Differential Equations (92.236)

Homework Assignment #14 Spring 2007

Numerical and Symbolic Solution of Higher Order Systems

Part A: Solve Problems 1-3 (see below) using the following **numerical** solution procedure:

a. Convert each 2nd order equation into two 1st order equations.

b. Solve the system of 1st order ODEs with Matlab's ODE solver, **ode23**.

c. Plot and interpret the solutions and compare these to the analytical result that is given in the problem description.

d. Comment on the ease of use of the numerical scheme relative to the analytical solution method and on the accuracy of the solutions.

Note: All three problems should be solved in a single Matlab script file, with three separate ODE function files (of course). Plot the numerical and analytical solutions for x(t) (Prob. 1 & 2) and I(t) (Prob. 3), as appropriate, on the same axes, but create a separate, properly labeled plot for each problem (i.e. you should have three separate plots for Part A.).

Part B: Now solve Problems 1-3 (see below) using Matlab's **symbolic** processing capability. Again all three problems should be solved in a single Matlab script file. Make each solution "**pretty**", and include a copy of the command window output as part of your documentation for this problem. Do the solutions using **dsolve** agree with the analytical solutions given below? What do you think of the general symbolic capability that is available in the Symbolic Toolbox?

Problem 1: $x'' + 10x' + 125x = 0$ with $x(0) = 6\text{ m}$ and $v(0) = x'(0) = 50\text{ m/s}$

with analytical solution: $x(t) = e^{-5t}\left(6\cos 10t + 8\sin 10t\right)$

Problem 2: $x'' + 4x' + 5x = 10\cos 3t$ with $x(0) = 0\text{ m}$ and $v(0) = x'(0) = 0\text{ m/s}$

with analytical solution: $x(t) = \frac{5\sqrt{2}}{4}e^{-2t}\cos(t - 4.8543) + \frac{\sqrt{10}}{4}\cos(3t - 1.8925)$

Problem 3: $I'' + 8I' + 25I = 0$ with $I(0) = 0\text{ amps}$ and $I'(0) = -75\text{ amps/s}$

with analytical solution: $I(t) = -25e^{-4t}\sin 3t$

Documentation:

Along with the brief comments requested above, also include the derivations required to convert each 2nd order equation into two 1st order equations (Part A). Also attach a listing of the Matlab programs and output plots as part of your solutions for this HW assignment.