Differential Equations (92.236)

Homework Assignment #7 Spring 2007

Numerical Approximations: Euler's Method

Problem 1

a. Solve the following IVP analytically. The exact solution obtained here will serve as the basis for addressing the accuracy of various numerical schemes:

$$\frac{dy}{dx} = \frac{2e^{2x} - y}{x} \qquad \text{with} \quad y(1) = e^2 \approx 7.389$$

b. Using the Euler method with a step size of h = 0.2, perform a series of hand calculations to estimate y(x) for $1 \le x \le 2$. Create a short table of results and compare the Euler solution to the exact result determined in Part a.

c. Use the **euler.m** Matlab function to solve the above problem using step sizes of $h_1 = 0.2$ and $h_2 = 0.1$, again comparing the analytical and numerical solutions. You should modify the **ndemo1.m** sample program discussed in the Matlab lab, as necessary, to perform these calculations (**ndemo1.m** is the main program which calls **euler.m** to actually implement the Euler method). Do the Matlab results with h = 0.2 agree with your hand calculations from Part b? With the errors at the final value of x from the two cases with two different step sizes denoted as ε_1 and ε_2, address how the error in the Euler method varies with the step size, h. Is this what you expected? Explain…

Problem 2

Do the same computations and analyses as for Prob. #1 for the following IVP:

$$\frac{dy}{dx} = y - x - 1 \qquad \text{with} \quad y(0) = 1$$

a. Develop the analytical solution.

b. Do hand calculations using the Euler method with h = 0.1 over the range $0 \le x \le 0.5$.

c. Perform Matlab calculations with $h_1 = 0.1$ and $h_2 = 0.05$ and develop a relationship between error and step size for the Euler method for this problem.

Note: Documentation for this HW assignment should include the analytical solutions, the hand calculations performed using the Euler method, a copy of the tabulated output results from the Matlab calculations, and a brief discussion of your observations concerning the numerical error associated with the Euler method. There is a lot of stuff here, so try to keep it as organized as possible…