Differential Equations (92.236)

Homework Assignment #1 Spring 2007

Differential Equations and Mathematical Models

Problem 1

State whether the ODE is linear or nonlinear and the order of the equation. Also determine if the system has variable or constant coefficients and if the equation is homogeneous or non-homogeneous. Thus, you should specify four attributes for each equation.

a. $y' + xy = x^2$

b. $y''' + 3y'' + y = e^x$

c. $y'' + 3yy' + 7x = 0$

d. $3(y')^2 + 2xy = 0$

e. $y' = \sqrt{2-5y}$

Problem 2

a. Show by direct substitution that $y(x) = \dfrac{1}{1+x^2}$ is a solution to $y' + 2xy^2 = 0$.

b. Show that $y_1 = e^{-2x}$ and $y_2 = xe^{-2x}$ are solutions to $y'' + 4y' + 4y = 0$. Also show that $y = c_1y_1 + c_2y_2$ is a solution, where c_1 and c_2 are arbitrary constants.

c. Verify that the general solution to $xy' + 3y = 2x^5$ is given by $y(x) = \dfrac{1}{4}x^5 + cx^{-3}$. If $y(2) = 1$, find the value of c in the general solution.

d. Show that $x(t) = e^{-2t} + 3e^{6t}$ and $y(t) = -e^{-2t} + 5e^{6t}$ are solutions to the system of 1st order ODEs given by

$$\frac{dx}{dt} = x + 3y \quad \text{and} \quad \frac{dy}{dt} = 5x + 3y$$

Also determine a single 2nd order ODE that involves only x(t) and t, and show that the x(t) solution given above also satisfies this 2nd order equation.

Problem 3

a. In a particular application, the time rate of change of temperature of an object is proportional to the difference between the object's temperature and the temperature of the surroundings. Write an equation that represents the mathematical model for this situation.

b. In a particular fluid flow problem, the time rate of change of fluid mass within a container is equal to the difference between the inlet mass flow rate, $\dot{m}_i$, and the exit mass flow rate, $\dot{m}_e$ (with units of kg/s, for example). For a container with vertical sides, the fluid mass at any time is given by $m(t) = \rho A h(t)$, where ρ is the fluid density (kg/m^3), A is the cross sectional area of the container (m^2), and h(t) is the time-dependent fluid height (m). In many gravity-driven flow applications, the exit mass flow rate is proportional to the square root of the fluid height within the tank. The inlet mass flow rate is an independent forcing function.

For the situation described here, write the appropriate mass balance equation with h(t) as the dependent variable of interest.