Differential Equations (92.236)

Exam #1 Spring 2006

Part A. Solution Techniques: Each problem in this section is worth 20 points.

Solve the following 1st order equations:

1. $\dfrac{dy}{dx} = \dfrac{1-2xy}{x^2+1}$ with $y(0)=1$

2. $\dfrac{dy}{dx} = \dfrac{y^2+yx}{x^2}$

3. $\dfrac{dy}{dx} = \dfrac{5x+4y}{8y^3-4x}$

4. Three students, Bob, Glen, and Paula, were asked to find a solution to the following ODE:

   $$\frac{dy}{dt} = \frac{y+1}{t+1}$$

   After some thought, Bob says $y(t)=t$, Glen says $y(t)=2t+1$, and Paula states simply that $y(t)=-1$!

   a. Which student or students are correct? Explain!

   b. Find the general solution to the given ODE and show that the correct solutions from above are simply special cases of the general solution.

   c. If we impose an additional constraint on the ODE, say $y(0)=1$, is it possible to have more than one correct solution to this problem? Explain!

Part B. Mathematical Modeling: The single problem in this section is worth 20 points.

A parachutist falling through the air experiences a downward force due to gravity and an upward force due to air friction. Two possible models for the drag force are proposed:

Model A: **Drag force is proportional to the velocity.**

Model B: **Drag force is proportional to the square of the velocity.**

a. Set up the appropriate balance equations for this physical situation for both Models A and B based on Newton's 2nd law of motion. Explain/identify each term in the resultant equations. For consistency, use m as the mass of the parachutist and his/her equipment and use g as the acceleration due to gravity.

b. In both cases, develop an expression for the ***terminal velocity*** of the parachutist (note that this does ***not*** require solution of the ODEs). Explain your logic here.

c. What are the units of the drag coefficients (i.e. the proportionality constants) in Models A and B if mass is expressed in kg, velocity in m/s, and acceleration in m/s^2? Show/explain your reasoning!