CHEN.3030 Fluid Mechanics
Short Quiz: Viscous Internal Flows

Part a: In class we showed that the velocity distribution in a circular pipe under laminar flow conditions is given by

$$u(r) = -\frac{\left(R^2 - r^2\right)}{4\mu}\frac{d}{dx}\left(P + \gamma h\right)$$

If the pipe inside radius is R, for a given pressure plus elevation gradient, determine expressions for the volumetric flow rate, Q, and the average fluid velocity, v, within the pipe. Be formal!!!

$$Q = \int_A \vec{v}\cdot\hat{n}\,dA = \int_0^R u(r)\,2\pi r\,dr$$

$$= -\frac{2\pi}{4\mu}\frac{d}{dx}(P+\gamma h)\underbrace{\int_0^R (R^2 - r^2)\,r\,dr}_{\frac{R^2 r^2}{2} - \frac{r^4}{4}\Big|_0^R = \frac{R^4}{2} - \frac{R^4}{4} = \frac{R^4}{4}}$$

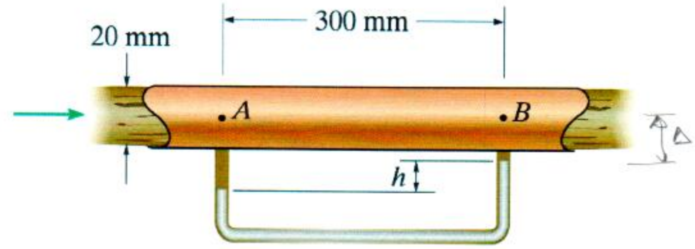
$$\therefore \boxed{Q = -\frac{\pi R^4}{8\mu}\frac{d}{dx}(P+\gamma h)}$$ ans

and $V = \frac{Q}{A} = \frac{Q}{\pi R^2}$ or $\boxed{V = -\frac{R^2}{8\mu}\frac{d}{dx}(P+\gamma h)}$ ans

Part b: For the specific case shown in the sketch with oil as the working fluid, determine Q if the mercury manometer reads h = 4 cm, ρ_{oil} = 880 kg/m^3, μ_{oil} = 0.068 N-s/m^2, and ρ_{Hg} = 13550 kg/m^3. Note also that, at the end, you should always validate the original laminar flow assumption. Use the back side of the page, as needed, for you work…

Note: If you are unsuccessful with Part a, use

$$Q = -\frac{d}{dx}(P+\gamma h)\frac{\pi R^4}{16\mu}$$ to do the Part b calculations.



for horizontal pipe

$$-\frac{d}{dx}(P+\gamma h) = -\frac{dP}{dx} = -\frac{(P_B - P_A)}{L} = \frac{P_A - P_B}{L} = \frac{\Delta P}{L}$$

from the manometer eqn

$$P_A + \cancel{\gamma_{oil}\Delta} + \gamma_{oil} h - \gamma_{Hg} h - \cancel{\gamma_{oil}\Delta} = P_B$$

$$P_A - P_B = (\gamma_{Hg} - \gamma_{oil})h = \underset{(kg/m^3)}{(13550 - 880)}\underset{(m/s^2)}{(9.81)}\underset{m}{(0.04)}$$

$$= 4972 \frac{N}{m^2}$$

$$= 4.972 \text{ kPa}$$

$D = 20\text{ mm} = 2\text{ cm} = 0.02\text{ m}$

$R = D/2$

Now

$$Q = \frac{\pi R^4}{8\mu}\frac{\Delta P}{L} = \frac{\pi(0.01\text{ m})^4}{8(0.068 \frac{N\text{-}s}{m^2})}\frac{(4972\, N/m^2)}{0.3\text{ m}}$$

$$\boxed{Q = 9.571 \times 10^{-4}\ m^3/s}$$ ans

finally

$$Re = \frac{\rho V D}{\mu} = \frac{\rho Q D}{\mu \frac{\pi}{4} D^2} = \frac{4\rho Q}{\pi \mu D}$$

$$= \frac{4(880\, kg/m^3)(9.571\times10^{-4}\, m^3/s)}{\pi(0.068\, kg/m\text{-}s)(0.02\, m)} = 788.5$$ ok

units ok

$$\frac{N\text{-}s}{m^2} \rightarrow \frac{kg\frac{m}{s^2}s}{m^2} = \frac{kg}{m\text{-}s}$$

laminar flow since $Re < 2300$