Fluid Mechanics
University of Massachusetts, Lowell – Department of Chemical Engineering
CHEN.3030 – Spring 2017

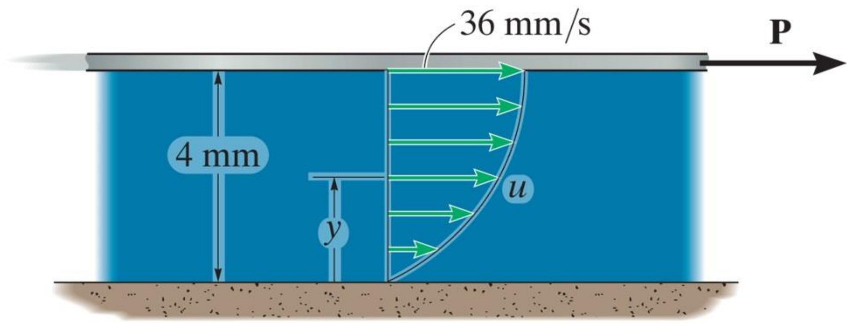
Name:______________________ Student ID: ____________________________

Final Exam; Time: 3:00PM – 6:00PM; Exam type: Open book (Only text book is allowed) **without any additional writing or attachment**. (Do all six (6) problems, with each problem having equal weight)

Q1. Basic Concepts and Fluid Properties

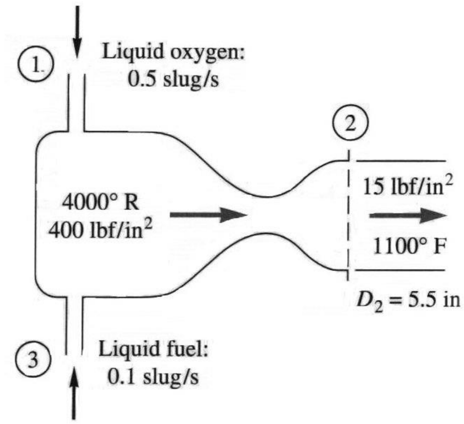
a. Kerosene is mixed with 10 ft^3 of ethyl alcohol so that the volume of the mixture in the tank becomes 14 ft^3. Both fluids are at room temperature. Determine the specific gravity and the specific weight of the mixture.

b. The velocity profile for a thin film of a Newtonian fluid with $\mu = 0.56$ N–s/m^2 that is confined between the plate and a fixed surface as shown in the sketch is defined by $u = (10y - 0.25y^2)$ mm/s, where y is in mm. Determine the force **P** that must be applied to the plate to give this motion. The plate has a surface area of 4000 mm^2 in contact with the fluid.



Q2. A rocket motor is operated in steady state with the conditions shown in the diagram. In particular, the combustion chamber operates at 4000 ^oR and 400 psia but the gas exit conditions are near atmospheric pressure (15 psia) and the temperature in Fahrenheit units is 1100 ^oF. The products of combustion flowing out the exhaust nozzle approximate a perfect gas with gas constant R = 1775 ft-lbf/slug-R.

For the given conditions, calculate the exit velocity, v_2, in ft/s.



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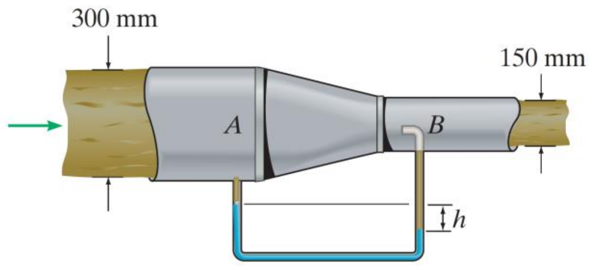
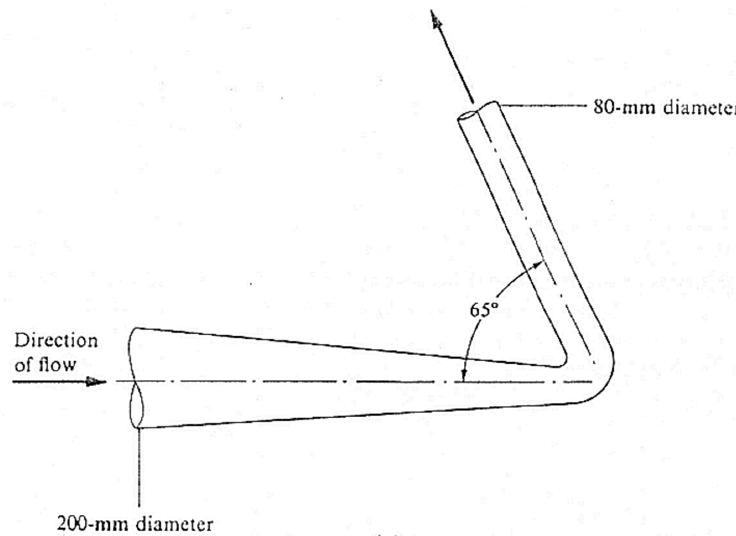
Q3. Determine the difference in height *h* of the water column in the manometer if the flow of oil through the pipe is 0.04 m^3/s. At flow conditions, the oil density is 875 kg/m^3 and the water density is 1000 kg/m^3.

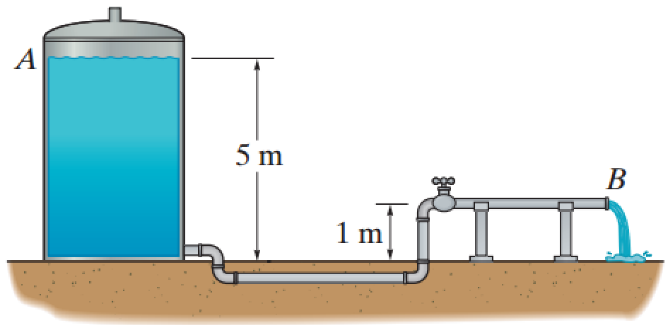


Q4. The reducing elbow shown in the sketch lies in the horizontal plane. It is only one component of a long piping system within a chemical processing plant. A fluid with specific weight $\gamma = 8.615$ kN/m^3 enters the bend with a velocity of 3.5 m/s and a pressure of 280 kPa. The inlet pipe diameter to the reducer is 200 mm and the outlet diameter is 80 mm, and the bend forms a 65^o angle as shown.
Neglecting energy losses in the bend, estimate the x and y-directed reaction force components needed to hold the elbow section in place.
Note: There are no net gravity effects here since the elbow is in the horizontal x-y plane (i.e. the sketch is a top view of the elbow).



(Extra Page 1)

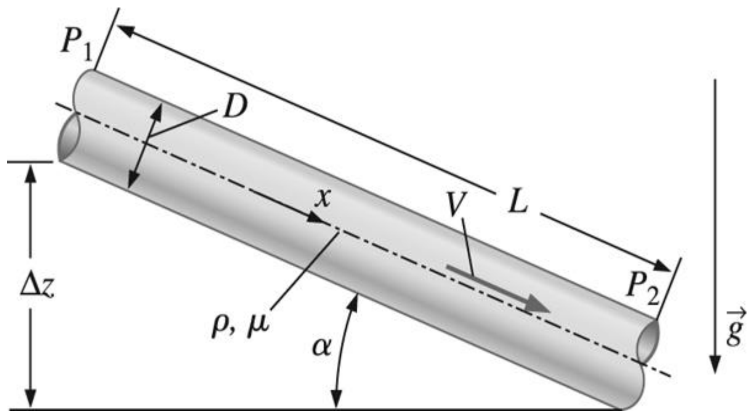
Q5. Water at $T = 20°C$ flows from the open tank through the 50-mm-diameter galvanized iron pipe. Determine the discharge at the end B if the globe valve is fully opened. The length of the pipe is 50 m. Include the minor losses of the flush entrance, the four elbows, and the globe valve. *Use moody diagram to find the friction factor along the pipe length.*



Q6. You can choose one of the following two problems, but not both (if you do both, only one will be counted)

Part a. A smooth horizontal pipe with D = 4 inches is 12 ft long and transports 70 ^{o}F water. The measured pressure drop along the pipe is 0.170 psi/ft. With this information, estimate the shear stress on the walls of the pipe, the velocity along the pipe's centerline, and the thickness of the viscous sublayer for this turbulent flow problem. **Hint:** Use the turbulent flow correlations given in Chapter 9 of your text by Hibbeler then, at the end, always check the Reynolds number to make sure that the flow is really turbulent.

Q6. Part b. Water flows down a long, straight, inclined pipe of diameter *D* and length *L*. There is no forced pressure gradient between points 1 and 2; in other words, the water flows through the pipe by gravity alone, and $P_1 = P_2 = P_{atm}$. The flow is steady, fully developed, and laminar. We adapt a coordinate system in which *x* flows the axis of the pipe. We would like to generate an expression for average velocity *V* as a function of the given parameters, ρ, g, D, Δz, μ, and *L*. (b) Use dimensional analysis to generate a dimensionless expression for *V* as a function of the given parameters. Construct a relationship between your results that matches the exact analytical expression.



(Extra Page 2)

DIMENSION	METRIC	METRIC/ENGLISH
Viscosity, kinematic	1 m^2/s = 10^4 cm^2/s 1 stoke = 1 cm^2/s = 10^{-4} m^2/s	1 m^2/s = 10.764 ft^2/s = 3.875 × 10^4 ft^2/h 1 m^2/s = 10.764 ft^2/s
Volume	1 m^3 = 1000 L = 10^6 cm^3 (cc)	1 m^3 = 6.1024 × 10^4 in^3 = 35.315 ft^3 = 264.17 gal (U.S.) 1 U.S. gallon = 231 in^3 = 3.7854 L 1 fl ounce = 29.5735 cm^3 = 0.0295735 L 1 U.S. gallon = 128 fl ounces
Volume flow rate	1 m^3/s = 60,000 L/min = 10^6 cm^3/s	1 m^3/s = 15,850 gal/min = 35.315 ft^3/s = 2118.9 ft^3/min (CFM)

*Exact conversion factor between metric and English units.

Some Physical Constants

PHYSICAL CONSTANT	METRIC	ENGLISH
Standard acceleration of gravity	g = 9.80665 m/s^2	g = 32.174 ft/s^2
Standard atmospheric pressure	P_{atm} = 1 atm = 101.325 kPa = 1.01325 bar = 760 mm Hg (0°C) = 10.3323 m H_2O (4°C)	P_{atm} = 1 atm = 14.696 psia = 2116.2 lbf/ft^2 = 29.9213 inches Hg (32°F) = 406.78 inches H_2O (39.2°F)
Universal gas constant	R_u = 8.31447 kJ/kmol · K = 8.31447 kN · m/kmol · K	R_u = 1.9859 Btu/lbmol · R = 1545.37 ft · lbf/lbmol · R

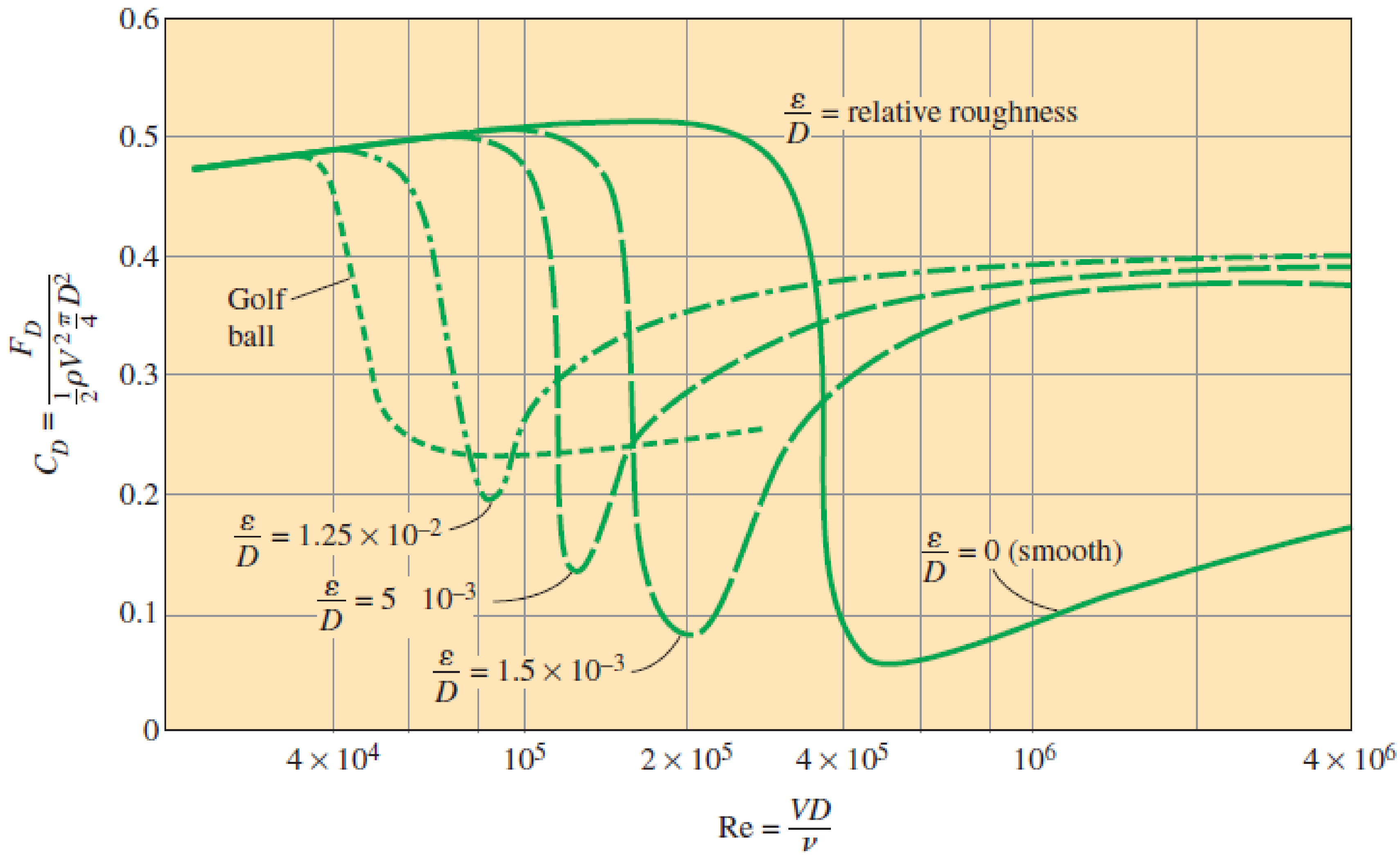
Commonly Used Properties

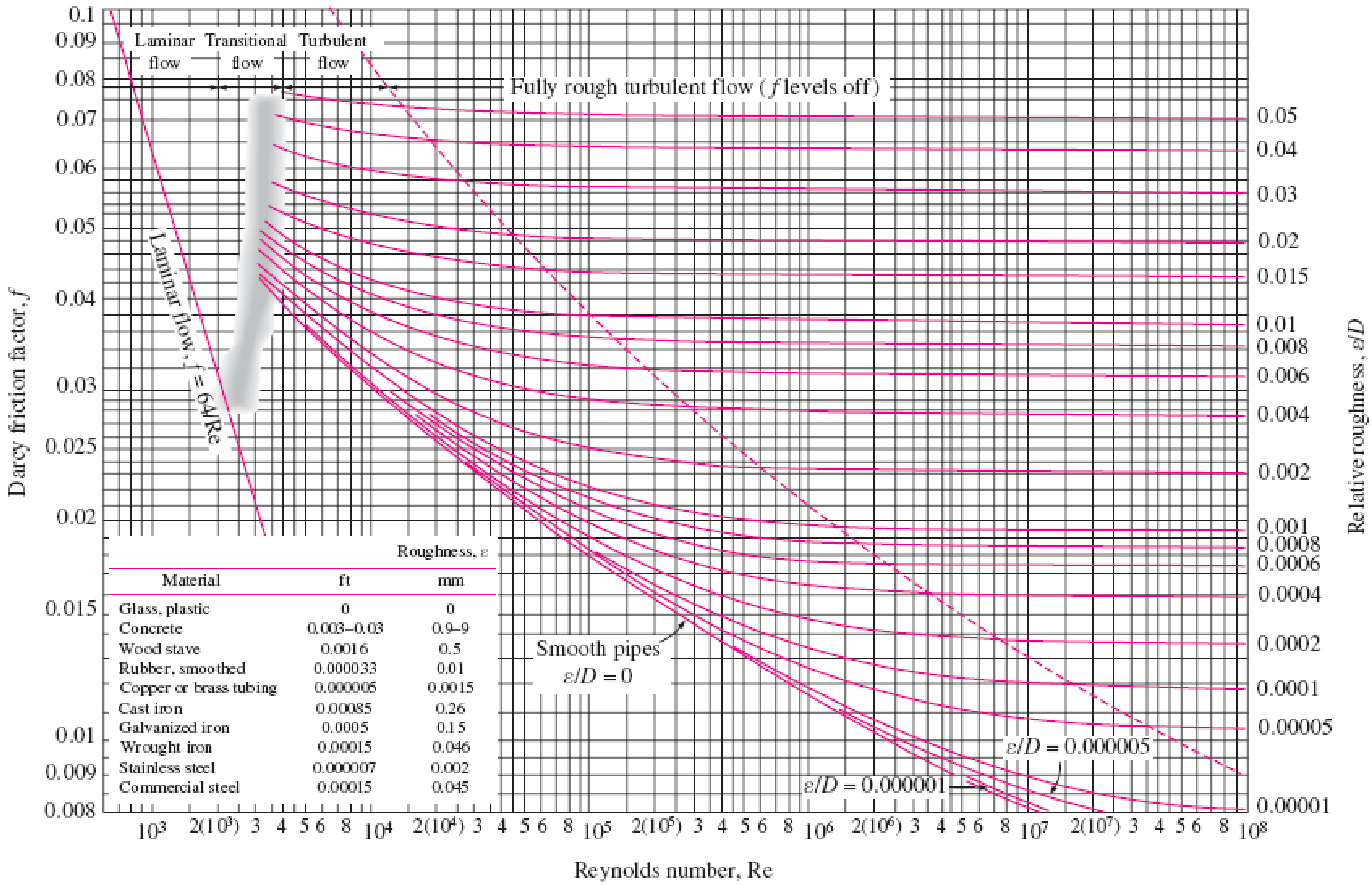
PROPERTY	METRIC	ENGLISH
Air at 20°C (68°F) and 1 atm		
Specific gas constant*	R_{air} = 0.2870 kJ/kg · K = 287.0 m^2/s^2 · K	R_{air} = 0.06855 Btu/lbm · R = 53.34 ft · lbf/lbm · R = 1716 ft^2/s^2 · R
Specific heat ratio	$k = c_p/c_v$ = 1.40	$k = c_p/c_v$ = 1.40
Specific heats	c_p = 1.007 kJ/kg · K = 1007 m^2/s^2 · K c_v = 0.7200 kJ/kg · K = 720.0 m^2/s^2 · K	c_p = 0.2404 Btu/lbm · R = 187.1 ft · lbf/lbm · R = 6019 ft^2/s^2 · R c_v = 0.1719 Btu/lbm · R = 133.8 ft · lbf/lbm · R = 4304 ft^2/s^2 · R
Speed of sound	c = 343.2 m/s = 1236 km/h	c = 1126 ft/s = 767.7 mi/h
Density	ρ = 1.204 kg/m^3	ρ = 0.07518 lbm/ft^3
Viscosity	μ = 1.825 × 10^{-5} kg/m · s	μ = 1.227 × 10^{-5} lbm/ft · s
Kinematic viscosity	ν = 1.516 × 10^{-5} m^2/s	ν = 1.632 × 10^{-4} ft^2/s
Liquid water at 20°C (68°F) and 1 atm		
Specific heat ($c = c_p = c_v$)	c = 4.182 kJ/kg · K = 4182 m^2/s^2 · K	c = 0.9989 Btu/lbm · R = 777.3 ft · lbf/lbm · R = 25,009 ft^2/s^2 · R
Density	ρ = 998.0 kg/m^3	ρ = 62.30 lbm/ft^3
Viscosity	μ = 1.002 × 10^{-3} kg/m · s	μ = 6.733 × 10^{-4} lbm/ft · s
Kinematic viscosity	ν = 1.004 × 10^{-6} m^2/s	ν = 1.081 × 10^{-5} ft^2/s

* Independent of pressure or temperature

Conversion Factors

DIMENSION	METRIC	METRIC/ENGLISH
Acceleration	1 m/s^2 = 100 cm/s^2	1 m/s^2 = 3.2808 ft/s^2 1 ft/s^2 = 0.3048* m/s^2
Area	1 m^2 = 10^4 cm^2 = 10^6 mm^2 = 10^{-6} km^2	1 m^2 = 1550 in^2 = 10.764 ft^2 1 ft^2 = 144 in^2 = 0.09290304* m^2
Density	1 g/cm^3 = 1 kg/L = 1000 kg/m^3	1 g/cm^3 = 62.428 lbm/ft^3 = 0.036127 lbm/in^3 1 lbm/in^3 = 1728 lbm/ft^3 1 kg/m^3 = 0.062428 lbm/ft^3
Energy, heat, work, and specific energy	1 kJ = 1000 J = 1000 N · m = 1 kPa · m^3 1 kJ/kg = 1000 m^2/s^2 1 kWh = 3600 kJ	1 kJ = 0.94782 Btu 1 Btu = 1.055056 kJ = 5.40395 psia · ft^3 = 778.169 lbf · ft 1 Btu/lbm = 25,037 ft^2/s^2 = 2.326* kJ/kg 1 kWh = 3412.14 Btu
Force	1 N = 1 kg · m/s^2 = 10^5 dyne 1 kgf = 9.80665 N	1 N = 0.22481 lbf 1 lbf = 32.174 lbm · ft/s^2 = 4.44822 N 1 lbf = 1 slug · ft/s^2
Length	1 m = 100 cm = 1000 mm = 10^6 μm 1 km = 1000 m	1 m = 39.370 in = 3.2808 ft = 1.0926 yd 1 ft = 12 in = 0.3048* m 1 mile = 5280 ft = 1.6093 km 1 in = 2.54* cm
Mass	1 kg = 1000 g 1 metric ton = 1000 kg	1 kg = 2.2046226 lbm 1 lbm = 0.45359237* kg 1 ounce = 28.3495 g 1 slug = 32.174 lbm = 14.5939 kg 1 short ton = 2000 lbm = 907.1847 kg
Power	1 W = 1 J/s 1 kW = 1000 W = 1 kJ/s 1 hp† = 745.7 W	1 kW = 3412.14 Btu/h = 1.341 hp = 737.56 lbf · ft/s 1 hp = 550 lbf · ft/s = 0.7068 Btu/s = 42.41 Btu/min = 2544.5 Btu/h = 0.74570 kW 1 Btu/h = 1.055056 kJ/h
Pressure or stress, and pressure expressed as a head	1 Pa = 1 N/m^2 1 kPa = 10^3 Pa = 10^{-3} MPa 1 atm = 101.325 kPa = 1.01325 bar = 760 mm Hg at 0°C = 1.03323 kgf/cm^2 1 mm Hg = 0.1333 kPa	1 Pa = 1.4504 × 10^{-4} psi = 0.020886 lbf/ft^2 1 psi = 144 lbf/ft^2 = 6.894757 kPa 1 atm = 14.696 psi = 29.92 inches Hg at 30°F 1 inch Hg = 13.60 inches H_2O = 3.387 kPa
Specific heat	1 kJ/kg · °C = 1 kJ/kg · K = 1 J/g · °C	1 Btu/lbm · °F = 4.1868 kJ/kg · °C 1 Btu/lbmol · R = 4.1868 kJ/kmol · K 1 kJ/kg · °C = 0.23885 Btu/lbm · °F = 0.23885 Btu/lbm · R
Specific volume	1 m^3/kg = 1000 L/kg = 1000 cm^3/g	1 m^3/kg = 16.02 ft^3/lbm 1 ft^3/lbm = 0.062428 m^3/kg
Temperature	T(K) = T(°C) + 273.15 ΔT(K) = ΔT(°C)	T(R) = T(°F) + 459.67 = 1.8T(K) T(°F) = 1.8 T(°C) + 32 ΔT(°F) = ΔT(R) = 1.8* ΔT(K)
Velocity	1 m/s = 3.60 km/h	1 m/s = 3.2808 ft/s = 2.237 mi/h 1 mi/h = 1.46667 ft/s 1 mi/h = 1.6093 km/h
Viscosity, dynamic	1 kg/m · s = 1 N · s/m^2 = 1 Pa · s = 10 poise	1 kg/m · s = 2419.1 lbm/ft · h = 0.020886 lbf · s/ft^2 = 0.67197 lbm/ft · s


0.6
0.5
0.4
0.3
0.2
0.1
0
$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 \frac{\pi}{4} D^2}$
$\frac{\varepsilon}{D}$ = relative roughness
Golf ball
$\frac{\varepsilon}{D} = 1.25 \times 10^{-2}$
$\frac{\varepsilon}{D} = 5 \quad 10^{-3}$
$\frac{\varepsilon}{D} = 1.5 \times 10^{-3}$
$\frac{\varepsilon}{D} = 0$ (smooth)
4×10^4
10^5
2×10^5
4×10^5
10^6
4×10^6
$\mathrm{Re} = \frac{VD}{\nu}$



	Roughness, ε	
Material	ft	mm
Glass, plastic	0	0
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

Solution

Q1.

Q2.

Q3.

Solution: Apply Bernoulli's equation between points A an B

$$\frac{P_A}{\rho_o g} + \frac{V_A^2}{2g} + Z_A^{\nearrow 0} = \frac{P_B}{\rho_o g} + \frac{V_B^2}{2g}^{\nearrow 0} + Z_B^{\nearrow 0}$$

$$\Rightarrow \frac{P_A}{\rho_o g} + \frac{V_A^2}{2g} = \frac{P_B}{\rho_o g}$$

$$\Rightarrow \frac{P_A}{\rho_o} + \frac{V_A^2}{2} = \frac{P_B}{\rho_o} \quad [\text{Multiply by } 'g']$$

$$\Rightarrow \frac{P_A}{875\,kg/m^3} + \frac{(0.5661\,m/s)^2}{2} = \frac{P_B}{875\,kg/m^3}$$

$$\Rightarrow \frac{P_B - P_A}{875\,kg/m^3} = 0.1602\ m^2/s^2$$

$$\Rightarrow P_B - P_A = (0.1602\,m^2/s^2)(875\,kg/m^3)\frac{(1N)}{(1\,kg\,m/s^2)}$$

$$\Rightarrow \boxed{P_B - P_A = 140.2\ N/m^2} \quad \text{——— (1)}$$

Given,

At stagnant point, B, $v_B = 0$

Datum is passing through points A and B.

$Z_A = Z_B = 0$

Oil density, $\rho_o = 875\,kg/m^3$

Water density, $\rho_w = 1000\,kg/m^3$

Dia, $D_A = 300\,mm$

Dia, $D_B = 150\,mm$

Flowrate, $Q = 0.04\,m^3/s$

$Q = V_A A_A$

$\Rightarrow 0.04\,m^3/s = V_A\left(\frac{1}{4}\pi\left(\frac{300}{1000}\right)^2\right)m^2$

$\Rightarrow \boxed{V_A = 0.5661\,m/s}$

Manometer equation:

$$P_A + \rho_o g h_{AC} + \rho_w g h_{CD} - \rho_o g h_{DB} = P_B$$

$$\Rightarrow \rho_o g a + \rho_w g h - \rho_o g(h+a) = P_B - P_A$$

$$\Rightarrow \rho_o g a + \rho_w g h - \rho_o g h - \rho_o g a = P_B - P_A$$

$$\Rightarrow h(\rho_w - \rho_o) g = P_B - P_A$$

$$\Rightarrow h = \frac{P_B - P_A}{(\rho_w - \rho_o) g}$$

$$\Rightarrow h = \frac{140.2\,N/m^2}{(1000-875)\,kg/m^3 \times 9.81\,m/s^2} \quad [\text{Replace } (P_B - P_A) \text{ from eq}^n (1)]$$

$$\Rightarrow h = 0.1143\,m$$

$$\Rightarrow \boxed{h = 114.3\,mm} \quad (Ans)$$

Here,

$h_{AC} = a$

$h_{CD} = h$

$h_{DB} = h + a$

Q4.

Q5.

Solution:

Assumption: ① Water is incompressible.

From Appendix A, at 20°C

$\rho = 998.3\, kg/m^3$

$\nu = 1\times10^{-6}\, m^2/s$

Length of pipe, $L = 50m$

Dia, $D = 50\, mm$

The Reynold's number, $Re = \frac{DV}{\nu}$

$= \frac{(50\times10^{-3}m)V}{1\times10^{-6}m^2/s}$

$= (5\times10^4)V$

Major head loss from A to B:

$$h_L = f\frac{L}{D}\cdot\frac{V^2}{2g}$$

$$\Rightarrow h_L = f\left(\frac{50m}{0.05m}\right)\frac{V^2}{2.(9.81m/s^2)}$$

$$\Rightarrow h_L = 50.968 f V^2$$

Minor head loss:

1 Fully opened globe valve, $K_L = 10$

4 90° Elbows, $K_L = 0.9$

1 Flush entrance, $K_L = 0.5$

$\Sigma K_L = K_{L,globe\ valve} + 4\, K_{L,elbows} + K_{L,flush\ entrance}$

$= 10 + 4\times0.9 + 0.5$

$= 14.1$

For energy eq^n:

$P_B = 0$

$P_A = 0$ (open tank)

$V_A = 0$ (Large tank)

$Z_A = 5m$

$Z_B = 1m$

$h_{pump} = 0$

$h_{turbine} = 0$

$V_B = V$

$$(h_L)_{minor} = \Sigma K_L\frac{V^2}{2g} = 14.1\times\frac{V^2}{2(9.81m/s^2)} = 0.7187V^2$$

Applying Energy equation:

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + z_A + h_{pump} = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{Turbine} + h_L + (h_L)_{minor}$$

$$\Rightarrow 0 + 0 + 5m + 0 = 0 + \frac{V_B^2}{2g} + 1m + 0 + h_L + (h_L)_{minor}$$

$$\Rightarrow \frac{V^2}{2\times(9.81m/s^2)} + h_L + (h_L)_{minor} = 4m$$

$$\Rightarrow 0.05097V^2 + 50.968fV^2 + 0.7187V^2 = 4$$

$$\Rightarrow 0.7697V^2 + 50.968fV^2 = 4$$

$$\Rightarrow V = \frac{2}{\sqrt{50.968f + 0.7697}}$$

For galvanized iron:

$$\varepsilon = 0.15\,mm$$
$$D = 50\,mm$$

$$\therefore \frac{\varepsilon}{D} = \frac{0.15\,mm}{50\,mm} = 0.003$$

Iteration	Assumed f	$V = \frac{2\ m/s}{\sqrt{50.968f + 0.7697}}$	$Re = (5\times10^4)V$	f (Moody diagram)
1	0.026	1.382	6.91×10^4	0.278
2	0.0278	1.353	6.76×10^4	0.028
3	0.0280	1.349	6.75×10^4	0.028

For 3rd iteration, the assumed 'f' is the same as the 'f' from Moody diagram. Thus, $V = 1.349\,m/s$

$$Q = VA = (1.349\,m/s) \times \frac{\pi}{4}\left(\frac{50}{1000}\right)^2 m^2$$

$$\Rightarrow Q = 0.00265\,m^3/s$$

Ans.

Q6a.

Q6b.

We are to analyze this problem three ways: with the control volume technique, with the differential technique, and with dimensional analysis, and we are to compare the results.

(*a*) An exact analysis of this flow was performed in Problem 9-100. We refer to the solution of that problem and do not show the details here. The average velocity through the pipe was found to be

$$V = \frac{R^2}{8\mu}\rho g \sin\alpha$$

But $R = D/2$, and from the figure provided in the problem statement we see that $\sin\alpha = \Delta z/L$. Thus, our result is

V from differential analysis:
$$\boxed{V = \frac{\rho g D^2 \Delta z}{32\mu L}} \quad (4)$$

(*b)* we perform a dimensional analysis. There are 7 parameters in the problem: V as a function of ρ, g, D, Δz, μ, and L. There are three primary dimensions represented in the problem, namely m, L, and t. Thus we expect 7-3 = 4 Πs. We choose three repeating variables, ρ, g, and D. The Πs are

Dimensionless parameters:
$$\Pi_1 = \frac{V}{\sqrt{gD}} \qquad \Pi_2 = \frac{\rho D\sqrt{gD}}{\mu} \qquad \Pi_3 = \frac{\Delta z}{D} \qquad \Pi_4 = \frac{L}{D}$$

The first Π is a Froude number and the second Π is a Reynolds number. The dimensionless relationship is

Result of dimensional analysis:
$$\boxed{\frac{V}{\sqrt{gD}} = f\left(\frac{\rho D\sqrt{gD}}{\mu}, \frac{\Delta z}{D}, \frac{L}{D}\right)} \quad (5)$$