac{\pi}{4} D^2 L g$$

surface tension must be greater than weight for needle to "float"

or $\boxed{D \leq \sqrt{\frac{8 \sigma \cos\theta}{\pi \rho g}}}$ ans

(b) Determine D_{max} for a steel needle (sg = 7.84) and water at 20°C. Does your result appear reasonable? Assume a contact angle of 0 degrees.

from Mott Table 1.5 and Wikipedia

$$\sigma_{\substack{water\\20°C}} = 72.8 \frac{mN}{m} = 0.073 \frac{N}{m} = 0.073 \frac{kg}{s^2}$$

$$\therefore D_{max} = \sqrt{\frac{8 (0.073 \text{ kg}/s^2)}{\pi (7.84)(1000 \text{ kg}/m^3)(9.81 \text{ m}/s^2)}}$$

$$= 1.555 \times 10^{-3} \text{ m}$$

$$= \boxed{1.56 \text{ mm}} \text{ ans}$$

{ quite small (as expected)