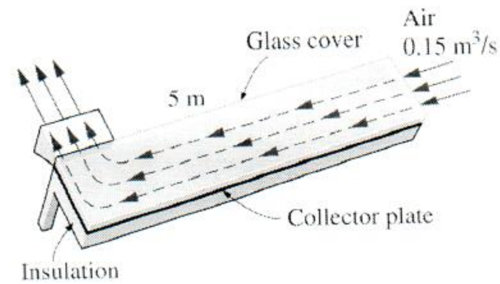
Consider a solar collector that is 1m wide and 5m long and has a constant spacing of 3cm between the glass cover and the collector plate. Air flows within the rectangular channel at an average temperature of 45°c at a rate of 0.15 m^3/s as shown in the sketch.



Disregarding the entrance and roughness effects, estimate the pressure drop in the collector.

Writing the energy eqn between the inlet and exit gives

$$\frac{P_1}{\gamma} + \frac{\alpha_1 V_1^2}{2g} + z_1 - h_L = \frac{P_2}{\gamma} + \frac{\alpha_2 V_2^2}{2g} + z_2$$ } no mechanical devices

but $z_1 \simeq z_2$ same elevation

$\alpha_1 \simeq \alpha_2$ and $V_1 = V_2$ from continuity eqn with no area change

$$\therefore \quad \frac{P_1 - P_2}{\gamma} = h_L \qquad \text{or} \quad \boxed{\Delta P = P_1 - P_2 = \gamma h_L}$$

but $\boxed{h_L = f \frac{L}{D_h} \frac{V^2}{2g}}$

$$V = \frac{Q}{A} = \frac{0.15\ m^3/s}{.03\ m^2} \qquad A = (1m)(.03m) = .03\ m^2$$

W width, H height

$$\boxed{V = 5\ m/s}$$

$$\frac{V^2}{2g} = \frac{25\ m^2/s^2}{2(9.8\ m/s^2)} = 1.28\ m$$

$L = 5\ m$ given

hydraulic diameter

$$D_h = \frac{4A_f}{P_w} = \frac{4WH}{2W + 2H} = \frac{4(.03\ m^2)}{2(1.03\ m)} = 0.0583\ m$$

$$\frac{L}{D_h} = \frac{5m}{.0583m} = 85.8$$

interpolated in Hibbeler

$$Re = \frac{\rho V D_h}{\mu} = \frac{V D_h}{\nu}$$

Cengel Table A-9 — close enough
air 1 atm T = 45°C
$\rho = 1.109\ kg/m^3$
$\mu = 1.941 \times 10^{-5}\ kg/m\text{-}s$
$\nu = 1.750 \times 10^{-5}\ m^2/s$

$$Re = \frac{(5 m/s)(0.0583 m)}{1.750 \times 10^{-5} m^2/s}$$

or $\boxed{Re = 16657}$ Turbulent Flow

Haaland eqn

$$\frac{1}{\sqrt{f}} = -1.8 \log_{10}\left[\frac{6.9}{Re} + \left(\frac{\epsilon/D}{3.7}\right)^{1.11}\right]$$

$$= -1.8 \log_{10}\left[\frac{6.9}{16660} + \underset{\substack{\uparrow \text{smooth} \\ \epsilon/D \to 0}}{0}\right]$$

$$\frac{1}{\sqrt{f}} = 6.089$$

$$\therefore f = \left(\frac{1}{6.089}\right)^2 = \boxed{0.0270}$$

Swamee Jain

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{\epsilon/D}{3.7} + \frac{5.74}{Re^{.9}}\right)\right]^2}$$

$$= \frac{0.25}{\left[\log_{10}\left(0 + \frac{5.74}{(16660)^{.9}}\right)\right]^2}$$

$$f = \frac{0.25}{9.245} = \boxed{0.0270}$$

Now, we can compute h_L

$$h_L = f \frac{L}{D_h}\frac{V^2}{2g} = (.027)(85.8)(1.28 m)$$

$$\boxed{h_L = 2.965 m}$$

and $\Delta P = P_1 - P_2 = \gamma h_L = \rho g h_L$

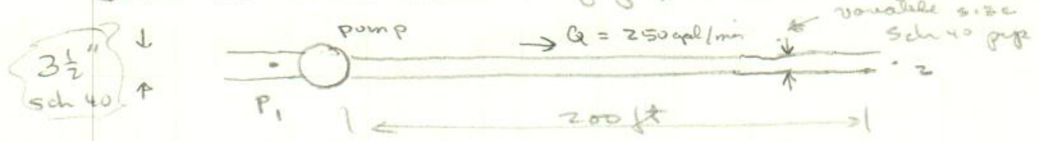
$$= \left(1.109 \frac{kg}{m^3}\right)\left(9.8 \frac{m}{s^2}\right)(2.965 m)$$

$$= 32.2 \frac{N}{m^2} = \boxed{32.2\ Pa}$$

ans

A positive displacement pump delivers essentially constant discharge flow rate independent of the discharge pressure. In a particular flow system, the suction pressure of the pump is 10 psig, the desired water flow rate is 250 gal/min, and the horizontal discharge line is 200 ft long and is made from Sch 40 pipe. It is open to the atmosphere at the end, the suction line has a relatively short $3\frac{1}{2}$" sch 40 pipe. (assume 60°F)

(a) Compute the power added by the pump if the discharge line is a 2" Sch 40 pipe.



The energy eqn becomes

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + \cancel{z_1} + h_A - \underbrace{h_R}_{0} - h_L = \underbrace{\frac{P_2}{\gamma}}_{\text{free jet}} + \frac{V_2^2}{2g} + \cancel{z_2}$$

$$z_1 = z_2$$

solving for h_A gives

$$h_A = \frac{V_2^2}{2g} + h_L - \left(\frac{P_1}{\gamma} + \frac{V_1^2}{2g}\right)$$

Since the suction line parameters are known, we have

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \frac{(10\ \text{lbf/in}^2)\left(\frac{144\ \text{in}^2}{\text{ft}^2}\right)}{62.4\ \text{lbf/ft}^3} + \frac{(8.1\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)}$$

$$V_1 = \frac{Q}{A_1} = \frac{(250\ \text{gal/min})\left(\frac{1\ \text{ft}^3/\text{s}}{449\ \text{gal/min}}\right)}{(0.06868\ \text{ft}^2)} = 8.1\ \text{ft/s}$$

$3\frac{1}{2}$" sch 40
$A = .06868\ \text{ft}^2$
$ID = .2957\ \text{ft}$

Mott App F

$$= 23.1\ \text{ft} + 1.0\ \text{ft} = \boxed{24.1\ \text{ft}}$$

head at pump inlet

Now $V_2 = \dfrac{Q}{A_2} = \dfrac{0.557\ \text{ft}^3/\text{s}}{(0.02333\ \text{ft}^2)} = \boxed{23.9\ \text{ft/s}}$

2" sch 40
$A = .02333$
$ID = .1723\ \text{ft}$

Mott App F

fluid properties

$\rightarrow \gamma = 62.4\ \text{lbf/ft}^3$

$\rho = 62.4\ \text{lbm/ft}^3$

$\mu = 2.35 \times 10^{-5}\ \text{lbf-s/ft}^2$

$\rightarrow \nu = 1.21 \times 10^{-5}\ \text{ft}^2/\text{s}$

$$Re = \frac{VD}{\nu} = \frac{(23.9)(.1723)}{1.21 \times 10^{-5}} = \boxed{3.4 \times 10^5}$$

$\varepsilon_{steel} = 1.5 \times 10^{-4}$ ft (commercial steel)

$\therefore \ \varepsilon/D = \frac{1.5 \times 10^{-4} \text{ ft}}{0.1723 \text{ ft}} = 0.0087$

$\therefore f \simeq 0.020.$ from Moody chart

and

$$h_L = f \frac{L}{D} \frac{V_2^2}{2g}$$

$$= 0.020 \left(\frac{200}{.1723}\right)\left(\frac{(23.9)^2}{2(32.2)}\right)$$

$$= (0.020)(8573)(8.87 \text{ ft})$$

$$= 205.9 \text{ ft}$$

$$\therefore \ h_A = \left(\frac{V_2^2}{2g} + h_L\right) - \left(\frac{P_1}{\gamma} + \frac{V_1^2}{2g}\right)$$

$$= (8.87 + 205.9) - 24.1 \text{ ft}$$

$$= 190.7 \text{ ft}$$

and

$$P_A = h_A \gamma Q = (190.7)(62.4)(0.557)$$

$$= 6627 \ \frac{\text{ft-lbf}}{\text{s}} \times \frac{1 \text{ hp}}{550 \ \frac{\text{ft-lbf}}{\text{s}}}$$

$$\boxed{P_A = 12.1 \text{ hp}}$$ ans

for 2" discharge line

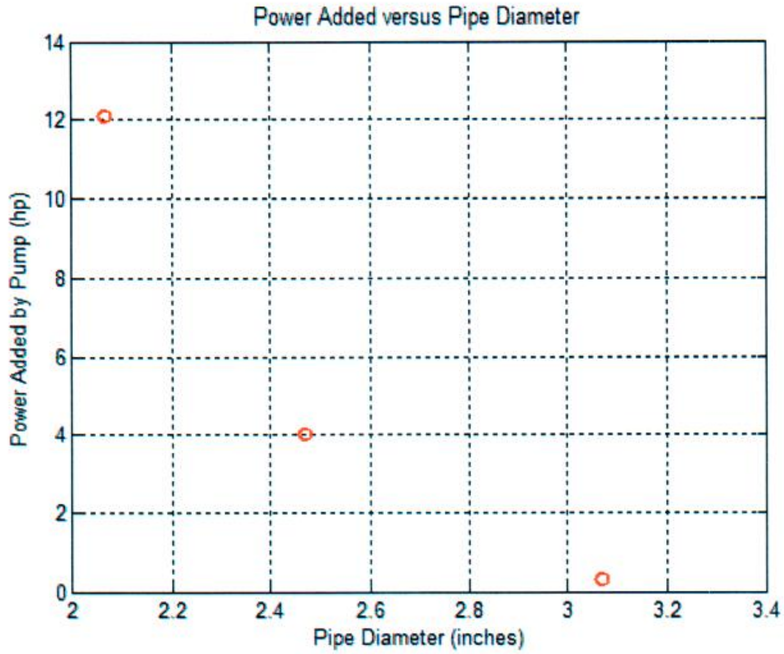
(b) Now redo the calculation from Part A using your favorite computer analysis tool (Excel, Matlab, mathcad, ...) Validate the computer calculation using the 2" Sch 40 discharge line as a benchmark.

Once you get this working redo the calculation using both 2½" and 3" Schedule 40 pipes

With these data, discuss how the energy delivered by the pump changes versus discharge line size.

see hL-vs-size1.xls

and hL-vs-size1.m




```
>> hl_vs_size1
   Results for "hL vs size1" Example

   Calculated Parameters:
     Sch 40  diameter    ave vel  Reynolds # fric factor  vel head  hL major   hL minor   head added Power Added
      (in)     (ft)      (ft/s)                             (ft)      (ft)       (ft)       (ft)        (hp)
     2.000  1.723e-01  2.389e+01  3.402e+05  2.007e-02  8.862e+00  2.065e+02  0.000e+00  1.912e+02  1.208e+01
     2.500  2.058e-01  1.674e+01  2.848e+05  1.959e-02  4.354e+00  8.289e+01  0.000e+00  6.315e+01  3.991e+00
     3.000  2.557e-01  1.085e+01  2.292e+05  1.917e-02  1.827e+00  2.739e+01  0.000e+00  5.117e+00  3.234e-01
```

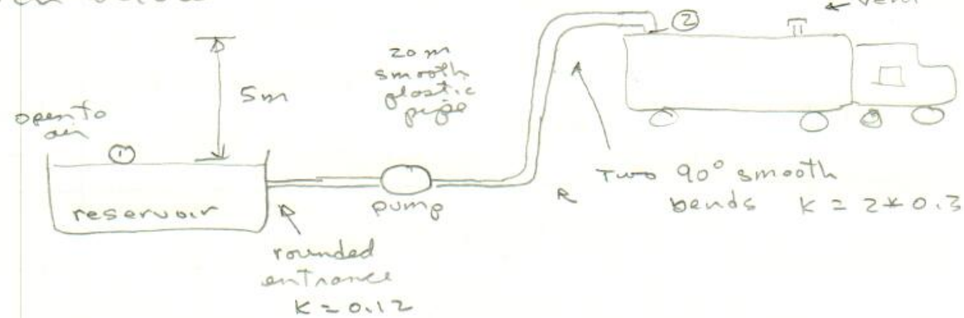
```
%
%   hL_vs_size1  Solve for Pump Size and Head Loss vs. Pipe Size for a horizontal pipe
%
%   This file computes the PA and hL vs. Sch 40 pipe size for a given system.  This
%   calculation was done by hand for one pipe size and here, we simply want to
%   automate this computation and look at a range of pipe sizes...
%
%   All the geometry information and fluid properties must be identified.
%   Depending on flow regime, either the laminar or turbulent flow relationship
%   is used to find the friction factor.  Minor losses need to be included
%   explicitly (none for this case).  The total head loss is found from Darcy's Eqn.
%   Finally, the energy equation is then used to compute the power added by the pump
%   for a given set of conditions (flow rate, pipe length, etc.) for different pipe
%   diameters.
%
%   File prepared by J. R. White, UMass-Lowell (April 2017)
%

      clear all;   close all;   nfig = 0;
%
%   set the geometry parameters
      g = 32.2;                     % gravitational constant (ft/s^2)
      gc = 32.2;                    % conv factor (1 lbf = 32.2 lbm-ft/s^2)
      D1 = 0.2957;                  % diameter of 3.5" Sch 40 inlet suction line (ft)
      A1 = pi*D1*D1/4;              % flow area of inlet pipe (ft^2)
```

```
      Sch = [2  2.5  3];            % Sch 40 nomimal pipe diameters (inches)(edit only)
      D2 = [0.1723 0.2058 0.2557]; % diameter for 2, 2.5, & 3" Sch 40 exit line (ft)
      ND2 = length(D2);             % number of pipe diameters to study
      A2 =  pi*D2.*D2/4;            % flow area of exit line(ft^2)
      L2 = 200;                     % exit channel length (ft)
      e = 0.00015;                  % surface roughness for commerical steel pipe (ft)
      eoD = e./D2;                  % relative roughness of exit pipe
%
%   set fluid (water) properties and volume flow rate
      rho = 62.4;                   % fluid density (lbm/ft^3)
      gam = rho*g/gc;               % specific weight (lbf/ft^3)
      nu = 1.21e-5;                 % fluid kinematic viscosity (ft^2/s)
      mu = nu*rho/gc;               % fluid dynamics viscosity (lbf-s/ft^2)
      Qgpm = 250;                   % volumetric flow rate (gpm)
      Q = Qgpm/448.83;              % volumetric flow rate (ft^3/s)
      P1 = 10*144;                  % pressure on suction side (lbf/ft^2)
%
%   treat minor losses
      K = 0;                        % total resistance coeff
%   minor losses: none for this case
%
%   compute variety of parameters
      v1 = Q/A1;                        % ave fluid velocity in suction line (ft/s)
      v2 = Q./A2;                       % ave fluid velocity in exit line (ft/s)
      Re = rho*v2.*D2/mu/gc;            % Reynolds number
      f = zeros(size(Re));              % initialize and calc the friction factor
      for i = 1:ND2
        if Re(i) <= 2300,  f(i) = 64/Re(i);  end
        if Re(i) > 2300 && Re(i) < 4000
          disp('  *** Warning: Flow in Transition Region ***');  f(i) = 0.05;  end
        if Re(i) >= 4000, f(i) = 0.25/(log10(eoD(i)/3.7 + 5.74/Re(i)^0.9))^2;  end
      end
      vhead = v2.*v2/(2*g);             % dynamic or velocity head (ft)
      hLmajor = f.*(L2./D2).*vhead;     % major head loss (ft)
      hLminor = K*vhead;                % minor head loss (ft)
%
%   now evaluate the energy equation (calc head added by pump)
      hA = vhead + hLmajor + hLminor - (P1/gam + v1*v1/2/g); % head added (ft)
%
%   compute the power added by the pump
      PA = rho*g*Q.*hA/gc/550;          % power added (hp)
%
%   let's plot power added versus pipe size
      nfig = nfig+1;   figure(nfig)
      plot(D2*12,PA,'ro','LineWidth',2),grid
      title('Power Added versus Pipe Diameter')
      xlabel('Pipe Diameter (inches)')
      ylabel('Power Added by Pump (hp)')
%
%   print results
      fid = 1;
      fprintf(fid,'   Results for "hL vs size1" Example \n');
      fprintf(fid,'    \n');
      fprintf(fid,'   Calculated Parameters: \n');
      fprintf(fid,'     Sch 40  diameter    ave vel  Reynolds # fric factor  vel head  hL major
hL minor   head added Power Added \n');
      fprintf(fid,'       (in)     (ft)      (ft/s)                                (ft)       (ft)
(ft)        (ft)       (hp)\n');
      for i = 1:ND2
        fprintf(fid,'     %6.3f %10.3e %10.3e %10.3e %10.3e %10.3e %10.3e %10.3e %10.3e %10.3e\n',
...
                 Sch(i), D2(i),v2(i),Re(i),f(i),vhead(i),hLmajor(i),hLminor(i),hA(i),PA(i));
      end
      fprintf(fid,'    \n');
%
%   end of file
```

A vented oil truck is to be filled with fuel oil ($\rho = 940$ kg/m^3 and $\mu = 0.035$ kg/m-s) from a vented underground reservoir using a 20 m long, 5 cm diameter, smooth plastic hose. The connection to the reservoir has a slightly rounded entrance (K = 0.12) and the hose to the tanker has two smooth 90° bends (K = 0.3 for each bend). The elevation difference between the oil level in the reservoir and the top of the tanker where the hose is connected is 5 m. A pump in the system between the reservoir and the tanker provides a constant flow rate of 0.015 m^3/s.

Taking the kinetic energy correction factor at the hose discharge (note that this is a free jet) to be 1.05, and assuming an overall pump efficiency of 82 percent, estimate the required power input to the pump to operate this system. change $\alpha_2 = 1.0$

A sketch of the system based on the above description is given below



We will write the energy eqn from pt 1 at the reservior surface (free surface) to the hose exit in the vented oil truck (free jet). this gives

$$\frac{P_1}{\gamma} + \frac{\alpha_1 V_1^2}{2g} + z_1 + h_A - h_L = \frac{P_2}{\gamma} + \frac{\alpha_2 V_2^2}{2g} + z_2$$

$\frac{P_1}{\gamma} \to 0$ free surface, $\frac{\alpha_1 V_1^2}{2g} \to 0$ free surface, $\frac{P_2}{\gamma} \to 0$ free jet

Solving this for h_A (energy per unit weight added to fluid by the pump) gives

$$h_A = \frac{\alpha_2 V_2^2}{2g} + (z_2 - z_1) + h_L$$

$$= \frac{\alpha_2 V_2^2}{2g} + (z_2 - z_1) + \left(\frac{fL}{D} + \sum_i k_i\right)\frac{V_2^2}{2g}$$

For the data given, we have

$$A = \frac{\pi D_p^2}{4} = \frac{\pi}{4}(0.05\,m)^2 = 1.963 \times 10^{-3}\ m^2$$

$$V = \frac{Q}{A} = \frac{0.015\ m^3/s}{1.963 \times 10^{-3}\ m^2} = 7.639\ m/s$$

$$\frac{V^2}{2g} = \frac{(7.639\ m/s)^2}{2\,(9.8\ m/s^2)} = 2.977\ m$$

$$Re = \frac{\rho V D}{\mu} = \frac{(940\ kg/m^3)(7.639\ m/s)(0.05\ m)}{0.035\ kg/m\text{-}s}$$

$$= 1.026 \times 10^4$$ Turbulent flow

∴ use the Swamee Jain correlation for a smooth pipe ($e/D \to 0$)

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{e/D}{3.7} + \frac{5.74}{Re^{0.9}}\right)\right]^2} = \frac{0.25}{\left\{\log_{10}\left[\frac{5.74}{(1.026\times10^4)^{0.9}}\right]\right\}^2}$$

$$= 0.0308 \simeq 0.031$$

This agrees with the Moody Chart which has $f \simeq 0.03$

let's use $f = 0.03$

Now putting in all these values into the energy eqn. gives

$$h_A = \alpha_2 \frac{V^2}{2g} + (z_2 - z_1) + \left(f\frac{L}{D} + \sum_i k_i\right)\frac{V^2}{2g}$$

$0.12 + 2 \times 0.6$

$$= 1.00(2.977) + 5 + \left[0.03\left(\frac{20}{0.05}\right) + 0.72\right](2.977)$$

$$= 2.977 + 5 + 37.87$$

$$\boxed{h_A = 45.85\ m}$$

Now, the input power to the pump is given by

$$P_I = \dot{W}_{pump} = \frac{P_A}{\eta} = \frac{\rho g Q h_A}{\eta}$$

$$= \frac{(940\ kg/m^3)(9.8\ m/s^2)(0.015\ \frac{m^3}{s})(45.85\ m)}{0.82}$$

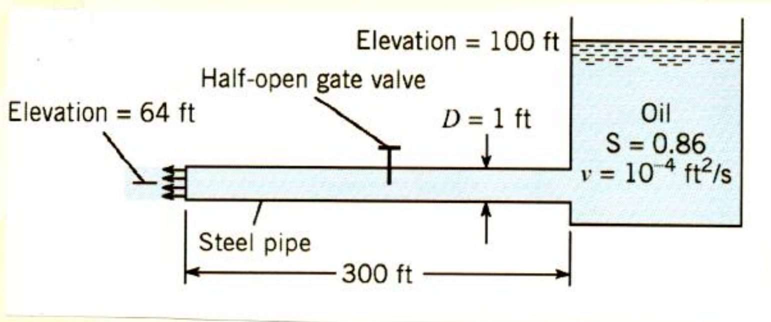
$$= 7727\ kg\,\frac{m}{s^2}\,\frac{m}{s} = 7727\ \frac{N\text{-}m}{s}$$

$\frac{J}{s} = W$

$$\boxed{P_I = \dot{W}_{pump} = 7.73\ kW}$$

ans

[10.94] Estimate the discharge of oil in the configuration shown in the sketch. Assume a sudden contraction at the tank-pipe interface.



Energy Eqn

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_A - h_R - h_L = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

$\frac{P_1}{\gamma} \to 0$, $\frac{V_1^2}{2g} \to 0$ free surface; $h_A \to 0$; $h_R \to 0$; $\frac{P_2}{\gamma} \to 0$ free jet

$$z_1 - z_2 = \frac{V_2^2}{2g} + h_L = \left(1 + \frac{fL}{D} + \sum_i k_i\right)\frac{V_2^2}{2g}$$

writing this in terms of Q

$$z_1 - z_2 = \left(1 + \frac{fL}{D} + \sum_i k_i\right)\frac{Q^2}{2gA^2}$$

vary Q until LHS = RHS
see Crowe10_94.xLS
compute ΔZ vs Q

$$V = \frac{Q}{A}$$

$$Re = \frac{VD}{\nu}$$

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{\epsilon/D}{3.7} + \frac{5.74}{Re^{0.9}}\right)\right]^2}$$

data:
$D = 1$ ft
$L = 300$ ft
$z_1 = 100$ ft
$z_2 = 64$ ft
$\epsilon = .00017$
$\frac{\epsilon}{D} = .00017$
$\frac{L}{D} = 300$
$\gamma = 0.86\,(62.4\ \text{lbf/ft}^3) = 53.7\ \text{lbf/ft}^3$
$\nu = 10^{-4}\ \text{ft}^2/\text{s}$
$\sum_i k_i = 6.1$

$$\sum_i k_i = \text{sudden contraction} + \text{half open gate valve} = 0.5 + 5.6 = 6.1$$

Table 10.3 Crowe

$\epsilon = 0.002$ in $= 0.00017$ ft — steel pipe, Table 10.2 Crowe

$$\frac{\epsilon}{D} = \frac{.00017}{1} = 0.00017$$

Hibbeler $\epsilon = 0.00015$ ft — close enough

Note that we should also be able to do this calculation by hand, as follows:

$$z_1 - z_2 = \left(1 + \frac{fL}{D} + \sum_i K_i\right)\frac{Q^2}{2gA^2}$$

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}\,ft^2$$

$$36\,ft = (1 + 300f + 6.1)\,\frac{Q^2}{2(32.2\,ft/s^2)\left(\frac{\pi}{4}\,ft^2\right)^2}$$

$$36 = \frac{(7.1 + 300f)\,Q^2}{39.73}$$

$$\therefore\ Q^2 = \frac{36(39.73)}{7.1 + 300f} = \frac{1430}{7.1 + 300f}$$

$$\boxed{Q = \left(\frac{1430}{7.1 + 300f}\right)^{\frac{1}{2}}}$$

f	Q	$V = \frac{Q}{A}$	$Re = \frac{VD}{\nu}$	$f = \frac{0.25}{\left[\log_{10}\left(\frac{\epsilon/D}{3.7} + \frac{5.74}{Re^{0.9}}\right)\right]^2}$
0.020	10.45	13.30	1.33×10^5	0.0180
0.018	10.70	13.62	1.36×10^5	0.0179

close enough

where $V = \frac{Q}{\frac{\pi}{4}(1)} = 1.273\,Q$

$$Re = \frac{VD}{\nu} = \frac{V(1)}{10^{-4}} = 10000\,V$$

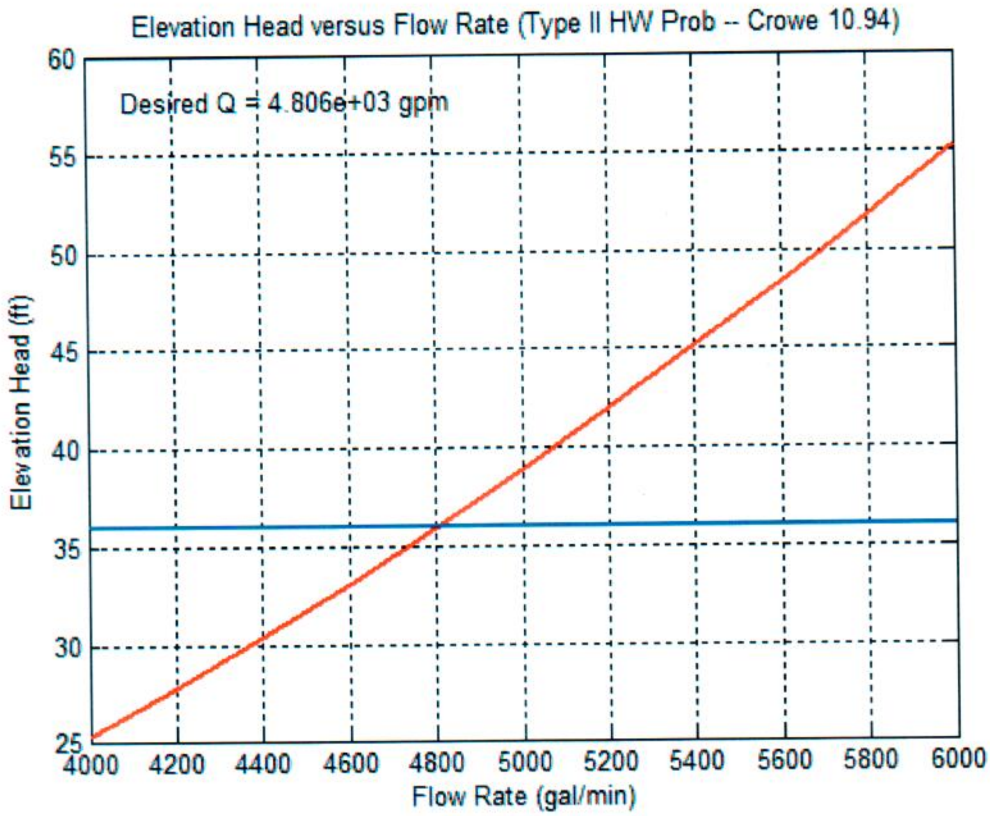
$$f = \frac{0.25}{\left[\log_{10}\left(\frac{\epsilon/D}{3.7} + \frac{5.74}{Re^{0.9}}\right)\right]^2} = \frac{0.25}{\left[\log_{10}\left(4.6\times10^{-5} + \frac{5.74}{Re^{0.9}}\right)\right]^2}$$

$\therefore$ $\boxed{Q = 10.7\ ft^3/s}$ and $\boxed{V = 13.6\ ft/s}$

ans

OK – same result as with Crowe10_94.xls / .m

$$Q = (10.7\ ft^3/s)\left(\frac{448.8\,gpm}{ft^3/s}\right) = \boxed{4802\ gpm}$$




```
>> crowe10_94
   Results for Type II Class Example
     Geometry:
       pipe dia (ft):                1.000e+00
       flow area (ft^2):             7.854e-01
       channel length (ft):          3.000e+02
       surface roughness (ft):       1.700e-04
       relative roughness :          1.700e-04
     Fluid Properties:
       density (slugs/ft^3):         1.668e+00
       viscosity (lbf-s/ft^2):       1.668e-04

     Calculated Parameters:
```

flow rate (gpm)	ave vel (ft/s)	Reynolds #	fric factor	vel head (ft)	hL major (ft)	hL minor (ft)	elev head (ft)
4.000e+03	1.135e+01	1.135e+05	1.844e-02	2.000e+00	1.106e+01	1.220e+01	2.526e+01
4.200e+03	1.192e+01	1.192e+05	1.829e-02	2.205e+00	1.210e+01	1.345e+01	2.775e+01
4.400e+03	1.248e+01	1.248e+05	1.816e-02	2.420e+00	1.318e+01	1.476e+01	3.036e+01
4.600e+03	1.305e+01	1.305e+05	1.803e-02	2.645e+00	1.430e+01	1.613e+01	3.308e+01
4.800e+03	1.362e+01	1.362e+05	1.791e-02	2.879e+00	1.547e+01	1.756e+01	3.591e+01
5.000e+03	1.418e+01	1.418e+05	1.779e-02	3.124e+00	1.668e+01	1.906e+01	3.886e+01
5.200e+03	1.475e+01	1.475e+05	1.769e-02	3.379e+00	1.793e+01	2.061e+01	4.192e+01
5.400e+03	1.532e+01	1.532e+05	1.758e-02	3.644e+00	1.922e+01	2.223e+01	4.510e+01
5.600e+03	1.589e+01	1.589e+05	1.749e-02	3.919e+00	2.056e+01	2.391e+01	4.839e+01
5.800e+03	1.645e+01	1.645e+05	1.740e-02	4.204e+00	2.194e+01	2.565e+01	5.179e+01
6.000e+03	1.702e+01	1.702e+05	1.731e-02	4.499e+00	2.337e+01	2.744e+01	5.531e+01

```
     Interpolated Result for actual Elevation Head of 36.0 ft:  Q =  4.806e+03 gpm
```

```
%
%   Crowe10_94  Type II Problem -- Solve for Flow Rate given the Elevation Head
%
%
%   This file computes the elevation head associated with a vector of flow rates.
%   Then a plot of Elevation Head versus Flow Rate is made.  With the desired
%   elevation head given in the problem statement, one can determine the actual flow
%   rate in the given system.
%
%   All the geometry information and fluid properties must be identified.
%   Depending on flow regime, either the laminar or turbulent flow relationship
%   is used to find the friction factor.  Minor losses need to be included
%   explicitly.  The total head loss is found from Darcy's Eqn.  Finally, the
%   energy equation is then used to compute the elevation head for several flow
%   rates.
%
%   File prepared by J. R. White, UMass-Lowell (last update: April 2017)
%

      clear all;   close all;   nfig = 0;
%
%   set the geometry parameters
      g = 32.2;                         % gravitational constant (ft/s^2)
      D =  1;                           % pipe diameter (ft)
      A =  pi*D*D/4;                    % flow area (ft^2)
      L = 300;                          % channel length (ft)
      e = 0.00017;                      % surface roughness (ft)
      eoD = e/D;                        % relative roughness
      act_ehead = 100-64;               % actual elevation head (z1-z2)  (ft)
%
%   set oil properties and volume flow rate
      rho = 0.86*1.94;                  % fluid density (slugs/ft^3)
      nu = 1.0e-4;                      % fluid kinematic viscosity (ft^2/s)
      mu = nu*rho;                      % fluid dynamics viscosity (lbf-s/ft^2
      Qgpm = 4000:200:6000;             % vector of volumetric flow rates (gpm)
      Q = Qgpm/448.8;                   % vector of volumetric flow rates (ft^3/s)
      NQ = length(Q);                   % number of flow rates
%
%   treat minor losses
      K = 0.5 + 5.6;                    % total resistance coeff
%   minor losses: sudden contraction + half open globe value
%
%   compute variety of parameters
      v = Q/A;                          % ave fluid velocity (ft/s)
      Re = rho*v*D/mu;                  % Reynolds number
      f = zeros(size(Q));               % initialize and calc the friction factor
      for i = 1:NQ
        if Re(i) <= 2300,  f(i) = 64/Re(i);  end
        if Re(i) > 2300 && Re(i) < 4000
          disp('  *** Warning: Flow in Transition Region ***');  f(i) = 0.05;  end
        if Re(i) >= 4000, f(i) = 0.25/(log10(eoD/3.7 + 5.74/Re(i)^0.9))^2;  end
      end
      vhead = v.*v/(2*g);               % dynamic or velocity head (ft)
      hLmajor = f*(L/D).*vhead;         % major head loss (ft)
      hLminor = K*vhead;                % minor head loss (ft)
%
%   now evaluate the energy equation
      ehead = vhead + hLmajor + hLminor; % desired elevation head
%
%   interpolate to find the desired Q
      Qfinal = interp1(ehead,Q,act_ehead)*448.8;
%
%   let's plot elevation head versus Qgpm
```

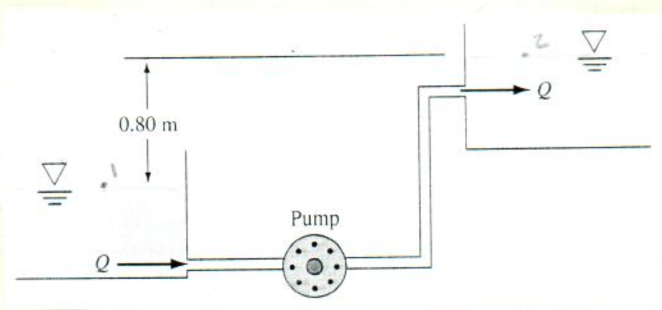
```
    nfig = nfig+1;    figure(nfig)
    plot(Qgpm,ehead,'r-','LineWidth',2),grid,hold on
    r = axis;
    plot(r(1:2),[act_ehead act_ehead],'b-','LineWidth',2),hold off
    title('Elevation Head versus Flow Rate (Type II HW Prob -- Crowe 10.94)')
    xlabel('Flow Rate (gal/min)')
    ylabel('Elevation Head (ft)')
    gtext(['Desired Q = ',num2str(Qfinal,'%10.3e'),' gpm'])
%
%   print results
    fid = 1;
    fprintf(fid,'   Results for Type II HW Problem (Crowe 10_94)  \n');
    fprintf(fid,'     Geometry: \n');
    fprintf(fid,'       pipe dia (ft):           %10.3e \n',D);
    fprintf(fid,'       flow area (ft^2):        %10.3e \n',A);
    fprintf(fid,'       channel length (ft):     %10.3e \n',L);
    fprintf(fid,'       surface roughness (ft):  %10.3e \n',e);
    fprintf(fid,'       relative roughness :     %10.3e \n',eoD);
    fprintf(fid,'     Fluid Properties: \n');
    fprintf(fid,'       density (slugs/ft^3):    %10.3e \n',rho);
    fprintf(fid,'       viscosity (lbf-s/ft^2):  %10.3e \n',mu);
    fprintf(fid,'    \n');
    fprintf(fid,'     Calculated Parameters: \n');
    fprintf(fid,'       flow rate    ave vel   Reynolds #  fric factor  vel head    hL major
hL minor   elev head \n');
    fprintf(fid,'          (gpm)       (ft/s)                                    (ft)         (ft)
(ft)        (ft) \n');
    for i = 1:NQ
      fprintf(fid,'      %10.3e  %10.3e  %10.3e  %10.3e  %10.3e  %10.3e  %10.3e  %10.3e \n', ...
                 Qgpm(i),v(i),Re(i),f(i),vhead(i),hLmajor(i),hLminor(i),ehead(i));
    end
    fprintf(fid,'    \n');
    fprintf(fid,'     Interpolated Result for actual Elevation Head of %4.1f ft:  Q = %10.3e
gpm\n', ...
                 act_ehead,Qfinal);
%
%   end of file
```

C11.1

Manufacturer data for a small aquarium pump are given below:

Q (m^3/s)	0	1E-6	2E-6	3E-6	4E-6	5E-6
Head (m)	1.10	1.00	0.80	0.60	0.35	0.0

Using this pump, what is the flow rate achieved in the system shown below? The tubing between the two reservoirs is a smooth plastic tube with an inner diameter of 5 mm and a total length of 29.8 m. The water is at room temperature. Also note that the minor losses can be ignored.



water properties at 20°C

$\rho = 998 \text{ kg/m}^3$

$\gamma = 9.79 \text{ kN/m}^3$

$\nu = 1.02 \times 10^{-6} \text{ m}^2/\text{s}$

Taking pts 1 and 2 as the surface of the lower and upper reservoirs, we have

$$\cancel{\frac{P_1}{\gamma}} + \cancel{\frac{\alpha V_1^2}{2g}} + z_1 + h_A - \cancel{h_R} - h_L = \cancel{\frac{P_2}{\gamma}} + \cancel{\frac{\alpha V_2^2}{2g}} + z_2$$

free surface — $h_R \to 0$ no energy removed — free surface

$$\therefore \; h_A = z_2 - z_1 + h_L$$

$$\boxed{h_A = \Delta z + f \frac{L}{D} \frac{V^2}{2g}}$$

no minor losses

note also that kinetic energy correction factor is not needed in h_L

Note: because of the low flow rates and the small pipe diameter, we expect that the flow will be laminar

check

for $Q_{max} = 5 \times 10^{-6} \text{ m}^3/\text{s}$

$$V = \frac{Q}{A} = \frac{5 \times 10^{-6}}{1.963 \times 10^{-5}} \qquad A = \frac{\pi D^2}{4} = \frac{\pi (.005\,m)^2}{4} = 1.963 \times 10^{-5} \text{ m}^2$$

$$\boxed{V = 0.255 \text{ m/s}}$$

$$\therefore \; Re = \frac{VD}{\nu} = \frac{(0.255)(.005)}{1.02 \times 10^{-6}} = 1250 < 2000$$

$\therefore$ flow is laminar

for laminar flow

$$f = \frac{64}{Re}$$

thus the system curve, h_A vs Q, can be easily developed

$$h_A = \Delta z + f \frac{L}{D}\frac{V^2}{2g} \qquad f = \frac{64}{Re} = \frac{64\nu}{VD}$$

$$h_A = \Delta z + \frac{32\nu L}{gD^2} V \qquad V = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}D^2}$$

$$\boxed{h_A = \Delta z + \frac{128\nu L}{\pi g D^4} Q}$$

This is just a linear function of Q

→ Now we plot this system curve and the pump curve data given in the problem description.

→ The intersection of these two curves is the desired operating point.

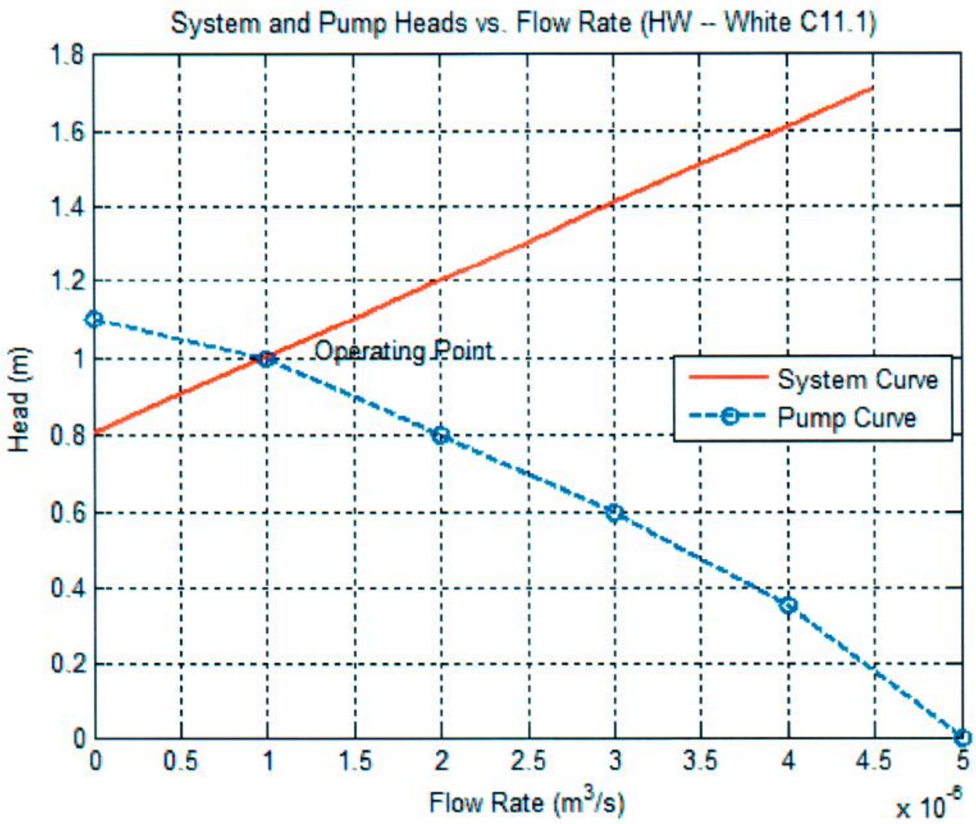
See White C11-1.XLS {also see WhiteC11-1.m}

[note: I actually implemented the standard procedure for a type I system with the laminar flow correlation for f — $f = 64/Re$]

→ Summary Result

$$\boxed{\begin{array}{l} Q \simeq 1.0\times10^{-6}\ m^3/s \\ h_A \simeq 1\ m \end{array}}$$

This is the operating point {see Excel plots}




```
>> whiteC11_1
   Results for System/Pump Operating Point (White C11.1)
     Geometry:
       pipe dia (m):              5.000e-03
       flow area (m^2):           1.963e-05
       channel length (m):        2.980e+01
       surface roughness (m):     0.000e+00
       relative roughness:        0.000e+00
     Fluid Properties:
       density (kg/m^3):          9.980e+02
       viscosity (N-s/m^2):       1.018e-03

     Calculated Parameters:
```

flow rate (m^3/s)	ave vel (m/s)	Reynolds #	fric factor	vel head (m)	hL major (m)	hL minor (m)	head added (m)
2.220e-16	1.131e-11	5.543e-08	1.155e+09	6.518e-24	4.485e-11	0.000e+00	8.000e-01
5.000e-07	2.546e-02	1.248e+02	5.127e-01	3.305e-05	1.010e-01	0.000e+00	9.010e-01
1.000e-06	5.093e-02	2.497e+02	2.564e-01	1.322e-04	2.020e-01	0.000e+00	1.002e+00
1.500e-06	7.639e-02	3.745e+02	1.709e-01	2.975e-04	3.030e-01	0.000e+00	1.103e+00
2.000e-06	1.019e-01	4.993e+02	1.282e-01	5.288e-04	4.040e-01	0.000e+00	1.204e+00
2.500e-06	1.273e-01	6.241e+02	1.025e-01	8.263e-04	5.050e-01	0.000e+00	1.305e+00
3.000e-06	1.528e-01	7.490e+02	8.545e-02	1.190e-03	6.060e-01	0.000e+00	1.406e+00
3.500e-06	1.783e-01	8.738e+02	7.324e-02	1.619e-03	7.070e-01	0.000e+00	1.507e+00
4.000e-06	2.037e-01	9.986e+02	6.409e-02	2.115e-03	8.080e-01	0.000e+00	1.608e+00
4.500e-06	2.292e-01	1.123e+03	5.697e-02	2.677e-03	9.090e-01	0.000e+00	1.709e+00

```
%
%    whiteC11_1  Find the System/Pump Operating Point
%
%
%    This file computes the head added from the energy equation for a specific flow
%    system.  The given pump curve is then superimposed on the system curve
%    to find the operating point for the overall system.
%
%    hA from the energy equation is a function of the flow rate Q.  However, the actual
%    measured head vs flow rate curve for the pump was given.  The two curves are
%    plotted and the intersection represents the "operating point".
%
%    For the system curve, all the geometry information and fluid properties must be
%    given.  Depending on flow regime, either the laminar or turbulent flow expression
%    is used to find the friction factor.  Minor losses need to be included explicitly.
%    The total head loss is found from Darcy's Eqn.  Finally, the energy equation is
%    then used to compute hA for several flow rates.
%
%    File prepared by J. R. White, UMass-Lowell (April 2017)
%

      clear all;   close all;   nfig = 0;
%
%    set the geometry parameters
      g = 9.81;                          % gravitational constant (m/s^2)
      D =  0.005;                        % pipe diameter (m)
      A =  pi*D*D/4;                     % flow area (ft^2)
      L = 29.8;                          % channel length (m)
      e = 0.00000;                       % surface roughness (m) -- smooth plastic tube
      eoD = e/D;                         % relative roughness
      ehead = 0.8;                       % elevation head (z2-z1) (m)
%
%    set oil properties and volume flow rate
      rho = 998;                         % fluid density (kg/m^3)
      gam = rho*g;                       % specific weight (N/m^3)
      nu = 1.02e-6;                      % fluid kinematic viscosity (m^2/s)
      mu = nu*rho;                       % fluid dynamics viscosity (N-s/m^2)
      Q = eps:0.5e-6:5e-6;               % vector of volumetric flow rates (m^3/s)
      NQ = length(Q);                    % number of flow rates
%
%    treat minor losses
      K = 0;                             % total resistance coeff
%    minor losses: none for this problem
%
%    compute variety of parameters
      v = Q/A;                           % ave fluid velocity (m/s)
      Re = rho*v*D/mu;                   % Reynolds number
      f = zeros(size(Re));               % initialize and calc the friction factor
      for i = 1:NQ
        if Re(i) <= 2300,  f(i) = 64/Re(i);  end
        if Re(i) > 2300 && Re(i) < 4000
          disp('  *** Warning: Flow in Transition Region ***');  f(i) = 0.05;  end
        if Re(i) >= 4000, f(i) = 0.25/(log10(eoD/3.7 + 5.74/Re(i)^0.9))^2;  end
      end
      vhead = v.*v/(2*g);                % dynamic or velocity head (m)
      hLmajor = f*(L/D).*vhead;          % major head loss (m)
      hLminor = K*vhead;                 % minor head loss (m)
%
%    now evaluate the energy equation
      hAsys = ehead + hLmajor + hLminor; % head added from energy eqn (syst curve)
%
%    pump curve data (given in problem)
      Qp = [0 1 2 3 4 5]*1e-6;                    % pump flow rates (m^3/s)
```

```
      hApump = [1.10 1.00 0.80 0.60 0.35 0.0];  % measured pump head (m)

%   let's plot the two curves
      nfig = nfig+1;   figure(nfig)
      plot(Q,hAsys,'r-',Qp,hApump,'b--o','LineWidth',2),grid
      title('System and Pump Heads vs. Flow Rate (HW -- White C11.1)')
      xlabel('Flow Rate (m^3/s)')
      ylabel('Head (m)')
      legend('System Curve','Pump Curve','Location','East');
      gtext('Operating Point')
%
%   print results
      fid = 1;
      fprintf(fid,'   Results for System/Pump Operating Point (White C11.1)  \n');
      fprintf(fid,'     Geometry: \n');
      fprintf(fid,'       pipe dia (m):             %10.3e \n',D);
      fprintf(fid,'       flow area (m^2):          %10.3e \n',A);
      fprintf(fid,'       channel length (m):       %10.3e \n',L);
      fprintf(fid,'       surface roughness (m):    %10.3e \n',e);
      fprintf(fid,'       relative roughness:       %10.3e \n',eoD);
      fprintf(fid,'     Fluid Properties: \n');
      fprintf(fid,'       density (kg/m^3):         %10.3e \n',rho);
      fprintf(fid,'       viscosity (N-s/m^2):      %10.3e \n',mu);
      fprintf(fid,'    \n');
      fprintf(fid,'     Calculated Parameters: \n');
      fprintf(fid,'       flow rate    ave vel    Reynolds #  fric factor  vel head    hL major
hL minor   head added \n');
      fprintf(fid,'        (m^3/s)      (m/s)                                    (m)         (m)
(m)         (m) \n');
      for i = 1:NQ
        fprintf(fid,'      %10.3e  %10.3e  %10.3e  %10.3e  %10.3e  %10.3e  %10.3e  %10.3e \n', ...
                   Q(i),v(i),Re(i),f(i),vhead(i),hLmajor(i),hLminor(i),hAsys(i));
      end
%
%   end of file
```